Number Theory Down Under 2016
Dedicated to Professor Richard Brent on his 70th birthday

Harbourview Function Centre, Newcastle
23–26 September 2016

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From 23–26 September 2016, Number Theory Down Under celebrated the 70th birthday of Professor Richard P. Brent. The conference was held at the Harbourview Function Centre in Newcastle (Australia).

Event Description

This was the fourth annual instalment of Number Theory Down Under, and the first major meeting since Number Theory Down Under was incorporated as a Special Interest Group of the AustMS in 2015.

Number theory is interconnected with many other disciplines in mathematics. The focus on this workshop was to explore the many open problems lying at the intersection of number theory with harmonic analysis, measure theory, computation, and algebra.

Another feature of the conference was to celebrate the 70th birthday of Professor Richard Brent, who is widely acknowledged for his work in computational number theory. Many distinguished overseas and domestic speakers presented their work, which was in one way or another connected with Professor Brent's.

Key Research Themes

We addressed four questions at this conference.

1. Diophantine approximation and its applications. This brings together Diophantine approximation, probabilistic number theory, measure theory and conformal dynamical systems. The main research question that was addressed from different perspectives was 'how well can a real number be approximated by rational numbers quantitatively?'. In other words, the focus was on 'shrinking target problems'. Our

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speakers presented state-of-the-art results in these areas, and made them more accessible by shedding light on their connections with each other.

2. Special values of L-series and questions of transcendence. Perhaps the most well-known example of this topic is Apéry's proof that $\zeta(3)$ is irrational. One question that has been receiving more attention in recent years is 'What role can computation play in predicting and verifying the irrationality or transcendence of a special value?'. Results were presented on both theoretical and computational approaches to irrationality.

3. Zeroes of the zeta-function and of L-functions. A fruitful approach to understanding the analytic properties of the Riemann zeta-function has been to examine L-functions in more exotic settings. For example, the Riemann hypothesis has been proved over finite fields. Our speakers addressed the current gaps in the literature — between 'easy' settings, such as finite fields, and 'hard' settings, such as Hecke L-functions.

4. Computational number theory. The main focus in this topic was the research theme related to the works of Professor Brent. Specifically, speakers talked about probabilistic bounds on maximal determinants of binary matrices, computer algebra algorithms, and multiple precision arithmetic inspired by Brent's MP package.

Overseas keynote speakers
- Shi Bai (ENS Lyon, France)
- David Bailey (California, USA)
- Youngju Choie (POSTECH, Korea)
- Shaun Cooper (Massey, NZ)
- Lassina Dembele (Warwick, UK)
- Karl Dilcher (Halifax, Canada)
- Hidenori Katsurada (Muroran, Japan)
- Simon Kristensen (Aarhus, Denmark)
- Yohei Tachiya (Hirosaki, Japan)
- Bao-Wei Wang (Wuhan, China)
- Paul Zimmermann (INRIA, France)

Report

With a focus on the use of computational techniques for exact problem solving, this event stimulated robust discussion, opening avenues for national and international collaboration. There were 24 talks spread equally over four days. Of these, 11 talks were given by distinguished overseas speakers. The conference was attended by almost 50 participants and everyone had plenty of opportunities for discussions related to mutual research interests.

Postgraduate students and early career researchers were given a platform to present and discuss their research in a collaborative and stimulating environment. They additionally benefited from engagement with national and international field leaders and new academic networks.

This year we dedicated the conference to celebrate Professor Richard Brent's 70th birthday. Many of the lectures were given by his collaborators and colleagues in number theory. We are sure everyone knows about Professor Brent and his legacy, but in case not, here is his bio.

Biography

Richard was born in 1946, the only child of Oscar and Nancy Brent, a doctor and nurse in the small country town of Maffra, Victoria. He spent his early childhood playing with the family dogs and the children next door, walking to school, and reading books whilst on fishing trips with his father. He enjoyed the cubs (and later scouts). In about 1955 the family moved to Melbourne, where Richard attended Caulfield North Central School, followed by a scholarship to Melbourne Grammar. When bored in senior school, Richard used to surreptitiously play chess with one of his classmates, passing a very small chess set under the desk, or sometimes by memory, without a set.

Chess has been one of Richard's interests for most of his life. He played competitively for the first time in the Melbourne Grammar team. He came second in the Australian Junior Chess championship in 1964 and third in the Australian Open Chess championship in 1965, at the age of 18. Later that year he went overseas for the first time to play in the World Junior championship in Barcelona. Richard continued to play chess in later years in Canberra and Oxford, winning the ACT championship several times. Richard's second son Rohan was later to win the ACT Under 8's Championship, at which point he wisely retired from chess.

Music was an interest that led indirectly to Richard's first marriage. As a school student, Richard had played the violin. Although he didn't play at University, he attended his old music teacher's daughter's 21st birthday, where he met Erin O'Connor, the young woman who was to become his wife from 1969 until her death in 2005. At that stage Erin was a third year student of 'Computation' (a degree that would later become Computer Science) at the University of Melbourne. Erin went on to obtain an Honours degree in Computation, one of the first women in Australia to do so.

