Book Reviews

Count Like an Egyptian

David Reimer
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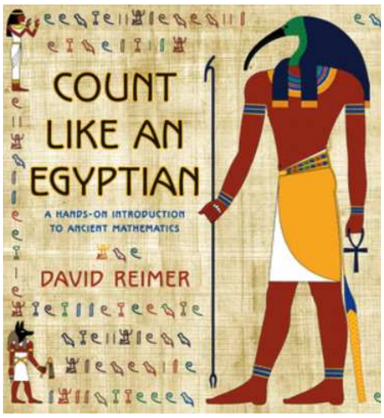
It was great to see the dedication and work David Reimer put into this book. In the Preface (but who reads that part of a book these days) he mentions he read many books on the history of Egyptian computation, though mainly to no avail. Then he took the radical step of going back to the original source, by obtaining a translation of the Rhind Mathematical papyrus. For those unfamiliar with that text, it dates from around 1,500 BC, or thereabouts, as we can't be exact, but that date puts it at the beginning of the New Kingdom era. Written by a Vizier named Ahmose, who was preserving an earlier document from the Middle Kingdom, perhaps as early as 2,000 BC, it contains most of our knowledge of Egyptian mathematical computation and geometry. Reimer goes through this document not once, but at least three times, coming to grips with how the Egyptians did their maths, working through their examples until he understood how they obtained their results as well as how they used their short cuts and maths tables.



What a great approach and a dedicated effort. So one hopes the book will reflect that persistence and it does. It is a pity that other scientists don't follow suit. Only recently I watched on TV a well known astronomer repeat the perennial myths about Galileo and the Catholic Church on the conflict between science and religion. It is a myth that the trial of Galileo was about his book on heliocentricity, it is a myth it was the church being anti-science, it is a myth that instruments of torture were presented before him, which is probably the result of the myth maker confusing the Catholic Church's Inquisition with the Spanish Inquisition and he probably didn't mumble under his breath at the end of the very long and protracted trial. Another example of this lack of consulting original documents is that these days social media has a lot to answer for the obsessive scrutiny of Ada Lovelace whose two hundredth anniversary of her birth happens this December. She has been promoted above her station by some mathematicians and computer programmers in this country and overseas. However, going back to original documents her misguided fame is based on excellent mathematical instruction by some big name mathematicians of the day, but for small gain, as her knowledge of maths was only elementary at best. She contributed no mathematical papers and is essentially known for just one thing, her translation of an engineering book from Italian into English, to which was added some notes. This appendix mentioned an algorithm that today we would call a program. Given her level of maths and close association with Babbage it is suggested in current research that it is more likely the latter suggested the content of the notes and was happy to have another person, high up in society, indirectly arguing his case as he pursued more money from the government. As scientists we need to know our limitations and let the science historians handle the history of science.

So it was refreshing to see Reimer had gone back to basics to write this book and not repeat 'a few trivial examples followed by abstract discussions filled with equations completely out of context' that the books he found in his library had done previously. After reading the book one can appreciate that he has managed, as best can be expected from his limited material, to get inside the mind of Egyptians who carried out computations for their country three, or four thousand years ago, possibly even further back. Also after reading it, it makes sense why the Greeks referred back to the Egyptians as the basis of their maths knowledge. One comes away with a respect for their maths and geometry, even though the latter is mainly mentioned in passing, as the book concentrates on mostly numeric computation. Reimer succeeds in imbuing one with the simplicity of the Egyptian computation, even comparing it to the complex Babylonian system in a chapter titled Base-Based Mathematics. The book is not without its faults, but looking at the forest and not the trees this book does achieve its goals.

