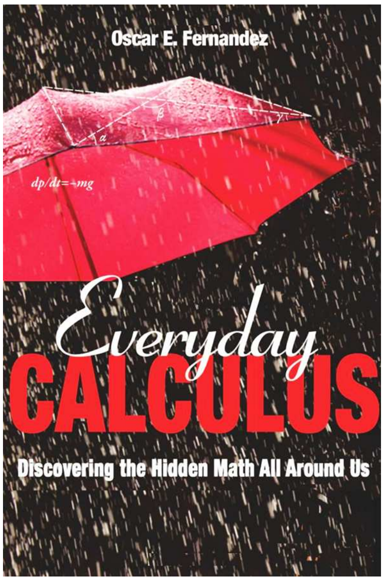
Book Reviews

Everyday Calculus

Oscar E. Fernandez
Princeton University Press, 2014, ISBN 978-0-691-15755-9

The target audience of this book is clearly stated right at the start, in the Preface. This book is for people who have asked 'When am I ever going to use [calculus]?' How many times do lecturers and teachers hear that question? And the book certainly promises much: The author's goal is that the reader 'should have a hard time figuring out what [calculus] *can't* be used for after reading this book.'

To achieve this goal, Fernandez takes the reader through a typical day and finds mathematics and calculus everywhere: while listening to the radio (Chapter 1), drinking his coffee (Chapter 2), using a GPS (Chapter 3), and selecting a seat at the movies (Chapter 7). This everyday journey also manages to discuss time travel (Chapter 3), why blood vessels branch at certain angles (Chapter 5) and the expanding universe (Chapter 7).



On the way, Fernandez manages to cover topics such as linear, polynomial, trigonometric, exponential and logarithmic functions (all in Chapter 1!), rates of changes, limits, derivatives, continuity (Chapter 2), second derivatives, linear approximations (Chapter 3), rules of differentiation, related rates (Chapter 4), differentials, optimisation, the mean value theorem (Chapter 5), Riemann sums, areas under curves, definite and indefinite integrals, the Fundamental Theorem of Calculus (Chapter 6), the average value of a function, and arc length (Chapter 7). All in 150 pages (and pages 119–150 are the appendices, end notes and index…). Compare that to the size of your favourite calculus textbook!

Clearly, to cover so many topics in 150 pages requires compromising on rigour and relegating much of the mathematical steps to appendices; Appendix A contains a refresher on graphs and functions, while Appendices 1–7 provide the intermediate steps and the calculations behind the mathematics that appears in each of the seven chapters.

The result is an interesting book, but perhaps not for the intended audience; those with less mathematical training would find the book frustrating I believe.

The book certainly includes plenty of complicated-looking formulae, such as

$$\theta(x) = \arccos\left(\frac{a^2 + b^2 - 576}{2ab}\right),$$

where

$$a^2 = (10 + x\cos\beta)^2 + (30 - x\sin\beta)^2$$
$$b^2 = (10 + x\cos\beta)^2 + (6 \quad - x\sin\beta)^2.$$

This formula appears while trying to select the best seat at a theatre (where the variables are defined on pages 101 and 102); a lay person would probably be amused to see such a complicated formula appear to help make such a mundane decision as selecting which row of seats in which to sit to watch a movie. Equations, admittedly not all as complicated as this (though some are!), appear regularly throughout the book, and surely would be too intimidating for someone without much mathematical training. And I suspect the pace at which the new topics emerge is too rapid for someone without mathematical training also. The book does, however, showcase many of the areas in which mathematics is applicable. In some ways the book reads as if it is preaching to the converted, and is providing evidence to the already-converted.

Many examples contain material that appears too involved for a lay reader, and unnecessarily so. For example, Newton's second law of motion is presented on p. 37 as

$$F_{\text{net}} = p'(t), \quad \text{where } p(t) = m(t)v(t) \text{ is the object's momentum,}$$

which I imagine is not how most of the intended reading audience would recall this Law. The more familiar $F = ma$ is given in a footnote.

Obviously, American units of measurement are used, so we see acceleration due to gravity given as 32 ft/s^2. There are some minor annoyances, such as the occasional loose use of language. On p. 54, for example, people are divided into two groups, I (those infected) and S (those susceptible to infection), and the next sentence talks about I and S as the *number* of people in those groups.

Some major quibbles do exist however, especially in the probability discussion of Chapter 6. For example, the picture of a normal distribution of the heights of American women on p. 94 (Figure 6.6) is certainly not normal in shape (surely producing a somewhat-accurate diagram would only have taken a few minutes of work?). Regrettably, only the mean of the distribution is labelled; no indication of the variation in the heights is provided at all.

