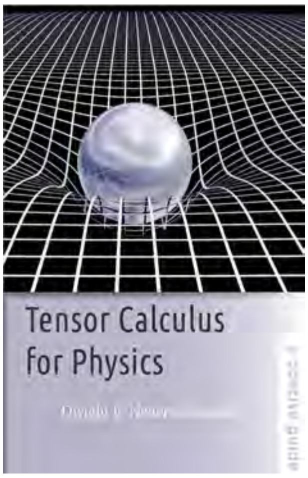
Book Reviews

Tensor Calculus for Physics

Dwight E. Neuenschwander
John Hopkins University Press, 2015, ISBN 978-1-4214-1565-9

Neuenschwander's opening section is titled, 'Why aren't tensors defined by what they *are*', echoing many a student's plaintive cry. Along with the blurb and preface this made me wonder if Neuenschwander was attempting to disprove the old joke known as The Tensor Uncertainty Principle: you can understand tensors, or work with them, but not both. To a certain degree he is, and he makes a fair fist of it. His approach is to build up to tensors from ideas that students reading this book should know: vectors in 3-space, vector fields and the inertia tensor. He states up front exactly the key point that a tensor is something independent of how it is described in terms of coordinates, even if they are primarily used in a particular coordinate system.



This idea does lead to some idiosyncrasies in both presentation and notation. I like the idea of using the Dirac bra and ket for row and column vectors, and the idea that while a tangent vector is a displacement divided by a scalar, a gradient is a scalar divided by a displacement. I am less sure of his heavy use of the concept of a dual vector. Maybe I'm just set in my ways, but I expect a vector-like object with one index downstairs to be a covector (1-form), i.e. a linear map on (tangent) vectors. We first meet dual vectors properly in Section 3.3, but in the midst of a discussion of the metric, when duals appear as the index lowered version of tensors, and so the 1 index ones are actually vectors (not 1-forms, despite how they transform). This is technically correct of course, and fits in with his approach, but leaves Neuenschwander searching for something that naturally transforms like a dual vector when there is no metric. He finds it in the gradient, which is in fact a covector. I wonder if students reading this book will feel he has left the question of what dual vectors actually are hanging.

Unfortunately, in the midst of so much good material, with interesting discussion questions as well as more traditional questions, there are a number of errors in the mathematics.

Chapter 1 is a good introduction and recap of vectors, setting up Neuenschwander's ideas, and we meet tensors in Chapter 2, in the form of specific types that students ought to have met. I felt there was some haziness over index positioning here, which could have easily been sorted out at the end of Section 3.3.

Chapter 3 begins with a good discussion on the difference between coordinate displacement and distance, an idea that is also useful in general relativity when the factors of c are replaced: it is great to see this consistently stressed. Unfortunately, this section is spoiled by an egregious error in the definition of a Riemannian metric as it is usually understood. The components of a positive definite metric do not have to be all non-negative (e.g. $dx^2 - 2dxdy + 3dy^2$) and a pseudo-Riemannian metric can have all non-negative components (e.g. $2dudv$). This error does not actually make any impact on the rest of the book, fortunately. Later on (and in Chapter 4) a good discussion is given on how the 'ordinary' vector components are to be related to the contra/co- variant components of vectors, and how these differ in non-cartesian coordinates: this is very useful material.

The affine connection is introduced in Chapter 4, using the local equivalence principle (in free fall special relativity is locally valid). This is a nice approach, but does mean there is no real indication of what a connection is, and why it has that name. The covariant constancy of the metric is an exercise, leaving me to wonder where Neuenschwander had sneaked in the equivalent requirement, and the familiar Christoffel symbols appear out of nowhere.

We meet curvature in Chapter 5, which Neuenschwander chooses to introduce through the concept of holonomy (although he avoids that term). Namely, parallel transport an arbitrary vector around an arbitrary loop in flat space and it returns unchanged; a space is curved if this does not happen. Neuenschwander makes an unfortunate logical error in stating the negative, but gets it right when he later uses the concept to prove that his definition does lead to the Riemann tensor, which he has by then introduced in the usual way by commuting covariant derivatives. He closes the chapter with an attempt to prove that the Riemann tensor is the only tensor linear in second derivatives of the metric. This is surely false without more restrictions (such as vanishing divergence). His proof does not hold up at least: nowhere in his argument does Neuenschwander introduce a general tensor linear in 2nd derivatives of the metric.

Chapter 6 is on applications, and 6.1 is a useful look at electromagnetism in tensor notation. However, Section 6.2 on general relativity skates over almost all the issues Einstein actually faced in his quest for a covariant theory of gravitation. It almost works as a post hoc derivation, but as history it is very inaccurate, see Pais [1].

In Chapter 7 Neuenschwander outlines the mathematics of manifolds, tying them in to his approach. He makes the same error about Riemannian metrics as in Chapter 3, and I had the same issue with the meaning of dual vectors: they are defined on a manifold in Section 7.3 but again under the assumption of a metric on the tangent space, so are tangent vectors, not covectors. His derivation of the covariant derivative for dual basis vectors in Section 7.4 follows the usual definition from surface theory, which to me misses the whole point of a connection. His account also begins by claiming that an n-manifold can always be (isometrically) embedded in an $(n+1)$-manifold, which is false. He does give a nice derivation for the Christoffel symbols from the completeness relation for the metric.

