Classroom notes

Spreadsheets: an overlooked technology for mathematics education

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Electronic spreadsheets have been with us for more than 25 years, yet in Australia, they are not common in mathematics classes. This paper discusses the author's personal experiences at Bond University in using Microsoft Excel to assist with mathematics instruction. It goes on to present a brief overview of what other mathematics educators have said about this technology, and offers some suggestions for its incorporation into the mathematics classroom, starting at grade one.

Background

There is a software tool, on essentially every desktop or laptop computer, which is often routinely ignored for mathematical modelling and instruction. It is, of course, the modern graphical spreadsheet program. My references here to *Excel* are to Microsoft Excel 2007 or 2003 as *exemplars par excellence* of the modern electronic spreadsheet. I do this for the sake of simplicity and fully realise that some may choose to use another spreadsheet program, or none at all. One advantage of Excel is its ubiquity. Most students have already had some exposure, and the absolute basics can in most cases be assumed known. What is used in secondary schools? There seems to be a love affair with *graphics calculators* (GCs) in Australia. When superior tools such as Excel are widely available, it is difficult to understand the GC choice.

In Australia, we have significant numbers of tertiary students enrolled in subjects that, I suspect, most readers of the *Gazette* would regard as rather trivial high-school mathematics. I refer to students enrolled in BCom, BIT or similar. Here, a rather basic level of mathematics is required. While many such students have a reasonable background in mathematics, there still remains a significant proportion who do not. They have somehow escaped from secondary school with a very poor understanding of mathematical concepts, and very weak algebraic skills. At Bond University, I have often found this to be about half the class, and sometimes even more (perhaps 60%).

Teaching mathematics at Bond

Firstly, there is very little mathematics at Bond. Apart from a few units of statistics, there is only *Analytical Toolkit* (details below) and *Business Mathematics*. Secondly, Bond is private and fees are high (a typical 24-unit degree costs very close to $72 000). This is a substantial sum, although, as public university students are now being required to shell out

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increasingly large sums for their tertiary studies, the difference has narrowed significantly in the past decade.

I feel under some obligation to assist all of my students as much as I can to reach the 'desired outcomes' for the subject I am teaching, regardless of their 'entry knowledge'. I am sure that many others also aspire to this lofty goal. However, I have essentially reached the view that, even with the best of intentions, this goal is today an impossible dream. I say this, even though my enthusiasm for teaching has not waned, and also given very small classes. The principal reasons for my negativity are that:

(1) In a large class (at Bond this is anything above about 30), I typically find several very good students, but also a (usually) much greater number of very weak students. To cover the advertised syllabus at a level with which I feel comfortable then becomes almost impossible. I usually have to omit a few sub-topics as the pace required to keep the majority of the class with me is significantly below that which I consider reasonable. Even a small rise in the level of rigour will leave half the class (roughly speaking) in the dust. Remember, they are paying around $3000 per subject, and I feel obliged to make an effort to cater for such weak students. At the same time, I try to imagine the position of the brighter students. How would a bright student of mathematics feel, having to pay such an amount if it were for a rehash of grade 11 or 12 material?

(2) The mathematical knowledge of entering students seems to decline each year. Example: this semester I am teaching a subject known as *Analytical Toolkit.* It is two-thirds discrete mathematics and one-third statistics, being the amalgam of two former units. In a lecture recently, I was about to complete an example of proof by induction, and I had the equation $2^k - 1 = 3\alpha$. 'I need to solve it for 2^k', I said to the class (only six students). 'I wish to transpose the -1. What will the right-hand side then look like?' *No-one could answer that question.* Finally, one ventured: 'the 3α will become 4α'. Note to self: I am totally wasting my time trying to teach induction. Better to omit it, and give them something that they have some chance of learning with the short 12-week Bond semester. This will be my recommendation to my Head of School at the end of semester.

Math wars

In the USA, since 1989, there has been heated debate over the introduction of a new mathematics curriculum. In brief, traditional algorithms such as those for addition and multiplication of decimal strings are de-emphasised or omitted completely. This is done in favour of allowing students to rediscover such basic mathematical truths for themselves. It is associated with Piaget's philosophy of *constructivism.* Let me very briefly state my own view. In mathematics education, I see a place for constructivism. Rediscovery of mathematical truth is exciting and rewarding for students. I do not deny that some time spent on such discovery is worthwhile, even with the limited time available. But to spend each lesson reinventing wheels is, in my view, just plain silly.

The rich body of mathematics we now possess is a legacy obtained over thousands of years. It is the result of much contemplation and experimentation by great mathematical scientists — very gifted men and women. While mathematical discovery is exciting and rewarding, shall we expect, for example, our high-school seniors, whom university lecturers will ultimately expect to have some mastery of calculus, to reinvent the work of Newton or Leibniz? Once again, this would be wonderful, and their understanding would, no doubt, be more profound, if we had the time. But we do not. My own view leans strongly toward learning of the traditional algorithms, but yes, also add some discovery. In this regard, for the purposes of both tool and catalyst for the '*Eureka!*', I use Excel. It allows for very efficient construction of models and visualisation of structures that would take students with just pen and paper, or even graphics calculators, far too long to expose. To fail to teach the fundamental algorithms of arithmetic in the early years of schooling is, in my view, a fundamental mistake. The effects are pernicious.