The discussion about probability density functions in the same section is also found wanting. The peak of the normal distribution in Figure 6.6 occurs at the mean of 64 inches, and the vertical axis at this point has the value of 60; 60 what? The vertical axis is simply labelled as $f(x)$, which the text defines as the 'probability density function'. The text interprets this by stating that 'the graph shows that 60% of the sample has a height of 64 inches' which is clearly incorrect, and incredibly frustrating to read.

The very next sentence also makes me cringe: 'The fact that this is the *most frequent height* among adult women in the United States [i.e. the 64 inches just discussed] makes this the average height for that population' (emphasis added), which is defining the average as the mode. Later on the same page, however, in a discussion of the time spent waiting for a train to arrive, Fernandez defines the average of an exponential distribution in a different way: halfway between the lower bound of a 0 min waiting time and the 10 min waiting time which is the longest waiting time that the author has experienced; this sounds more like the median. Turn a few pages, into the next chapter, and the average is defined on p. 99 as

$$f_{\text{avg}} = \frac{1}{n}\left[\sum_{i=1}^{n} f(x_i)\right],$$

which is an arithmetic mean. I sincerely hope that some of these errors are fixed in any subsequent editions.

There are some places where details are glossed over, understandably so in a short volume intended for a wide audience. However, some technical details are actually openly discussed, such as point discontinuities in continuous functions in Chapter 2.

Superscripts abound and can become tedious: superscripts in Roman numerals are for footnotes, superscripts in Arabic numerals are for endnotes, and superscripts in Arabic numerals that are preceded by an asterisk (for example, like this^{*1}) point the reader to an appendix containing more steps of the mathematical calculations. The results is that some pages contain large numbers of these superscripts and they soon become intrusive.

I have managed to find a number of things to criticise about this text, but this is unfortunate; the book is interesting, a little technical, and I enjoyed reading the book. But what of a lay reader, the intended audience? A reader with very little mathematics background is likely to find that the book moves too fast through complicated topics, is technical (despite much of this being placed in the appendices), and has too many intimidating formulae; however, they will probably be impressed to see how calculus does appear in so many situations even if they do not understand the details. And a reader who does have some mathematical background is likely to get frustrated by the lack of rigour and some inaccuracies in the text, but still be impressed by the applications discussed. Teachers and lecturers of calculus can probably find some useful applications to use and discuss in class, but these people are not really the target audience. In this context, note that the index contains mathematical topics but not the applications (so 'Chain Rule' is indexed, but not 'coffee'; 'Gaussian distribution' but not 'GPS').

So, despite some reservations, I recommend this book; I'm just not sure to what audience. Rather than being appreciated by the author's intended audience (those who ask 'When am I ever going to use calculus?'), perhaps the book would be most appreciated by those who teach the students who are likely to ask this very question.

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Topics in Random Matrix Theory

Terence Tao
Graduate Studies in Mathematics Vol. 132, American Mathematical Society, Providence, RI, 2012, ISBN-13: 978-0-8218-7430-1

There are already several reviews of this book; in particular, there is the MathSciNet review [7] by Steven Joel Miller, and the DMV review by Benjamin Schlein [8]. The advantage of this present review, if there is one, is that it is written by non-experts. We run a fortnightly evening reading group over dinner and drinks for staff, students, and ex-students. Each year we have a topic or book, and each evening one person volunteers to present material. In 2014, our 17th year, we chose Terry Tao's book on Random Matrix Theory.

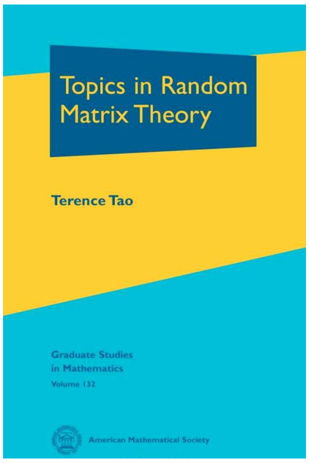
To begin this review, we can do no better than quote Terry himself, from his review of another book on random matrices:

> The field of random matrices is a sprawling one, which originated in statistics and nuclear physics, but which nowadays has many deep and interesting connections with combinatorics, complex analysis, high-dimensional geometry, concentration of measure, integrable systems, Lie groups, and number theory. As such, it is nearly impossible to write a text on the subject that covers all aspects of random matrix theory, especially given that many parts of the subject are still evolving and not yet at a mature state of understanding. Tao [9]

