Book reviews

Fundamental Groups and Covering Spaces

Elon Lages Lima
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Poincaré has been described as one of the last great universalists, being the most prolific author of his time, surpassed only by Euler in quantity: his legacy permeates many areas of contemporary mathematics. Algebraic topology is one of these, and his notion of the fundamental group of a space is one of many original and far-reaching ideas. The prototype is the exponential map $x \mapsto e^{ix}$ by which the circle is infinitely and evenly covered by the real line, more familiar to undergraduates in the form $\theta \mapsto (\cos\theta, \sin\theta)$. Poincaré's idea was to observe this as the circle "unwrapping" to produce the real line, and to abstract this idea so that any reasonable (topological) space X can be uniquely unwrapped to give a simply-connected universal covering space $\tilde{X} \to X$. This is achieved by observing that a set of equivalence classes of paths in a space, beginning and ending at the same point, naturally has a composition and group structure. This group – the so-called 'fundamental group' $\pi_1(X)$ – acts naturally on $\tilde{X}$, with X as its orbit space. Any subgroup H of $\pi_1(X)$ also gives a quotient X_H of $\tilde{X}$ with a canonical projection $X_H \to X$, with $\tilde{X} = X_{Id}$ corresponding to the trivial subgroup. Remarkably $\pi_1(X_H) \cong H$, and so $\tilde{X}$ has trivial fundamental group (ie is 'simply-connected'). Topological spaces and group theory are inextricably united.

Poincaré had also been the first to observe the relationship between the complex analysis of Riemann surfaces, Fuchsian groups, Klein's ideas of transformations and geometry, and discrete groups of hyperbolic isometries. Accordingly, the new subject of algebraic topology abstracted the underpinnings of the work of Gauss, Abel, Galois, Cauchy, Riemann and many other mathematicians of the preceding century. It is interesting to note that Poincaré's initial ideas preceded the concepts of a metric or topological space developed by Fréchet and Hausdorff more than a decade later, and eschewed the notions of sets developed by Cantor. Just as one can construct the real line by taking a circle, cutting at a point and gluing together infinitely many copies, there is a 'constructive' approach to producing the universal cover $\tilde{X}$ from X itself. One of the simplest applications of the fundamental group is Brouwer's fixed-point theorem, that every continuous map from the disc to itself has a fixed point.

This and many other applications are given in Lima's book, together with a number of historical asides and numerous exercises. This is a book that all libraries should have, not because it is encyclopedic, but because it introduces the student to a variety of accessible applications and insights too quickly skimmed over in standard texts. For an encyclopedic treatment, famous for its succinct impenetrability for a novice, Edwin Spanier's 'Algebraic Topology' is still classic. Elon Lima and Morris Hirsch were students of Spanier together at Chicago, both graduating in 1958. Interestingly Hirsch also has a proof of Brouwer's fixed point theorem credited to him, using differential topology (the title of his graduate level text). Lima's book contains many examples of the interplay between differential topology, covering spaces, fibre spaces, matrix Lie groups, immersions of curves in the plane and differential forms. It is these examples which make the book so useful for a student, since these are all ingredients of contemporary mathematics and mathematical physics. Fibre spaces are precursors of fibre bundles, and path liftings to covering spaces and fibre spaces are the precursors to liftings via connections in Riemannian geometry. The structure of the groups $SO(n)$, $SU(n)$, and $Sp(n)$ is described in a way understandable to students who may not have been exposed to a course in transversality and differential topology, and the description of winding numbers of immersed curves will prepare the reader for Smale's more sophisticated proof of the existence of a smooth eversion of the sphere in $\mathbb{R}^3$. (Amusingly there is an account of Eilenberg's initial disbelief of Smale's result reminiscent of the story attributed to Feynman – those who had met Eilenberg would not be surprised!)

A student often first encounters the fundamental group as a last topic in an introductory course in topology, or as part of the homotopy section of a course in algebraic topology. For example, Greenberg and Harper's classic 'Algebraic Topology: a first course' completes the discussion of $\pi_1(X)$ and covering spaces in about 30 pages. Lima's book spends a lot longer going deeper into conceptual applications, is a very nice read, and is highly recommended for students. It also provides additional material a lecturer might wish to present in a first course. A minor warning is that 'surface' should sometimes be construed as 'hyper-surface' or 'manifold'. A prior course in basic or metric topology would be desirable.

Incidentally and appropriately, the given proof of Brouwer's theorem is of course not the original, nor is it the one attributed to Hirsch. Those interested in historical anecdotes concerning Lima's own experiences might consider asking any Brazillian connected with IMPA; they may be surprised at what hot gossip they learn!