Amid the mountain of literature on the 'Math Wars', I have found [7] particularly lucid.

Why I use Excel in my mathematics classes

It is difficult in such a short space to convey the variety of uses to which I put Excel in my mathematics classes. My attempt at a summary appears here in the next section, while further detail may be found at the *Spreadsheets in Education* website [3]. One of the benefits of electronic journals is that live models can be downloaded by readers[1].

Because of the generally poor standard of students over the past 15 years, I have been forced to dumb-down my classes. As noted earlier, a principal component of my response to the diversity problem has been to use Excel for the illustration of mathematical concepts, and also for mathematical modelling and problem-solving. In general terms, I have found Excel to be very useful for:

- Conveying mathematical principles to students whose algebra is very deficient, and in many cases, essentially non-existent.
- Illustration of such principles to *all* students. This may include unsuspected connections between topics which had appeared to be unrelated. An example that immediately comes to mind is that of binary truth-table, sets, subsets, powersets, binomial coefficients. This example also beautifully illustrates, in Excel, the inductive step of the proof of the cardinality of 2^S [9].
- Description and demonstration of problem-solving principles/techniques and algorithms (for example, gcd).
- Providing the algebraically-able students with new material (examples include curve-fitting from first principles, simulation techniques).
- Observing patterns which may then suggest general principles or theorems. Using tables, graphs, colours and patterns to teach basics of mathematics is not the norm at Australian universities. Looking for patterns using colours and table may appear trivial. At first, I thought so. On some deeper reflection, I believe it is not. Humans are hardwired for pattern recognition. Learning and developing proficiency in mathematics makes use of this innate human facility. Some modern definitions of mathematics include the statement 'study of patterns'. As working mathematicians, we are certainly looking for patterns when we develop a conjecture which we hope to elevate to theoremhood after constructing a proof.

Topics illustrated using Excel

What are some of the topics, approaches and methods I use? Here are just a few examples.

[1]Please email me if there seems to be some implication here that a particular Excel model is online at SiE but you cannot find it. If I have it, then I shall be pleased to send it to you.

- *Sequences.* For the most basic of these, the most natural definition is the recursive one: start anywhere, then either keep adding a constant (arithmetic), or keep multiplying by a constant (geometric). In Excel, the recursive step is accomplished by a double-click. The challenge for the teacher is, of course, to relate the mathematical formalism of a recurrence relation to the readily comprehended spreadsheet notion of *fill-down.* I discussed this in [9]. Equation (1) represents a superannuation model with \$100 000 rollover (initial value), 1% interest per month, and \$500 contribution per month. It is interesting that many students have difficulty even *understanding* equation (1), let alone solving it, yet have very little trouble implementing the corresponding model in Microsoft Excel. Note: the alteration of *just one character* in the Excel model (+ to −) converts the superannuation model to a mortgage model.

$$a_n = \begin{cases} 1.01a_{n-1} + 500 & \text{if } n > 0, \\ 100,000 & \text{if } n = 0. \end{cases} \tag{1}$$

  Coupled recurrence relations (difference equations) relating to population dynamics, for example, are also easily handled [10]. These may be non-linear and/or stochastic.
- *Binary connections.* Truth table: comparison of two algorithms for constructing just the input bit-vectors (strings) is instructive. Then connections with set membership (the basic Boolean question in set theory) are examined. Each bit represents inclusion or exclusion of an element into the typical subset of a given set. This leads to easy construction of powerset for sets of modest cardinality. Students see very clearly the exponential nature of powerset, and the binomial coefficients emerge naturally [9]. The model can be extended to sampling theory: showing all possible samples of a small set, from which parameters may be computed.
- *Number theory.* Illustration of Euler's totient function $\varphi(n)$, and the number of divisors function $\tau(n)$, as well as the basic operations of modular arithmetic, including inverse, exponentiation. This culminates in a model for the RSA public key cryptosystem [11].
- *Stochastic modelling (deterministic and simulation).* Over the past 12 years I have done much consulting work on Keno. I use the more elementary parts in class to illustrate basics of probability, in particular, *expected value.* The students seem to appreciate the practical nature of the work — all done in Excel, of course.
- *Conditional formatting.* Use of *conditional formatting* to automatically highlight a cell based on its current value. The literature on the use of this feature for educational applications is scant indeed, although applications abound [2]:
  - solving $f(x) = 0$ or $f'(x) = 0$ without algebra, but by just observing change of sign (colour) [12].
  - computing modular inverse or verification of its non-existence by table lookup.
  - illustrating the solution of simultaneous linear congruences.

What have educators said about spreadsheets?

This section makes no pretence at being exhaustive or balanced — it is unashamedly biased toward positive comments about spreadsheets in education. For a much more comprehensive summary, including some negatives, see [3].

A very recent assessment is that of Haspekian [4]:

> An inventory of didactic research in this area shows the importance given by researchers to the potential of the spreadsheet for the teaching and learning of *algebra*.

