Enhance teaching and learning mathematics using DERIVE

Nethal K. Jajo

Abstract

This article proposes the use of an algebra computer package named DERIVE to enhance teaching and learning of mathematics. This proposal is based on our students' results, who were studying mathematics at the foundation (pre-university) level in China.

1 Introduction

Most college students feel mathematics is a boring and distant thinking subject. The negative feeling is increased when it is taught to students in a language different from their native language, which is the case in many Australian colleges. It is important to show that mathematics is not distant thinking. It can be a pleasure instead of boring and an adjacent thinking instead of distant thinking. To assure this change, in this article we investigate how mathematical software can be used as an educational tool, and its ability to change mathematical teaching/learning, both in methodological and technical aspects, see Patricia [3].

DERIVE is a mathematical computer program. It is an algebra, equations, trigonometry, vectors, matrices, and calculus processor, like a scientific calculator processing floating numbers, see Bernhard and Vlasta [1]. It can do both symbolic and numerical computations. These can also be visualized with numerous 2-dimensional and 3-dimensional graphical capabilities. Many problems that require extensive and laborious training at school can be solved with a single keystroke using DERIVE. Teachers and students can concentrate on the exciting and useful techniques of problem solving, which includes the use of pen and paper as well as the computer. This will enable students to work independently of the teacher and to engage more with other students. This doesn't undermine the need for a teacher, but changes his/her role.

The advantage of using DERIVE in teaching may be recognized from the positive effect of using this tool in providing fast solutions and good visualizations of applied mathematics problems, which may help students to identify patterns and see connections. This tool removes the needs of oppressive calculations and allows teachers to direct their students' attention to the implicit mathematics. In a simple example: Teaching the concepts of limit has always caused problems and frustrations to both students and teachers. Anyone who taught limits will probably have spent some time trying to convince a puzzled student that, for example,

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0} \frac{\sin x}{x^2} \quad \text{does not exist.}$$

Graphing both functions, Figure 1 and Figure 2, illustrates the limits of these two functions when x approaches 0. In using DERIVE to calculate limits of a function at a specified point, students will be given three options: calculating the limit from the left, the right or both directions. This makes the definition of the existence of the limit of a function at a specified point clearer and easier to remember.

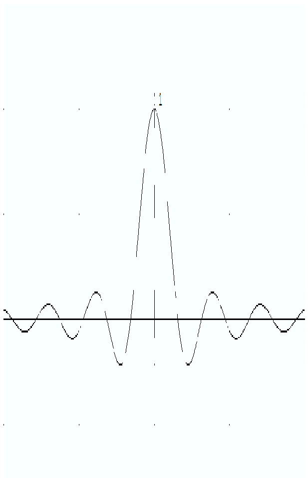

Figure 1: $f(x) = \frac{\sin x}{x}$

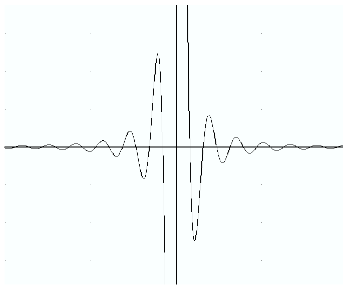

Figure 2: $f(x) = \frac{\sin x}{x^2}$

In learning, the advantage of using DERIVE might lie in solving problems where analysis is required and where the use of the tool removes the need for onerous calculations and allows students to direct their attention to the underlying mathematics. For example, in learning the trigonometric functions, it is necessary that students know that the $\sin x$ and $\cos x$ are odd and even functions, respectively. Mostly, students learn that by the obscure rule of (loosely saying): if $f(x) = f(-x)$ then f is even and if $f(-x) = -f(x)$ then f is odd. Another approach which is simple and obvious using DERIVE can be illustrated using Taylor series (say order 10) to have:

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$$

and

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800}$$

which show that $\sin x$ is an odd function as can be seen from the odd powers of x and $\cos x$ is an even function as can be noticed from the even power of x.

In a project which takes place in China to prepare Chinese students (Pre-University level) for studying abroad, I have been asked to teach mathematics in the English language using DERIVE to improve their mathematics skills. In this paper we present some results of this teaching. In Section 2, we will describe the main objectives and activities of this teaching, with some concluding remarks in Section 3.