Throughout the book Reimer gives some history to put the computations that follow into context. We are quickly taken through addition then into how the Egyptian carried out their multiplication by doubling. Like the rest of the book there are several examples followed by some practice exercises. None of this calculation is difficult, but the simplicity of their algorithm is impressive. One ends up considering if this method could be used for primary school students who just struggle with their rote learning of multiplication tables, in those schools that still use this learning method. A nice touch is that all the examples are presented as an image on a papyrus scroll. Division, essentially the reverse of multiplication, is very ably illustrated by the simplicity of the Egyptian algorithm which is the same as the one for multiplication with a very minor twist. In fact, it highlighted for me how complex our division is (well at least what I was taught) when say we want to divide 133 into 2261 for argument's sake. We work it out as follows,

$$
\begin{array}{r@{\,}r}
 & 17 \\ \cline{2-2}
133 \,\big) & 2261 \\
 & 133 \\ \cline{2-2}
 & 931 \\
 & 931
\end{array}
$$

Not that easy when it is all said and done. After all how does a primary school student work out that 133 divides into 931 seven times? Well, not easily. The

Egyptian student only has to double and either add or subtract. In the examples I did, I think it is faster and certainly easier for division with 3-, 4- and 5-digit numbers.

Another interesting aspect of their maths is fractions, where their numerator was always 1, with the exception of the fraction two-thirds. There is always an exception! Reimer writes their fractions without the numerator, so one-tenth is $\overline{10}$. So our Egyptian student or Vizier would write two-fifths as $\overline{3}\ \overline{15}$. Reimer then shows how this system is very similar to our decimal system, using the example of pi, 3.141, which would be written by them as $3\ \overline{10}\ \overline{25}\ \overline{1000}$. The beauty of this system, like our decimal system, is that it allows us to assess that a good approximation to Pi would be $3\ \overline{10}\ \overline{25}$ due to the small value of the third term. So while the reader isn't presented with any complicated mathematics we are presented with an elegant methodology and a system that is simplicity itself, that in some areas mirrors our own decimal system.

Reimer also seeks to put the practical examples into the context of the Egyptian system, which is appreciated. So he gives examples like the area of a triangle, which they knew to be half of the area of the rectangle, so the halving and doubling arises yet again. Other practical examples are working out shares of physical items like wheat, wages, or area of land. This involved some complex work with fractions and it was fascinating to see how the Egyptian calculated multiples of a fraction or divided that fraction into yet smaller fractions, like what is two-thirds of $39\frac{1}{2}$. The Rhind papyrus also gives us an insight into the tables they used to help in the simplification of their everyday computation. Not unlike our logarithm tables or Chambers Shorter Six Figure Mathematical Tables that went to 389 pages, which I'm sure some readers remember. A table introduced in the Rhind document is two times the odd fractions, so 2 times an eleventh is $\overline{6}\ \overline{66}$ and so on to a limit of two times $\overline{101}$. Why no table of two times the even fractions you ask? Because they knew that all they needed to do was halve the denominator. Halving and doubling was second nature to them, so no table was needed.

He considers how the Egyptians drew of human figures and how those Egyptians of equal status had to have equal heights, as measured to their foreheads. Consequently it was important not only to carve the figures correctly but to the right height. This meant that measuring was important along with marking out the grids in which to carve the figure. This in turn led to the need for rulers, as in marking or measuring devices, which were very different to our uniform 300 mm rulers of today.

As with all good mathematicians the Viziers had their short cuts or simplified workings to speed up their computations, as well as why did they choose a particular value in their tables from a list of multiples. So Reimer takes us through a series of such simplifications and choices. In fact, he devotes a full chapter to it. As an example from the book, he illustrates how the Egyptians selected a particular set of fractions for the two times the odd fractions table from a list of options. The odd fraction he chose was $\overline{15}$, so two times that can be expressed in four different ways:

- $\overline{8}\,\overline{120}$
- $\overline{9}\,\overline{45}$
- $\overline{10}\,\overline{30}$
- $\overline{12}\,\overline{20}$