From the preface of his own book, Terry writes: 'This text is *not* intended as a comprehensive introduction to random matrix theory, which is now a vast subject' [10, p. x]. Indeed, there is already a wealth of information available for the enthusiastic graduate student. In particular, we mention:

- the compilation of lectures delivered at the San Diego 2013 AMS Short Course on Random Matrices, containing a paper by Tao and Vu [11],
- the introduction to random matrices by Anderson, Guionnet, and Zeitouni [2], the MathSciNet review [9] of which being the one, by Tao, quoted from above,
- Peter Forrester's 791-page encyclopaedic work [4],
- the 919-page 'handbook' by Akemann, Baik and Di Francesco [1],
- the Courant Lecture Notes by Deift and Gioev [3],
- Mehta's book, now in its third edition [6],
- the oft cited book by Katz and Sarnak [5].

Terry's book of 282 pages comprises just three chapters. The first chapter (54 pages) is an entrée devoted to preliminary material: basic probability theory (34 pages), Stirling's formula (4 pages) and eigenvalues and sums of Hermitian matrices (16 pages). The notation adopted explicitly avoids set theoretic notation, and doesn't stipulate the underlying sample space. This natural notation is very flexible, but if you have grown up with your feet planted firmly in the sample space, you may find yourself tempted to regularly translate statements from the book into more conventional language. The second chapter (179 pages) is the main meal. A substantial part of this chapter leads the reader to Wigner's semicircular law. This passage passes through the Chernoff inequality, the weak and strong law of large numbers, McDiarmid's inequality, the Talagrand concentration inequality, the central limit theorem (the treatment of which provides a natural introduction to the moment method proof of Wigner's semicircular law), the Berry–Esséen theorem, the Carleman continuity theorem, Lindelberg's swapping trick, Stein's method, the Bai–Yin theorems, etc. Chapter 2 then proceeds with a discussion of free probability (where Wigner's semicircular law is reproved using the free central limit theorem), Gaussian ensembles, the least singular value of a matrix, and the circular law. This latter topic, where Terry himself has contributed to the theory, involves significant new ideas beyond those used earlier and was beyond the scope of our reading group. The final chapter (36 pages) is a dessert of 'related articles' on Brownian and Dyson Brownian motion (16 pages), the Golden–Thompson inequality (7 pages) and the Dyson and Airy kernels of the Gaussian unitary ensemble via semiclassical analysis (13 pages). If the book is not a comprehensive introduction to random matrix theory, it is certainly a detailed presentation of many important aspects of the subject.



Overall, the impact of the book is quite awe inspiring. Terry gives an erudite and masterful presentation of the material. Each new page gives new historically important theorems, complete with comprehensive proofs, insightful comments and enlightening exercises. The unrelenting tempo of profound ideas is quite breathtaking. In reading this book, there can be no doubt that one is reading the words of one of the world's great mathematicians.

Our evening reading group using this book did function well, although a smaller than usual proportion of members volunteered to talk, perhaps because of the level of preparation required. The book is possibly too demanding a read for a group that meets over dinner and, particularly, over drinks. For self-study, or as a source for a day-time seminar group, it would be ideal.

Finally, we mention that this book continues the series of books derived from Terry's blog. A pdf version of the book is available at http://terrytao.files.wordpress.com/2011/08/matrix-book.pdf. Errata and comments can be found at http://terrytao.wordpress.com/books/topics-in-random-matrix-theory/.

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Data-Driven Modeling & Scientific Computation: Methods for Complex Systems & Big Data

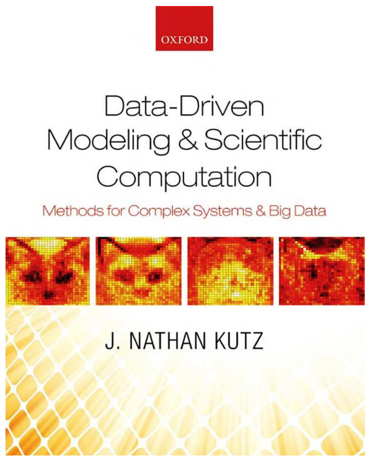
J. Nathan Kutz
Oxford University Press, 2013, ISBN 978-0-19-966033-9 (hardback)
Also available in eBook and paperback

The applied mathematician makes a three-stage attack on a problem: the creation of a model; analysis of the model by exact and approximate techniques; and simulation via scientific computation. Of the three, it is scientific computation that has spread the most into other disciplines and become an important tool for the researcher within them. Whether it is the engineer studying fluid flow over a complex geometry via finite elements, an ecologist solving a predator-prey ordinary differential equation (ODE) system with an adaptive Runge–Kutta scheme or the environmental scientist teasing out patterns from large data sets with time series analysis all increasingly require at least a passing acquaintance with the fruits of numerical analysis and some programming competence.