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Galileo Galilei: When the World Stood Still

Atle Næss, trans. James Anderson
Springer Berlin 2005
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They worked faster in those days, but an eye for a grant came in handy as much then as now. In June 1609 Galileo, then Professor of Mathematics at Padua but always hopeful of a move back home to Florence, heard an interesting rumour from one of his dubious intellectual contacts in Venice. It was said that someone in Holland had put lenses in a tube and was able to see things larger at a distance. Galileo did some geometry as to what shaped lenses might do that and over several months of experiment (and learning to polish glass) he achieved a result that magnified 9 times with good clarity. In August he exhibited it to the senators in Venice. The military implications were obvious and they doubled his salary. By the winter he had a version that magnified 20 times, but with rumours all over Europe it was going to be hard to keep ahead of the competition. On 7 January 1610 he turned the telescope on Jupiter. It had three unknown stars next to it, in itself not surprising as many new stars were evident. The next night, the tiny stars seemed to have got ahead of Jupiter instead of being left a little behind. 9 January was overcast. On 10 January, the stars were still with Jupiter but in a different position. Galileo realised he had discovered moons, the first celestial discovery in recorded history (except for transient events like comets). A fast enough printing job would ensure his priority, and he had the naming rights. As a result of a previous networking success, he had some years before tutored the young Grand Duke Cosimo de Medici of Florence, before his accession. On 13 February he wrote asking for gracious ducal permission to name the new moons the "cosmic" stars, suggesting Cosimo, but adding that "Medicean" might also be a possibility. The Grand Duke's marketing department replied graciously preferring the latter option, and with the necessary last-minute typesetting changes the book announcing the discovery of the Medicean stars appeared on 12 March. The several pages of dedication to Cosimo include the sentence "Scarcely have the immortal graces of your soul begun to shine forth on earth than bright stars offer themselves in the heavens which, like tongues, will speak of and celebrate your most excellent virtues for all time." There was then an urgent need to fend off a claim by a rival professor of mathematics that it was all made up, but an urgent request to Kepler and a rushed pamphlet from him (though he did not have a telescope powerful enough to see the new stars), sorted out that contretemps. Galileo was appointed grand ducal mathematician and philosopher at Florence on 10 July, needless to say at an acceptably huge salary. Galileo's people skills did not work so well on the Inquisition. They could work fast as well – too fast. When Galileo visited Rome in 1616 talking up Copernicus' heliocentric theory, the Inquisition, in a process lasting six days and without the benefit of any astronomical advice, declared it heretical. It is a classic case of the saying, "everything is run by idiots". Bizarrely, it seems to have occurred to no-one except Galileo and an obscure German visitor who had nothing to do with the case that the Church might get egg on its face by being proved wrong; it was then supposed that scientific questions, like legal and theological ones, were subject to never-ending debate. Galileo was instructed not to teach or defend the theory. He did the exact opposite, publishing the polemical *Dialogue Concerning the Two Chief World Systems*, which, though not explicitly favouring Copernicanism, made the alternatives look ridiculous. Galileo, then aged 70 and ill, was dragged before the Inquisition, found guilty of disobeying its previous orders, and ordered to recant his theory and live under house arrest. The end of the story is surely inevitable: Galileo lived out his few remaining years in illness, a broken man unable to accomplish anything further . . . well, actually, no. Though he was ill and eventually blind, Galileo's last years were amazingly productive. The Archbishop of Siena installed him in comfort and urged him to write something less controversial. In Siena and later at home in Florence, he produced his *Dialogue Concerning Two New Sciences*, the summary of his researches on motion and mechanics that is his true masterwork. Copernicanism was gaining ground anyway and might have been better off without his help, but his work on the quantification of motion is the beginning of the true Scientific (or perhaps better Mathematical) Revolution, the work on which Newton and all later applied mathematicians built. The search for laws of proportionality like Boyle's, Pascal's, Hooke's and Newton's laws, perhaps the central discoveries of the Scientific Revolution, was begun in earnest by Galileo. Atle Næss knows how to write a readable book. He works fast, too, and wastes no time on padding. He weaves the history, the personalities and the technicalities together expertly, with a sure eye for an interesting detail and an ability to cut through the myths, propaganda and speculation that surround the Galileo case. A mathematical audience might prefer him to have given more of the mathematics, such as Galileo's remarkable "bare hands" proof that it is impossible for anything to move from rest such that its speed is proportional to the distance covered. But the aim of the book is a short introduction to introduce the man and whet the appetite for his ideas. It is a resounding success.

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Lagrange-Type Functions in Constrained Non-Convex Optimization

A. Rubinov and X.Q. Yang
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Lagrange and penalty functions provide a powerful approach for study of constrained optimization problems. The applications of classical Lagrange and penalty functions are, however, restricted with some (although very important) special classes of constrained optimization problems, and there is a clear need to introduce and study more general types of such functions, which would have a wider range of applications. The book gives a systematic and unified presentation of many important results that have been obtained in this area during last several years. The authors introduce and examine non-linear Lagrange-type functions, which can be represented as a convolution of the objective and constraints. They also consider generalized augmented Lagrangians with a very broad class of augmenting functions and they pay a special attention to level-bounded and peak-at-zero augmenting functions. Both non-linear Lagrange-type functions and generalized Lagrangians are studied in the framework of abstract convexity (the area of expertise of the first author) and with use of tools of modern non-smooth analysis (the area of expertise of the second author) for examining the first and second order optimality conditions. Among the non-linear and generalized augmented Lagrangians that are considered there are generalized penalty functions and, in particular, the so-called lower order penalty functions that are useful in the study of error bounds and mathematical programs with equilibrium constraints.