2 The project

The project is designed to prepare a group of 60 Chinese students to study abroad in English speaking countries. The students were required to study Quantitative Methods, Business and

Economics, Social Studies, Physics, Biology, Chemistry and English for Academic purposes. Our concern in this paper is teaching quantitative methods using DERIVE. The teaching plan included five hours face to face teaching per week, for two fifteen-week semesters, made up of three hours using DERIVE in mathematics problem solving followed by two hours pen and paper mathematics study. A laboratory with 20 workstations was available for nine hours a week. The assessments included two parts; the first using computer, based on DERIVE projects, and the second a mathematics paper (without computer).

Aims

- To enhance their English language ability in mathematics, to enable the students to communicate and study mathematics effectively and confidentially at the undergraduate level in English speaking countries.
- To motivate students to study mathematics.
- To develop their quantitative skills in pure mathematics, applied mathematics and mathematics for commerce.

We were concerned about some points made by John [2], which we tried to influence positively (by setting up an appropriate DERIVE project) to improve students' progress. John mentioned: First, the language of the students is unrefined. It consists to a large part of 'this', 'that' and 'there'. Second, the students do not simply watch the screen, they use their books as directed and to check work. Third, DERIVE generates considerable group discussion and gives rise to the opportunity for students to explain to their follow students what is happening. We carefully wrote ten DERIVE projects (two samples of these projects included in Appendix A) for each semester to fulfill the aims, to avoid the first two undesirable results recorded by John [2] and to enhance the last desirable point. As a result of using this algebra package we noticed that:

- The students were able to quickly learn the English mathematics terms, like function, domain, limit, integration, etc.
- They overcome the problem of misunderstanding functions and variables. For example they are not cancelling x from the numerator and denominator of $x/\sin x$ anymore, where most of our college students do that.
- The students discussed their results with their classmates and at the same time they compared their work to the work they did without computer. This way students' independent and collaborative thinking was improved.
- High level students extended their work to more simulation studies using DERIVE, which ensured they were not bored when we were explaining simple processes to the rest of the class.
- The students claimed in their feedback that the subject was more interesting when it was accompanied by the Algebra package.
- There were some delays in the students' print-out of their DERIVE results. A high quality laser printer is required.

In addition to ten DERIVE projects for each semester, I gave the students 5 mathematics assignments for each semester. I marked the projects and the assignments and returned it to the students without awarding marks. The students had two exam papers at the end of each semester, one solving a mathematics problem using DERIVE, and the other without using a computer. Each paper weighted 50%. To pass the subject students needed to score 50% or over overall. The students' overall grades in this subject are listed in Table 1.

Table 1. The number of students at each grade; overall marks

H High Distinction $85 \leq$ mark ≤ 100	D Distinction $75 \leq$ mark < 85	C Credit $65 \leq$ mark < 75	P Pass $50 \leq$ mark < 65	F Fail mark < 50
6	7	18	21	8

This way of teaching was designed by the institute in a form that we explained earlier. Therefore, a control group was not available for statistical study. Table 2 summarizes the outcomes. D stands for the event that a student passes the DERIVE assessment and D^c for the complement of this event. M stands for the event that a student passes the mathematics paper (without using a computer).

Table 2. The number of students at each event

	D	D^c
M	36 (= 9P + 14C + 7D + 6H)	2 (= 2F)
M^c	21 (= 5F + 12P + 4C)	1 (= 1F)

As can be seen from Table 2, the eight students who failed this unit were distributed as follows: 5 students out of 21, who passed the DERIVE assessment but failed the mathematics paper, failed the unit. Two students passed the mathematics paper but failed the DERIVE test and one student failed both modules. Based on this, we believe DERIVE may have a positive impact on students' final achievement. In fact, D and M are highly correlated as a student who passes the DERIVE assessment is much more likely to also pass the mathematics paper then a student who fails the DERIVE assessment. A statistical analysis would only make sense when mathematics papers of several years would be compared. As these are not available at this stage a quantitative analysis was not possible. However, a qualitative analysis based on the institute official students' feedback (scale of 9; 1: strongly disagree, ..., and 9: strongly agree) was conducted and the mean value of responses were; 8.6 for the usefulness of introducing DERIVE in teaching mathematics, 8.1 for the comprehension of graphical representation of functions and 5.6 for the use of DERIVE by students apart from the practical lessons, and not only to solve questions related to mathematical subjects.