It is this requirement that prompts Professor Kutz's book *Data-Driven Modeling & Scientific Computation*. Kutz believes that the typical numerical analysis course, with its focus on rigour and establishing techniques, requires too much time to master and delivers material not clearly able to be implemented to solve real-world problems the broader research community wishes to solve. Kutz states that the goal of his book is to establish '. . . computing proficiency as the first and foremost priority above rigorous analysis'.

Computing requires a language in which to instruct the computer. The choice in *Data-Driven modeling* is MATLAB. On the whole this is a good choice. MATLAB is a commonly available software in universities and research institutions, is relatively friendly to the beginner (when compared to 3rd generation languages like FORTRAN), has a built-in development environment and visualisation, and comes with a large suite of code for all the common scientific computational tasks. The use of MATLAB is thus intended to allow one to reach coding and computation proficiency more quickly.

For individuals or poorer institutions the cost of MATLAB licences may be an issue. Kutz makes brief mention of one open source alternative (OCTAVE) and glibly claims that most MATLAB code is 'easily portable' to OCTAVE. Porting code presupposes a command of programming skills which may undercut the book's rationale somewhat.



Data-Driven modeling is organised into four parts. Chapter 1 of Kutz's 600+ page text is a MATLAB primer covering the basics of variable assignment, vectors, matrices, flow control, iteration, functions, data export and import, and plotting data. After the MATLAB primer in part 1 the reader is introduced to standard computational tasks: solving linear systems of equations, curve fitting, numerical differentiation and integration, optimization and more advanced visualisation. Each of these tasks has its own chapter where background information is given and theoretical results are stated but usually not derived. Snippets of MATLAB code and graphics are scattered throughout the text to illustrate how the problem is solved. Where there is a built-in MATLAB package for the task it is used and in general the MATLAB packages are treated 'to a large extent as blackbox operations'.

Part 2 deals with the solution of ODE, both initial (IVP) and boundary value problems (BVP), and partial differential equations (PDE). Standard IVP methods such as Runge–Kutta schemes are detailed. Shooting and relaxation methods are discussed for solving BVP. The section on ODE is rounded out with techniques for computing spectra. PDE are dealt with using finite differences, spectral methods and finite elements. Various time-stepping schemes are employed including the method of lines and the concept of operator splitting is demonstrated on the nonlinear Schrödinger equation.

There is nothing particularly novel about the material or its presentation in the first two parts. Other books cover similar material in a likewise pragmatic style e.g. *Numerical Methods in Engineering in MATLAB* by Kiusalaas, *Applied Numerical Methods Using MATLAB* by Yang *et al.* or *Applied Numerical Methods with MATLAB for Engineers and Scientists* by Chapra. What is novel is part 3, where the reader is introduced to methods for data analysis. This substantial section of the book covers methods for large and complicated data sets i.e. the fashionable idea of 'big data'. Thus part 3 is timely and it is uncommon to see it combined with traditional topics such as curve fitting or the solution of BVP.

Data analysis methods start with some basics of probability, move on to time-frequency analysis including the use of wavelets, and then look at image processing. Some background linear algebra is then covered to enable the introduction of the key tool of singular value decomposition (SVD). The breadth of part 3 is impressive. Image recognition is used as a vehicle to present machine learning. Kalman filtering is introduced in the chapter on data assimilation. The SVD is then used for doing principal component analysis of a PDE so proper orthogonal modes may be found and the dimensionality of the PDE reduced in order to understand the dynamics. All this and more is presented and embellished with Kutz's own research.

The last section looks at 'real world' applications — i.e. real pieces of research not sterile problems. This part is mainly to inspire but also to say *look these methods really work*. Again there is quite a range of case studies here which could be made into research projects for advanced undergraduates.

Kutz envisions that his book could be used for both undergraduate and graduate courses in scientific computing, a graduate course in methods for data analysis or as a practitioner's reference book. In my opinion the book could be overwhelming in an undergraduate course (particularly so if students are also learning programming at the same time from this text) and as mentioned above there are a number of alternative texts teaching the traditional topics of scientific computation in a pragmatic manner. Further, the book is large and more than half of it would not be put to use in an undergraduate course. However, being able to direct your new PhD student to *Data-Driven Modeling & Scientific Computation* would be a big boon. So it is here as a reading course or reference manual for a researcher that I see Kutz's excellent text as being of greatest value.

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