The book develops a unified approach to duality and penalization and to convergence analysis of the first and second order optimality conditions. The authors examine the properties of weak duality, zero duality gap, as well as those of existence of exact Lagrange multipliers, penalty parameters and saddle points, without usual assumptions about convexity and constraint qualification. It is known that the weak duality gives an estimation of a global minimum and that the zero duality gap property allows one to reduce a constrained problem to a series of unconstrained ones. Thus new dual models investigated in the book may provide a better estimation of the lower bound of the optimal value of the original problem. It is also known that the duality theory provides tools for analysis of optimal algorithms. It is yet to be seen how the new models studied in the book can be used to help developing this line of research.

A number of impressive new results on the existence of an exact penalty parameter have been obtained in the book, some of these being of interest even in a classical penalty functions setting. In particular, an analytic expression for the least exact penalty parameter for a large class of penalty functions has been obtained.

Along with a theoretical study, the book contains some recommendations concerning numerical solution of some optimization problems. In particular the authors have found that a special class of penalty functions is useful in solving some nonconvex problems including problems of concave minimization problems.

The outline of the book is as follows.

Chapter 1 contains motivation for studying Lagrange-type functions and provides an outline of main topics studied in the book.

Chapter 2 is devoted to abstract convexity and theory of IPH (increasing positively homogeneous) functions and, also, it provides a description of the technique, which is used for the examination of the zero duality gap property and penalty-type functions for problems with a single constraint.

Chapter 3 introduces a general scheme of Lagrange-type functions. This is based on a separation of certain sets in the image-space of the problem. Also, optimality conditions are discussed in this chapter.

In Chapter 4 penalty-type functions for problems with a single constraint are studied using an IPH convolution function. The estimation of the least exact penalty parameter for various problems is given.

In Chapter 5 the authors examine the zero duality gap property and exactness for various augmented Lagrangian functions.

Chapter 6 is devoted to the analysis of the first- and the second-order optimality conditions for non-linear penalty-type functions and augmented Lagrangian functions in relation to the corresponding conditions of the original constrained optimization problem.

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Coding Theory
A first course

San Ling and Chaoping Xing
Cambridge University Press 2004
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For some years I have taught a 12 lecture introductory course in coding theory to level 3 mathematics, computer science and engineering undergraduates, using as major reference the excellent text “A first course in coding theory”, by Raymond Hill (OUP, 1986). Since I am revising and extending the syllabus for this course I was interested to review this book to see if it would be suitable as a more up-to-date reference. The authors have used the book as a text for a 2 semester course to advanced undergraduates at the National University of Singapore.

The assumed prerequisite is some basic linear algebra, although a healthy dose of “mathematical maturity” would be needed to cope with some of the more advanced topics. The first two chapters introduce channel coding, the binary symmetric channel, redundancy, error detection and correction and nearest neighbour decoding. Chapter 3 gives a fairly complete and detailed treatment of finite fields. This treatment would be quite difficult for students with no previous course in modern algebra, although here and elsewhere there are many routine and less routine exercises. Chapter 4 treats the standard material on linear codes, encoding and syndrome decoding. The next chapter gives the main bounds in coding theory and also introduces the Hamming and extended binary Hamming codes, Golay codes (via the generator matrix) and some non-linear codes, including the Hadamard codes. Chapter 6, on the construction of linear codes, gives methods of forming new codes from old and introduces the Reed-Muller codes. Chapter 7 gives a full account of cyclic codes from an algebraic point of view and gives a relatively simple decoding algorithm via the division algorithm for polynomials. The burst-error correcting capabilities of cyclic codes are also treated. In chapter 8 we meet special cyclic codes, including BCH codes and their decoding algorithm, Reed-Solomon codes and Quadratic Residue codes. Finally, in chapter 9, we meet the generalized Reed-Solomon codes and their decoding algorithm, alternant codes and Goppa codes.

As can be seen from this description, most of the well-known classes of codes are treated. There are many examples and there is a wealth of well-chosen exercises, some of which provide significant extensions to the theory. Some topics that are not treated or are treated very briefly, in comparison to Hill, are perfect codes, codes and Latin squares, decimal codes and the weight enumerator.

Two of the strengths of Hill, which are less present in Ling and Xing, are firstly the way in which more complicated consructions, e.g., BCH codes, are introduced via simple special cases; secondly the simple and interesting examples of the practical use of codes, ranging from the transmission of photographs from deep-space to the use of ISBN numbers. Nevertheless, Ling and Xing is a valuable additional reference which covers a number of topics not present in Hill. The treatment of cyclic codes is especially useful.

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