So trying to reverse engineer the Egyptian decision he suggests the following methodology as to how Ahmose's predecessor selected one of the above, over the other three. Firstly, the Egyptians thought of multiple fractions as approximations that are refined with each term. So $\overline{8}\,\overline{120}$ is close to an eighth, differing by only $\overline{120}$ so it is a good approximation for an eighth, but $\overline{12}\,\overline{20}$ is a poor approximation because one twentieth is larger than half of one twelfth. So that option was dropped from contention. The Egyptians were doubling fractions and numbers all the time. If they chose the option $\overline{9}\,\overline{45}$ and needed to double that, then it meant consulting the tables twice, once for $\overline{9}$ and again for $\overline{45}$ giving $\overline{6}\,\overline{18}\,\overline{30}\,\overline{90}$, which would of course would have to be simplified. So much extra work! For even fractions it is so much easier. Need to double $\overline{8}\,\overline{120}$, easy, as mentioned above just halve the denominator, so it is $\overline{4}\,\overline{60}$ which is still a good approximation. Need to double $\overline{10}\,\overline{30}$, easy $\overline{5}\,\overline{15}$. Also a corollary from this means that $\overline{6}\,\overline{18}\,\overline{30}\,\overline{90}$ can be simplified to either $\overline{4}\,\overline{60}$ or $\overline{5}\,\overline{15}$. This would probably not have been lost on the experienced scribe either. So that option of odd fractions from the list is dropped from contention leaving us with just two options. Now a good many fractions and hence their use came about in the Egyptian system as a means of dividing up some quantities. Would you prefer to divide up a loaf or a bushel of wheat into thirtieths or one hundred and twentieths? Hmmm thought so. Reimer's guess is that the Egyptians were just like us in that too. So it doesn't come as a surprise then that the scribe chose $\overline{10}\,\overline{30}$ as the doubling of $\overline{15}$, which is exactly as it appears in the Rhind papyrus.

Another aspect that comes through in this book is not only their accuracy in calculation but also in measurement, with the prime example being the pyramids. Not only were the pyramids spot on with their location and height, the Viziers knew that the slope couldn't be too steep or it would slide away. The measurement of the slope of the Great Pyramid of Kufu was 51 degrees, 50 minutes and 34 seconds, while some others were 51 degrees and 50 minutes. Amazing accuracy that had to be reflected in their computation. However, although we aren't sure of what the Egyptian knew about the volume of a pyramid, we do know that they knew something even more complex — the volume of a truncated pyramid, which of course suggests they knew the former too. Reimer gives an interesting possibility of how it might have been worked out, but at the end of the day the scraps of information that we do have does hint at greatness. It is a pity that we don't have more. As mentioned previously they were greatly respected by the Greeks, so that might just have to be enough.

This book isn't perfect, there are some errors. Some of them are inconsequential like missing //3 symbol in a sentence where the omission is pretty obvious and didn't really affect the maths. Though this particular error was repeated a few times which was a bit disturbing. Then there were some mix-up of words in a sentence, like in Chapter 6, 'We base all we that know about Egyptian mathematics ...'

and so on. Consequently, these errors are pretty harmless, but there are some that are just wrong. As an example, again in Chapter 6, there are a sequence of fractions ‘$\overline{5}$, $\overline{15}$, $\overline{19}$, $\overline{15}$ and $\overline{285}$’ that should be $\overline{5}$, $\overline{15}$, $\overline{19}$, $\overline{95}$ and $\overline{285}$. Fortunately, I didn’t find any errors in the answers to the exercises in the book, but then, I didn’t check the Practical Solutions at the end of the book. Finally in some places Reimer does labour the point where it appeared to me not needed. The maths wasn’t complicated and his first explanation was clear enough.

The maths in this book isn’t taxing by any stretch of the imagination. An upper primary school student would be able to deal with most of it, while a high school student should have little difficulty. It would be interesting to see if teaching this Egyptian computation would be of help or hindrance to the high school student. I think it would be of help. At the end of the day the purpose of the book is to get into the mind of the Egyptian Vizier to try and understand how they carried out their computations to run everything in their empire for over 2,500 years. This is a book that comes recommended, for anyone who wants to know where our current basis of mathematics comes from through to those with an interest in maths and history.

Finally, I like that his book’s title is playfully based on the Bangles song ‘Walk Like An Egyptian’. It is good to see he likes to have some fun as well as have a bit of a play!

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