Puzzle Corner

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Welcome to Puzzle Corner 47 of the *Gazette* of the Australian Mathematical Society. I will start with the new problem and then include a solution of Puzzle Corner 46 in the March issue of the *Gazette*.

Finding the sum of the reciprocals of the squares, $S = \sum_{k=1}^{\infty} \frac{1}{k^2}$, is one of the surprising bonuses in store when you calculate the Fourier series for the function $\frac{1}{2}x^2$. The sum turns out to be $\frac{\pi^2}{6}$ but this famous series has another neat feature that may be news to my readers.

At least it was new to me and my friend, Martin Hawes, when we were housemates at ANU in 1976. Martin had got his hands on a newfangled HP programmable calculator, a really expensive item at the time. A very handsome device it was too, it nestled nicely in your hand, and it was all set up to run in reverse Polish notation, a truly efficient way to input arithmetic instructions that the Hewlett Packard engineers assured everyone represented the future for personal calculators.

To test it out Martin had written a programme to find how many terms of the series S are required in order to get within $\frac{1}{n}$ of its limit. He first ran it for $n = 10$ and being a touch surprised with the result, he ran it again for $n = 100$. The answer was once more rather striking so he pushed the little machine to its limits and executed the programme for $n = 1,000$. The result was quite astonishing and so he came and told me what had happened. Well, I replied, if that is true it can't be hard to prove.

After a couple of hours I triumphantly knocked on the door of his room to announce that I had a proof! Martin, (who has gone on to become a well-known wilderness photographer in Tasmania) quietly said that he had proved it himself ten minutes after he had been talking to me. 'Is it really that easy?' thought I, feeling not a little crestfallen. We duly swapped notes and pronounced one another's efforts to be sound, although our proofs were different. I had just taken the oracle at its word so to speak and proved the observation by induction on n. Martin, on the other hand, used a little calculus to derive the result even more quickly.

Our problem for this month then is to recreate this moment of discovery. Find a (very simple) expression for the number of terms k in the shortest partial sum S_k of S such that S_k lies within $\frac{1}{n}$ of the sum of the series and prove that it is so.

The value of S is not required to solve this problem. Indeed knowing S in terms of π will be of no help to you at all. Solutions will appear in the next issue of the *Gazette*.

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Solution to Puzzle Corner 46

Let me now turn to a solution of Puzzle Corner 46 in the March issue of the *Gazette*. (Angel Plaza of Departamento de Matemáticas Universidad de Las Palmas de Gran Canaria, España supplied a nice solution to the problem.)

Solution: Alex (who knows the month) knows that Barbara (who knows the day of the month) definitely does not know Caroline's birthday. It therefore cannot fall either on the 31st of March or the 11th of July, for in either of those cases Barbara would know the birthday month as well, as those numbers are only listed once. Hence the month *cannot* be either March or July, for if it were one of those months, Alex would be uncertain as to whether or not Barbara knew Caroline's birthday. We know therefore that the special day is in either August or December.

Having heard of Alex's deduction, Barbara proudly announces that she can now figure out the answer. This allows *us* to eliminate the 27th, by the following reasoning. Like us, Barbara has deduced from what Alex has said, that March and July are out. If the birthday were the 27th, Barbara would still not be able to infer which of the latter two months occasioned the birthday. However, she does know, so the birthday must be either August 30th or December 8th or 29th. These are all different dates, which is why B, who knows the day of the month, now knows the complete answer.

Finally, Alex says he now knows as well, which allows *us* to eliminate December, for if Caroline were born in December, then Alex still could not decide between the 8th and the 29th. The only remaining possibility is therefore August 30th, and so that is Caroline's birthday.

Peter Higgins is a Professor of Mathematics at the University of Essex. He is the inventor of Circular Sudoku, a puzzle type that has featured in many newspapers, magazines, books, and computer games all over the world. He has written extensively on the subject of mathematics and won the 2013 Premio Peano Prize in Turin for the best book published about mathematics in Italian in 2012. Originally from Australia, Peter has lived in Colchester, England with his wife and four children since 1990.