The 56th International Mathematical Olympiad Chiang Mai, Thailand

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The 56th International Mathematical Olympiad (IMO) was held from 4–16 July in Chiang Mai, Thailand.

This was the largest IMO in history with a record number of 577 high school students from 104 countries participating. Of these, 52 were girls.

Each participating country may send a team of up to six students, a Team Leader and a Deputy Team Leader. At the IMO the Team Leaders, as an international collective, form what is called the *Jury*. This Jury was chaired by Soontorn Oraintara.

The first major task facing the Jury is to set the two competition papers. During this period the Leaders and their observers are trusted to keep all information about the contest problems completely confidential. The local Problem Selection Committee had already shortlisted 29 problems from 155 problem proposals submitted by 53 of the participating countries from around the world. During the Jury meetings one of the shortlisted problems had to be discarded from consideration due to being too similar to material already in the public domain. Eventually, the Jury finalised the exam questions and then made translations into all the more than 50 languages required by the contestants. Unfortunately, due to an accidental security breach, the second day's paper had to be changed on the night before that exam was to be taken. This probably resulted in a harder than intended second day.

The six questions that ultimately appeared on the IMO contest are described as follows.

1. A relatively easy two-part problem in combinatorial geometry proposed by the Netherlands. It concerns finite sets of points in the plane in which the perpendicular bisector of any pair of points in such a set also contains another point of the set.
2. A medium classical number theory problem proposed by Serbia.
3. A difficult classical geometry problem in which it is asked to prove that a certain two circles are mutually tangent. It was proposed by Ukraine.
4. A relatively easy classical geometry problem proposed by Greece.

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5. A medium to difficult functional equation proposed by Albania.
6. A difficult problem in which one is asked to prove an inequality about a sequence of integers. Although it does not seem so at first sight, the problem is much more combinatorial than algebraic. It was inspired by a notation used to describe juggling. The problem was proposed by Australia.

These six questions were posed in two exam papers held on Friday 10 July and Saturday 11 July. Each paper had three problems. The contestants worked individually. They were allowed four-and-a-half hours per paper to write their attempted proofs. Each problem was scored out of a maximum of seven points.

For many years now there has been an opening ceremony prior to the first day of competition. HRH Crown Princess Sirindhorn presided over the opening ceremony. Following the formal speeches there was the parade of the teams and the 2015 IMO was declared open.

After the exams the Leaders and their Deputies spent about two days assessing the work of the students from their own countries, guided by marking schemes, which had been discussed earlier. A local team of markers called *Coordinators* also assessed the papers. They too were guided by the marking schemes but are allowed some flexibility if, for example, a Leader brings something to their attention in a contestant's exam script that is not covered by the marking scheme. The Team Leader and Coordinators have to agree on scores for each student of the Leader's country in order to finalise scores. Any disagreements that cannot be resolved in this way are ultimately referred to the Jury.

The IMO paper turned out to be quite difficult. While the easier problems 1 and 4 were quite accessible, the other four problems 2, 3, 5 and 6 were found to be the most difficult combination of medium and difficult problems ever seen at the IMO. There were only around 30 complete solutions to each of problems 2, 3 and 5. Problem 6 was very difficult, averaging just 0.4 points. Only 11 students scored full marks on it.

The medal cuts were set at 26 for gold, 19 for silver and 14 for bronze.[1] Consequently, there were 282 (=48.9%) medals awarded. The medal distributions[2] were 39 (= 6.8%) gold, 100 (= 17.3%) silver and 143 (= 24.8%) bronze. These awards were presented at the closing ceremony. Of those who did not get a medal, a further 126 contestants received an honourable mention for solving at least one question perfectly.

[1] This was the lowest ever cut for gold, and the equal lowest ever cut for silver. (This was indicative of the difficulty of the exam, not the standard of the contestants.)

[2] The total number of medals must be approved by the Jury and should not normally exceed half the total number of contestants. The numbers of gold, silver and bronze medals must be approximately in the ratio 1 : 2 : 3.

Alex Song of Canada was the sole contestant who achieved the most excellent feat of a perfect score of 42. He now leads the IMO hall of fame, being the most decorated contestant in IMO history. He is the only person to have won five IMO gold medals.[3] He was given a standing ovation during the presentation of medals at the closing ceremony.