Note the remark of Morishita *et al* [5]:

> ... the spreadsheet has allowed teachers to adopt a middle course, compared to the extremes of fully coding an algorithm in some programming language such as BASIC or Pascal, or using an off-the-shelf package with a canned solution. It is argued that neither of these methods is ideal for learning and the spreadsheet approach is recommended.

Neuwirth and Arganbright offer a similar viewpoint in their prologue of [6], and essentially the same points are made by Steward [8]:

> I would suggest that when both are possible, students find it easier and quicker to use a spreadsheet than write a computer program. Moreover, once written a program can often mask the mathematics that it is intended to represent, while on a spreadsheet the procedure is constantly exposed.

Sutherland and Rojano [14] investigate the potential of the spreadsheet to enable students to form a correct understanding of algebraic concepts. Sutherland [15] has much more recently used the spreadsheet environment to allow secondary school students in the UK to develop basic concepts of algebraic dependency. Since the students have trouble with the abstract nature of algebra, the spreadsheet is used to develop relationships with point-and-click [15]:

> Mouse pointing becomes a way of supporting pupils to express general relationships, which are then represented automatically in spreadsheet code. .... In this way the algebra-like spreadsheet code is learned effortlessly without explicit teaching. Pupils use the spreadsheet specific calculations to help in the construction of general rules and often verify their general rule with reference to specific numbers. In this way links between symbols and general numbers are established.

Similar comments are made by Abramovich [1]. Referring to a spreadsheet model to support an inductive proof, it is stated that the model:

> ... allows for the visualisation of an inductive proof of combinatorial identity, and it cognitively supports a transition from computing to a formal language of mathematics.

In hindsight it seems obvious, but probably one of the most profound, clear benefits of using spreadsheets is just that of *saving time*. The time gained may then be spent on investigating properties of the mathematical objects created in the spreadsheet environment.

In closing this section, I should mention the compilation of Vacher and Fratesi: a rather interesting collection of published examples of spreadsheets in the *Journal of Geoscience Education*[16].

SiE

John Baker and I created the electronic journal, *Spreadsheets in Education*, (*eJSiE*) in late 2002, with the first number appearing mid-2003. It is an open access journal: all articles are freely available from the website [3]. Content of the journal is divided into fully peer-reviewed articles and classroom articles, the latter typically accompanied by ready-to-use spreadsheet models. An annotated bibliography, up to Volume 2, #1 is given in [13].

Conclusion

Much of this article has described my response to a very undesirable situation, namely, large numbers of very poorly-prepared students in the tertiary mathematics classroom. The whole milieu is one in which mathematics is not valued, is downplayed, and treated as a necessary evil by both students and administrators. Both of these classes, directly or indirectly, now control the dollars. However, few members of either class seem to have much inkling of the true value of mathematics, as applied to a vast range of disciplines, or even what mathematics *is*. It is clear that large sections of the general public are ignorant of the broad scope of mathematics when, in the newspaper, we read '... to solve this Sudoku requires no mathematics', but fundamental logic, set theory, decision trees, *reductio ad absurdum* are all used in this game.

From the ground up, mathematics educators must become aware of the possibilities for *evangelism of mathematics*, or the public perception of our practical worth as mathematics professionals will plummet even further. I am not claiming that use of spreadsheets is a panacea, but I do suggest that spreadsheets must inevitably be part of any aggressive move by professional mathematicians as mathematics educators to reclaim ground, which, we had somewhat smugly assumed would never be ceded. It has slipped away. We must restore rigour, but shall inevitably do so in an environment where many of the teachers and students are themselves products of a flawed system. Rightly used, spreadsheets can help us.

We are in the 21st century, and like it or not, students (or their parents) paying the dollars expect Internet, video/DVD images and much more digital magic. They are wondering: how can boring old maths still be of any relevance? It is up to us to persuade them that it is. We sorely need *mathematical evangelists*. Of course, *Gazette* readers know that mathematics is more relevant than ever, being the very foundation of any quantitative science, including the basis of all modern IT wizardry, computer science! How could anyone seriously deny the value of mathematics? Well, the rot set in around 20 years ago, and we are now in dire straits, brought on by years of compounded negative effects of flawed federal and state funding systems, and our own complacency. We sat by and blithely assumed that no rational person could seriously deny the value of mathematics. We need positive moves, where benefits are made clear. I submit that spreadsheets can help enormously, as they offer a friendly vehicle for support of mathematics learning in the very earliest years of schooling.

I have put forth a very brief case for the use of electronic spreadsheets at all levels of mathematics education, but especially in the early years, right from grade 1. There is plenty of literature to support my case. My own view is that the case is clear: there is no longer any need to justify the value of spreadsheets for mathematics education — it should be obvious to anyone who cares to objectively examine the evidence. The next step is to convince the practitioners, decision makers and those who hold the purse-strings. We do not need to convince the students — they are already on board. Indeed, many of them are probably wondering why we, so often, fail to take advantage of such a useful and ubiquitous tool in the mathematics classroom.

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