Richard got his own start in computing whilst working at Mount Stromlo Observatory in 1965 as part of a vacation scholarship between the second and third years of his Bachelor of Science, studying mathematics, physics and chemistry at Monash University. At that stage he contemplated a career as an astronomer. His vacation project was the simulation of white dwarf stars. The machine was an IBM 1620, which featured built-in arbitrary precision decimal computation. Seduced by

the 1620 and Fortran, Richard changed tack and set his sights upon a career in mathematics or computer science.

Between the end of his Honours degree in Mathematics at Monash, and taking up a CSIRO overseas graduate scholarship to do a PhD in Computer Science at Stanford, Richard worked in the Monash Computer Centre on a project joint with the Bureau of Mineral Resources, to do with drawing contour maps of data obtained by helicopter surveys of gravity anomalies. In those days there was no off-the-shelf software to draw contour maps, and the work led to Richard's first academic paper

In September 1968 Richard set sail by ship to California. He spent his first year in California wooing Erin by letter, and studying the required coursework in mathematics and computer science at Stanford. One of the best lecturers was George Pólya (then aged 82) teaching Combinatorics. Another notable course was taught by Bob Floyd, who hardly lectured at all but instead set a sequence of difficult programming problems whose solutions were then discussed in class.

At the end of the year Richard had passed his qualifying exams, written his second and third papers (on fast algorithms for matrix multiplication, and addition of binary numbers), started computing gaps between primes, and convinced Erin to marry him. He flew back to Australia and the couple were wed on 13th September 1969 in Newman College Chapel at Melbourne University. Their honeymoon was the fortnight-long shipboard journey back to the USA.

Upon his return to Stanford Richard commenced working with thesis advisor Gene Golub, in Numerical Analysis, an area chosen to meet the requirements of his CSIRO scholarship. In doing so, he regretfully set aside an interest in Artificial Intelligence and hopes of writing a competitive chess-playing program. Erin also worked for a while for Numerical Analysis Professors George Forsythe and Gene Golub as a teaching assistant and programmer respectively, before getting a job as a systems analyst with a start-up company in nearby Silicon Valley.

Richard finished his PhD in April 1971, in a thesis that was signed off by Professor Forsythe, since Gene was overseas at the time. Curiously, both Richard's supervisors Golub and Forsythe trace their mathematical family tree back via Klein to ancestors including Dirichlet, Johann Bernoulli and Gauss. In 1972, Richard's thesis was published as a book *Algorithms for Minimization without Derivatives*, in a series of which Forsythe was the editor. The book was reprinted by Dover and is still regularly cited. It contains the zero-finding algorithm now known as 'Brent's Algorithm'.

Richard and Erin left Stanford, Silicon Valley and California in an old Ford Galaxy, immediately after Richard's thesis submission. They travelled East on Route 66, heading for a job (effectively a postdoc) that Richard had at IBM Research in Yorktown Heights. On the way they visited Joe Traub at Carnegie Mellon and his wife Pamela McCorduck who was later to write the classic book on Artificial

Intelligence, *Machines Who Think*. This was the first of many visits to Carnegie Mellon.

It was during Richard's 18-month stint at IBM Research (1971–1972) that he first got interested in computational complexity. Shmuel Winograd was one of the pioneers in the area, and he was at IBM Research in the same corridor (the maths corridor) as Richard. The walls of the offices were glass, and colleagues could see each other at work. Richard's supervisor was Phil Wolfe. Phil would often sleep on a couch in his office, so Richard would be careful not to knock on his door at such times. Nonetheless Winograd, Wolfe and Richard managed to talk regularly, and jointly published one of the first papers on analytic computational complexity.

Bob Anderssen from ANU (later CSIRO) visited Richard and Erin while they lived near Yorktown Heights. Bob insisted on going out onto the ice of Lake Mohegan, which was often strong and thick enough to drive upon. That year however was unseasonably warm, and the ice was very thin. Richard and Erin were worried, and anxiously called Bob to come back. Luckily for Australian mathematics, the ice did not break and Bob made it safely back to shore. Erin was less lucky with ice, and broke her wrist whilst learning to skate on one of Yorktown's skating rinks. It was then that Richard learnt to cook.

Richard and Erin came home to Australia in 1972, both to jobs in the Computer Centre at ANU, Richard as a Research Fellow and Erin as a programmer. This was before the Computer Science Department existed, and the Computer Centre, run by Mike Osborne, serviced all the computing needs of the University, from payroll to research.

1974 was a particularly productive year for Richard in his academic work. He was motivated to develop his Multiple Precision Package by conversations with Kurt Mahler at ANU. Mahler wanted a very accurate computation of $\exp(\pi\sqrt{163}/3)$, which for a reason related to the Monster Group is extremely close to the integer 640320, but not quite (since it is transcendental)! Richard was also reading the proofs for Volume 2 of Knuth's books (having known Knuth at Stanford). This led to the inspiration for Richard's random number generators.