Congratulations to the Australian IMO team on an absolutely spectacular performance this year. They smashed our record rank[4] to come 6th, and they also smashed our record medal haul, bringing home two Gold and four Silver medals.[5] This is the first time that each team member has achieved Silver or better. The team finished ahead of many of the traditionally stronger teams. In particular, they finished ahead of Russia, whom we would have considered as untouchable.

Congratulations to Gold medalist Alexander Gunning, year 12, Glen Waverley Secondary College, Victoria. He is now the most decorated Australian at the IMO, being the only Australian to have won three Gold medals at the IMO. On each of these occasions he also finished in the top 10 in individual rankings.[6] He is now equal 17th on the IMOs all-time hall of fame.

Congratulations to Gold medalist Seyoon Ragavan, year 11, Knox Grammar School, NSW. Seyoon solved four problems perfectly and was comfortably above the Gold medal cut. He was individually ranked 19th.

And congratulations to our four Silver medalists: Ilia Kucherov, year 11, Westall Secondary College, Victoria; Yang Song, year 12, James Ruse Agricultural High School, NSW; Kevin Xian, year 11, James Ruse Agricultural High School, NSW; and Jeremy Yip, year 12, Trinity Grammar School, Victoria.

Three members of this year's team are eligible for selection to the 2016 IMO team. So while it is unlikely we will be able to repeat this year's stellar performance, the outlook seems promising.

Congratulations also to Ross Atkins and Ivan Guo, who were IMO medalists with the Australian team when they were students.[7] They were the authors of the juggling-inspired IMO problem number six. In fact Ross is a proficient juggler.

The 2015 IMO was organised by: The Institute for the Promotion of Teaching Science and Technology; Chiang Mai University; The Mathematical Association

[3] In his six appearances at the IMO, Alex Song won a bronze medal in 2010, and followed up with gold medals in 2011, 2012, 2013, 2014 and 2015.

[4] The ranking of countries is not officially part of the IMO general regulations. However, countries are ranked each year on the IMO's official website according to the sum of the individual student scores from each country.

[5] Australia's best performance prior to this was the dream team of 1997. They came 9th, with a medal tally of two Gold, three Silver and one Bronze.

[6] In his four appearances at the IMO, Alexander won a bronze medal in 2012, and followed up with gold medals in 2013 (8th), 2014 (1st) and 2015 (4th).

[7] Ross and Ivan won Bronze at the 2003 IMO, and Ivan won Gold at the 2004 IMO.

of Thailand under the Patronage of His Majesty the King; and The Promotion of Academic Olympiad and Development of Science Education Foundation.

Venues for future IMOs have been secured up to 2019 as follows:

- 2017, Brazil
- 2018, Romania
- 2019, United Kingdom.

The 2016 IMO is scheduled to be held 6–16 July in Hong Kong.

Much of the statistical information found in this report can also be found at the official website of the IMO: https://www.imo-official.org.

IMO Papers

Day 1, Friday 10 July 2015

Problem 1. We say that a finite set $\mathcal{S}$ of points in the plane is *balanced* if, for any two different points A and B in $\mathcal{S}$, there is a point C in $\mathcal{S}$ such that $AC = BC$. We say that $\mathcal{S}$ is *centre-free* if for any three different points A, B and C in $\mathcal{S}$, there is no point P in $\mathcal{S}$ such that $PA = PB = PC$.

(a) Show that for all integers $n \geq 3$, there exists a balanced set consisting of n points.

(b) Determine all integers $n \geq 3$ for which there exists a balanced centre-free set consisting of n points.

Problem 2. Determine all triples (a, b, c) of positive integers such that each of the numbers

$$ab - c, \quad bc - a, \quad ca - b$$

is a power of 2. (*A power of 2 is an integer of the form* 2^n, *where* n *is a non-negative integer.*)

Problem 3. Let ABC be an acute triangle with $AB > AC$. Let Γ be its circumcircle, H its orthocentre, and F the foot of the altitude from A. Let M be the midpoint of BC. Let Q be the point on Γ such that $\angle HQA = 90°$, and let K be the point on Γ such that $\angle HKQ = 90°$. Assume that the points A, B, C, K and Q are all different, and lie on Γ in this order.