3 Conclusions

We find DERIVE helps to heighten the students' interest in mathematics. It takes the tedium out of algebraic manipulation and allows us to concentrate on concepts. The main benefit that the laboratory sessions can bring to the teaching process is the generation of enthusiasm among the students. They can actually find out themselves how the graph of a function changes as a variable is altered: they can draw so many curves of a particular type so quickly that they begin to "feel" what the mathematics is doing. This is mainly achieved through the feeling of discovery that the experiments have given them. We hope that this will stimulate their interest when the theory is taught, away from the computer, and that they will be encouraged to ask searching questions; why they are doing such things, why these things happen and how and why DERIVE gave such answers. For this reason, these experiments must be fully integrated with normal classroom work: they are of limited value in their own right. Students working with DERIVE were involved in mathematics discussions with their classmates. They learn a language to speak about forms, process and experience which improves their language and collaborative thinking. More detailed experimentally designed study is required to determine how and why this package enhances teaching/learning mathematics.

References

[1] K. Bernhard and K.V. Vlasta, *Introduction to Derive 5 The Mathematical Assistant for your PC.* Texas Instruments, Inc., Dallas, Texas, 2000.
[2] M. John, *Using A computer algebra system to teach quadratic functions*, in: *Proceedings of the International School on the Didactics of Computer Algebra* (Krems Austria 1992).
[3] R. M. Patricia, *A High-School Experiment Using the Mathematical Software DERIVE*, in: *Proceedings of the International School on the Didactics of Computer Algebra* (Krems Austria 1992).

Appendix A

DERIVE project (...)
Q1: Use DERIVE to factorize the expressions and solve the equations. All workings must be clearly shown as follows:

(1) DERIVE print-out to obtain answers.
(2) Put the answers in a table:

Equations	Factors	Solutions
....	(...)(...)	$x =$..... or ....

(3) Full explanations of your reading information.

Expressions:

(a) $x^2 + x + 30 = 0$
(b) $x^2 + 3x - 54 = 0$
(c) $x^2 - 7x - 18 = 0$
(d) $x^2 - 7x + 12 = 0$
(e) $x^2 - 11x + 30 = 0$

Q2: Solve the following equations without using the computer:

(1) $x^2 + 3x + 2 = 0$
(2) $x^2 + 8x + 15 = 0$
(3) $x^2 - 9x + 20 = 0$
(4) $x^2 + 2x - 35 = 0$
(5) $x^2 - 7x + 18 = 0$

Q3:

(1) Try to solve the following equation using DERIVE.
(2) How would you have to solve it without the computer?
(3) How can you tell by looking at the quadratic equation whether or not it has 2, 1, or 0 solutions?

$$x^2 + 2x - 6 = 0$$

....

DERIVE project (...)
Q1: Use the binomial series to expand the given function as a power series. Use DERIVE (Tayler series-10 terms only) to obtain the results and save it as your name file. All workings must be clearly shown as follows:

(1) DERIVE print-out to obtain answers.

(2) Compare the results that you will obtain without using the computer with your DERIVE results.
(3) State the radius of the convergence.
(4) Full explanations of reading information.
   (a) $\sqrt{1+x}$
   (b) $1/(1+x)^2$
   (c) $1/(1+2x)^4$
   (d) $x/\sqrt{1-x}$
   (e) $1/\sqrt{2+x}$
   (f) $x^2/\sqrt{1-x^3}$
   (g) $x^5/(1-x)^5$

Q2: Expand the following:
(1) Using DERIVE.
(2) Without computer; using Binomial Theorem.
   (a) $(1+x)^7$
   (b) $(1-x)^7$

School of Quantitative Methods and Mathematical Sciences, University of Western Sydney, Blacktown Campus, Locked Bag 1797, Penrith South DC NSW 1797
E-mail: n.jajo@uws.edu.au

Received 18 February 2005, accepted for publication 21 October 2005.