

# Workshop on Algebraic, Number Theoretic and Graph Theoretic Aspects of Dynamical Systems

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University of New South Wales

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*Arithmetical dynamical systems*, that is, dynamical systems generated by iterations of rational functions over fields of number-theoretic interest, have seen a significant explosion of work in recent years but still many algebraic, number theoretic and graph theoretic problems remain wide open. The interest in such dynamical systems comes also from connections that have been forged with many different areas of pure and applied mathematics. The purpose of this workshop was to further explore the complex algebraic and number theoretic behaviour, as well as to gain a better understanding of the structure of functional graphs of arithmetical dynamical systems.

## Topics covered

- Algebraic dynamical systems
- Dynamical systems of number theoretic origins
- Graph theory
- Number theory

## Special presenters

- Professor Alex Gamburd (City University of New York): expert in number theory and expander graphs
- Tony Guttman (University of Melbourne): expert in random matrices, lattices, random walks
- Cheryl Praeger (University of Western Australia): expert in graph theory and group theory

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- Klaus Schmidt (University of Vienna): expert in algebraic dynamics
- Joe Silverman (Brown University): expert in arithmetic dynamics and number theory
- Franco Vivaldi (Queen Mary University of London): expert in algebraic dynamics and number theory
- Thomas Ward (Durham University): expert in ergodic theory and algebraic dynamics
- Nick Wormald (Monash University): expert in graph theory and combinatorics

### Report

The ADS\_NT\_GT Workshop brought together researchers in dynamical systems, graph theory and number theory who were interested in learning about the latest advances in these fields, as well as expanding their own research areas. The choice of the invited speakers, who all had very broad research interests, reflected the goal of the workshop: finding new connections between the above areas. In fact each invited talk was covering several different topics. For example, Professor Cheryl Praeger was talking about problems of graph theory, which are related to dynamical systems on graphs (e.g. mixing) and also to such ‘hot’ areas of number theory as additive combinatorics. The workshop was very ‘dynamic’ not only by its name but also by its nature: one of the invited speakers, Professor Tony Guttman, influenced by some discussions at the workshop, changed the topic of his previously planned talk to another one to reflect these discussions and new developments.

The structure of the workshop, which allocated plenty of time for informal interaction (with appropriate logistical support: dedicated class rooms, internet, coffee, etc.), greatly stimulated such discussions and interactions. New contacts have been made and new collaborative links have been established. A number of participants provided positive feedback. It is expected that these links will mature to long-term collaborative projects with publications in high ranked outlets.

A very successful and well-attended outreach event embedded in the program and aimed at the general public and high school students was the public lecture on Tuesday evening by Professor Franco Vivaldi entitled ‘The Arithmetic of Chaos’.

Visit [http://web.maths.unsw.edu.au/~jagr/ADS\\_NT\\_GT.html](http://web.maths.unsw.edu.au/~jagr/ADS_NT_GT.html) for more details and slides of presented lectures and contributed talks.

### Summary of participants

There were 47 participants in total: 28 from Australia and 19 from overseas. Of the 47 participants, there were 9 student participants, 1 from overseas. There were 8 female participants within the total. The total number of participants was close to the expected number. However, we had fewer than expected Australian participants and more than expected international participants. The lower than expected number of Australian participants can be explained by an unavoidable overlap with the dates of the ANZIAM 2015 Conference: 1–5 February, Gold Coast, Queensland. In particular, many potential participants from the University of Newcastle and ANU, who initially expressed high interest in the ADS\_NT\_GT Workshop, attended the ANZIAM Conference instead.

### Organisers' opinion of success

We were very pleased with all aspects of the workshop: logistically, scientifically and socially. People mixed and discussed really well and we felt that they learnt from each other, in line with our aims to bring people from different areas together. Subsequent feedback also supports this. As an example, we report the recent appearance on arXiv of *Somos-4 and Somos-5 are arithmetic divisibility sequences* by Peter van der Kamp, one of the participants. It relates the work described in his contributed talk but mentions discussions at our workshop in the acknowledgements as a source of additional knowledge.

As mentioned above, 8 of the 47 participants were female, a little under 20%. As agreed with the AMSI Director prior to confirmation of the AMSI funding, we took several steps to increase female participation. Firstly, we individually approached female academics in Australia working in the conference topic areas. Secondly, we contacted the Secretary of the Women in Mathematics Special Interest Group of the Australian Mathematical Society to ask if we could have their membership list and research interests.

Increasingly, it is being recognised that the under-representation of females in mathematics needs to be addressed at the school level. Accordingly, we arranged a public lecture in the workshop program which we advertised extensively, in particular to the schools we deal with through our *Girls do the Maths* network. The lecture by Professor Franco Vivaldi (University of London) was a great success and, hopefully, an inspiration for those young and old who attended.

Some specific feedback from the public lecture via two unsolicited emails from high school teachers accompanying their students to the lecture:

I am a teacher at North Sydney Girls High School and attended the lecture yesterday. I am wondering if we could go on your mailing list to be advised of future public lectures. We could then promote it at our school.

It was fantastic to have such an erudite speaker talk so passionately about important research in mathematics whilst making it accessible to a school audience at the same time. I'd love to be able to promote future lectures to our students as a means of raising the profile of mathematics in their minds as they consider their future study and careers. Based on the feedback from colleagues from the school who attended, these lectures are also a great opportunity to reach out to school teachers. Let me know if I can assist in any way.