Prove that the circumcircles of triangles KQH and FKM are tangent to each other.

Language: English *Time: 4 hours and 30 minutes*
Each problem is worth 7 points

Day 2, Saturday 11 July 2015

Problem 4. Triangle ABC has circumcircle Ω and circumcentre O. A circle Γ with centre A intersects the segment BC at points D and E, such that B, D, E and C are all different and lie on line BC in this order. Let F and G be the points of intersection of Γ and Ω, such that A, F, B, C and G lie on Ω in this order. Let K be the second point of intersection of the circumcircle of triangle BDF and the segment AB. Let L be the second point of intersection of the circumcircle of triangle CGE and the segment CA.

Suppose that the lines FK and GL are different and intersect at the point X. Prove that X lies on the line AO.

Problem 5. Let $\mathbb{R}$ denote the set of real numbers. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying the equation

$$f(x + f(x + y)) + f(xy) = x + f(x + y) + yf(x)$$

for all real numbers x and y.

Problem 6. The sequence $a_1, a_2, \ldots$ of integers satisfies the following conditions:

(i) $1 \le a_j \le 2015$ for all $j \ge 1$;
(ii) $k + a_k \ne \ell + a_\ell$ for all $1 \le k < \ell$.

Prove that there exist two positive integers b and N such that

$$\left| \sum_{j=m+1}^{n} (a_j - b) \right| \le 1007^2$$

for all integers m and n satisfying $n > m \ge N$.

Language: English

Time: 4 hours and 30 minutes
Each problem is worth 7 points

Mark distribution by question

Mark	Q1	Q2	Q3	Q4	Q5	Q6
0	93	256	408	91	153	521
1	89	151	122	36	255	11
2	5	77	12	61	34	15
3	21	27	1	18	90	6
4	72	8	3	11	8	3
5	12	13	0	1	4	3
6	20	14	1	8	3	7
7	265	31	30	351	30	11
Total	577	577	577	577	577	577
Mean	4.3	1.4	0.7	4.8	1.5	0.4

Some country totals

Rank	Country	Total
1	United States of America	185
2	China	181
3	South Korea	161
4	North Korea	156
5	Vietnam	151
6	Australia	148
7	Iran	145
8	Russia	141
9	Canada	140
10	Singapore	139
11	Ukraine	135
12	Thailand	134
13	Romania	132
14	France	120
15	Croatia	119
16	Peru	118

Rank	Country	Score
17	Poland	117
18	Taiwan	115
19	Mexico	114
20	Hungary	113
20	Turkey	113
22	Brazil	109
22	Japan	109
22	United Kingdom	109
25	Kazakhstan	105
26	Armenia	104
27	Germany	102
28	Hong Kong	101
29	Bulgaria	100
29	Indonesia	100
29	Italy	100
29	Serbia	100

Australian scores at the 2015 IMO

Name	Q1	Q2	Q3	Q4	Q5	Q6	Score	Award
Alex Gunning	7	6	7	7	2	7	36	Gold
Ilia Kucherov	7	2	0	7	3	0	19	Silver
Seyoon Ragavan	7	7	1	7	7	0	29	Gold
Yang Song	7	2	1	7	3	0	20	Silver
Kevin Xian	7	3	1	7	3	0	21	Silver
Jeremy Yip	7	6	1	7	2	0	23	Silver
Totals	42	26	11	42	20	7	148	
Australian Average	7.0	4.3	1.8	7.0	3.3	1.2	24.7	
IMO Average	4.3	1.4	0.7	4.8	1.5	0.4	13.0	

The medal cuts were set at 26 for gold, 19 for silver and 14 for bronze.