1975 saw the birth of Richard's first son Geoffrey, while Richard was on sabbatical leave at Stanford. Once again Richard and Erin drove across the USA via Carnegie Mellon (Pittsburgh), in a different old Ford Galaxy, this time with Geoffrey in a basket on the back seat. Whilst on that sabbatical, Richard worked on fast algorithms for computing π and elementary functions. He also collaborated with H.T. Kung (now at Harvard) on parallel computing and computational complexity. Kung was to become a good friend and later visited the family in Australia.

From Australia Richard went back to the US once more on sabbatical in 1978, to the University of California, Berkeley. There he met Derrick and Emma Lehmer, with whom he later corresponded on the topic of factorisation. Richard also met Nobel Prize-winning physicist Ed McMillan at Lawrence Berkeley Labs, about a mile up the hill from UCB. Richard and Ed ended up writing a paper together on the computation of Euler's constant.

Back in Australia later in 1978 at the age of 32, Richard became a father again with the arrival of his second son Rohan, and was made Foundation Professor and Head of Computer Science at ANU. The Computer Science Department had just been established in the Science Faculty out of origins in the Statistics Department in the Faculty of Economics. The young Department had a lot of students, and not many staff.

Left to right: Rohan, Erin, Geoffrey and Richard, 1979

At the stage Richard became Head of Department, Computer Science was just getting established as a proper academic discipline. There was a lot to do. A curriculum had to be invented, and the courses to fill it. Richard's teaching included the second year Algorithms course, and third year Numerical Computing. During this time he also obtained a DSc from Monash University (1981), largely based on his work with H.T. Kung. In 1982 he became a Fellow of the Australian Academy of Science. Richard steered the new Department for five of its formative years.

In 1983 Richard took the short walk from Computer Science in the Copeland Building to Mathematics in the Hannah Neumann Building, at ANU. He was part of a group led by Neil Trudinger that had sought and gained funding from the ARC for a Centre for Mathematical Analysis. This group, which soon included Alan McIntosh, Frank de Hoog, Bob Anderssen, Nalini Joshi, Gerhard Huisken and others, did research in PDEs and several other areas. There was money for computing equipment, and Richard spent some time doing VLSI (Very Large Scale Integration) design. Erin's work in the Computer Centre around the same time, on computer graphics, contributed to Richard's work on VLSI design.

In 1985 Richard became Professor and Head of the Computer Sciences Lab within the Institute of Advanced Studies at ANU. He was to stay in that role until headhunted by Oxford University 13 years later. Whilst Head of the Computer Sciences Lab at ANU Richard continued his research, working mostly on Parallel Computing and Number Theory. In 1991 Richard was made a Fellow of the IEEE (Institute of Electrical and Electronics Engineers), and later of a number of other learned societies such as ACM, AustMS, IEAust, BCS, IMA and SIAM. During his career he has also received various awards, such as the BHP Prize (1963), AustMS Medal (1984), Hannan Medal (2005), and Moyal Medal (2014).

Computation and Number Theory continued to be key research interests for Richard whilst at Oxford, where he was a Professor of Computing Science and Fellow of St Hugh's College. At this time Erin worked for an Oxford University spin-off company called NAG (Numerical Algorithms Group). Richard taught undergraduate courses whilst at Oxford; in order to be allowed to do so he had to first be awarded a Masters degree (MA) from Oxford (on top of his Stanford PhD which was by itself regarded as insufficient). Living in the UK gave Richard an opportunity to expand his collaboration with Paul Zimmermann, who worked at INRIA (Institut National

de Recherche en Informatique et en Automatique) in Nancy. That collaboration led to several joint papers and a book, *Modern Computer Arithmetic*, published in 2010, after Richard's return to Australia.

Richard and Erin did not want to stay away from their family in Australia for too many years, so Richard left Oxford in 2005 to take up an ARC Federation Fellowship in Australia. It was after Richard and Erin returned to Australia that Erin passed away in December 2005, after a seven-year battle with breast cancer. The Erin Brent Computer Science Prize at ANU, for best results in the Honours year of Computer Science, was established in her honour.

In 2005 Richard also took up responsibility for leadership of the ANU node of the multi-institution ARC grant MASCOS (Mathematics and Statistics of Complex Systems). He led MASCOS at ANU for the five-year duration of his Federation Fellowship. During this time he set up and led a group in Computation and Combinatorics. One of the postdocs that he hired, Judy-anne Osborn, was to become his second wife.

Richard and Judy-anne on their wedding day

Richard is now Emeritus Professor at ANU and Conjoint Professor and Vice Chancellor's Visiting Fellow at the University of Newcastle, where Judy-anne was hired as a postdoc in the CARMA (Computer Assisted Research Mathematics and its Applications) Centre in 2011 and then as ongoing staff in 2013. The couple welcomed their first child, Richard's third son, Allan Brent, into the world on 11 September 2015. Allan is slightly younger than his two nephews, Oscar and Sam!

Left to right: Rohan (son) holding Sam (grandson),
Oscar (grandson) and Richard holding Allan (son)

In addition to spending time with his family, as a typical 'retired' academic, Richard still supervises PhD students, serves on committees, presents at conferences, teaches the occasional undergraduate course, and continues his research in mathematics and computing.