Distribution of awards at the 2015 IMO

Country	Total	Gold	Silver	Bronze	H.M.
Albania	37	0	0	0	3
Algeria	60	0	1	1	2
Argentina	70	0	0	1	4
Armenia	104	0	1	5	0
Australia	148	2	4	0	0
Austria	63	0	0	3	1
Azerbaijan	73	0	0	2	4
Bangladesh	97	0	1	4	1
Belarus	84	0	0	3	3
Belgium	67	0	1	0	3
Bolivia	5	0	0	0	0

Distribution of awards at the 2015 IMO (continued)

Country	Total	Gold	Silver	Bronze	H.M.
Bosnia and Herzegovina	76	0	0	2	4
Botswana	1	0	0	0	0
Brazil	109	0	3	3	0
Bulgaria	100	0	2	1	2
Cambodia	24	0	0	0	2
Canada	140	2	0	4	0
Chile	12	0	0	0	1
China	181	4	2	0	0
Colombia	72	0	0	4	0
Costa Rica	53	0	0	2	2
Croatia	119	1	3	1	0
Cuba	15	0	0	1	0
Cyprus	58	0	1	0	2
Czech Republic	74	0	0	3	3
Denmark	52	0	0	2	1
Ecuador	27	0	0	0	2
El Salvador	14	0	0	0	0
Estonia	51	0	0	1	3
Finland	26	0	0	0	1
France	120	0	3	3	0
Georgia	80	0	1	3	1
Germany	102	0	2	3	0
Ghana	5	0	0	0	0
Greece	71	0	1	2	2
Hong Kong	101	0	2	3	1
Hungary	113	0	3	3	0
Iceland	41	0	0	0	3
India	86	0	1	2	3
Indonesia	100	0	2	4	0
Iran	145	3	2	1	0
Ireland	37	0	0	0	3
Israel	83	1	0	2	2
Italy	100	1	2	0	0
Japan	109	0	3	3	0
Kazakhstan	105	1	1	2	2
Kosovo	24	0	0	0	1
Kyrgyzstan	17	0	0	0	0
Latvia	36	0	0	0	3
Liechtenstein	18	0	0	1	0
Lithuania	54	0	0	1	1
Luxembourg	12	0	0	0	1
Macau	88	0	1	2	3
Macedonia (FYR)	45	0	0	1	1
Malaysia	66	0	0	3	1
Mexico	114	1	2	3	0
Moldova	85	0	1	2	3
Mongolia	74	0	0	2	4
Montenegro	19	0	0	1	0
Morocco	27	0	0	0	1

Distribution of awards at the 2015 IMO (continued)

Country	Total	Gold	Silver	Bronze	H.M.
Netherlands	76	0	0	3	1
New Zealand	72	0	0	2	4
Nicaragua	26	0	0	0	3
Nigeria	22	0	0	0	2
North Korea	156	3	3	0	0
Norway	54	0	1	0	2
Pakistan	25	0	0	1	0
Panama	9	0	0	0	0
Paraguay	53	0	0	3	0
Peru	118	2	2	1	0
Philippines	87	0	2	2	1
Poland	117	1	1	4	0
Portugal	70	0	0	3	1
Puerto Rico	18	0	0	1	0
Romania	132	1	4	1	0
Russia	141	0	6	0	0
Saudi Arabia	81	0	1	3	2
Serbia	100	1	1	2	2
Singapore	139	1	4	1	0
Slovakia	97	0	2	3	0
Slovenia	46	0	0	1	1
South Africa	68	0	0	1	2
South Korea	161	3	1	2	0
Spain	47	0	0	1	2
Sri Lanka	51	0	0	0	4
Sweden	63	0	0	2	2
Switzerland	74	0	0	3	2
Syria	69	0	1	1	3
Taiwan	115	0	4	1	1
Tajikistan	57	0	1	1	2
Tanzania	0	0	0	0	0
Thailand	134	2	3	1	0
Trinidad and Tobago	26	0	1	0	0
Tunisia	41	0	0	1	2
Turkey	113	0	5	0	0
Turkmenistan	64	0	0	2	2
Uganda	6	0	0	0	0
Ukraine	135	2	3	1	0
United Kingdom	109	0	4	1	1
United States of America	185	5	1	0	0
Uruguay	16	0	0	0	1
Uzbekistan	64	0	0	3	2
Venezuela	13	0	0	0	1
Vietnam	151	2	3	1	0
Total (104 teams, 577 contestants)		**39**	**100**	**143**	**126**

NB: Not all countries sent a full team of six students.