Chapter 8 is brief overview of differential forms. Neuenschwander uses idea of a 1-form as 'parallel planes' to illustrate the difference between vectors and 1-forms. This is all fairly standard but modified to fit his overall approach. There are several mistakes in this chapter too. Neuenschwander twice confuses linear dependence with collinearity; defines an inner product of a p-form and a q-form that only works if $p = q$; uses continuity rather than smoothness to swap the order of partial derivatives and refers to the converse of the Poincaré Lemma as a corollary of it.

Most of the errors are easily fixed, and I trust will be in a second edition. However, I cannot recommend this edition as suitable for independent reading by students. It does address a need though, so with suitable guidance would be useful for students trying to get to grips with the fundamental ideas of tensors, what they mean and how they are used.

References

[1] Pais, A. (1982). *Subtle is the Lord.... The Science and Life of Albert Einstein*. The Clarendon Press, Oxford University Press, New York.

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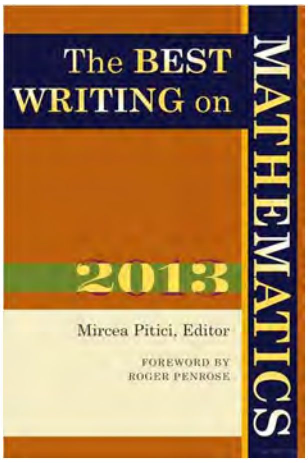
The Best Writing on Mathematics 2013 and **The Best Writing on Mathematics 2014**

Mircea Pitici (Editor)
Princeton University Press, 2013 and 2014,
ISBN 978-0-691-16041-2 and 978-0-691-16417-5

These two volumes are the fourth and fifth of an annual series edited by Mircea Pitici, who teaches maths and writing at Cornell University. I had the pleasure of reviewing earlier volumes in this *Gazette*, Vol. 39, No. 4, 2012 and Vol. 40, No. 4, 2013. The stated aim of the editor is to present to mathematicians and the general public accessible but non-trivial perspectives on pure and applied mathematics, historical and philosophical issues related to mathematics and its teaching, and social and institutional aspects.

Selecting examples of 'best writing' from the plethora which is published every year is a difficult task. Pitici takes the course of reading widely, selecting around 150 candidates which fit into his chosen format and present no copyright problems, and presenting a short list of a few dozen which his publisher asks independent referees to rate.

The essays in the current volumes, which were originally published in generalist journals such as *American Scientist* and *Plus* magazine, or in specialist mathematics, statistics, history, philosophy or education journals or simply in blog posts, are about ten pages in length and eschew calculations and technical details. Thus the level of mathematics presented is a step below that of expository papers in *The Mathematical Intelligencer* or the *Notices* and *Bulletin* of the American Mathematical Society. To this biased reviewer, the result is a mixed bag, so I will limit my comments to those which I found most enjoyable.



At the top end of the spectrum, the 2013 volume contains extracts from Terry Tao's blog on Complexity and Universality and Kevin Hartnett describes recent work on the ABC Conjecture, which is apparently still (in June 2015) open. The 2014 volume contains a vivid account, written only a few weeks after the result was announced, of Zhang's result on bounded gaps between successive primes, and John H. Conway describes recent work on the Collatz $3x+1$ conjecture and explains why it may well be unsettleable; that is, neither the conjecture nor its negation is provable in ordinary set theory.

At the lower end, there are fascinating articles on creative people who make no claim to be mathematicians, but who are inspired by mathematical ideas. For example, the artist Fiona Ross, assisted by her partner mathematician William T. Ross, and motivated by diagrams illustrating the Jordan Curve Theorem and space filling curves, creates striking unicursal ink drawings which are largely abstract but contain haunting figurative images. Kelly Delp describes how sketches illustrating Thurston's 3-dimensional geometries, or rather the singular sets in the orbifolds representing these geometries, were used by the fashion designer Dai Fujiwara to design scarves and other elements of Issey Miyaki's 2010 Paris Fashion Week presentation.

Architecture is a discipline which demands equal parts of mathematics and aesthetics. Renan Gross uses Bézier curves to analyse Spanish architect Santiago Calatrava's beautiful Jerusalem Chords Bridge, a suspension bridge whose span is supported by steel cables from a single inclined tower. The cables emanate from different levels of the tower, the lower ones being attached to more distant points on the deck so that their envelope forms a parabola with inclined axis.

Among the historical offerings, Ian Stewart describes Alan Turing's early work on morphogenesis; Daniel Silver discusses Dürer's work on projections of the conic sections, and his use of them in his paintings; John Pavlus recounts Gödel's early recognition of computational complexity and his exchanges with Von Neumann on the subject and David Lloyd describes the Scottish Neolithic stone balls which resemble the Platonic solids. Michael J. Barany discusses Cauchy's discoveries in analysis, particularly the Intermediate Value Theorem and Prakash Gorroochurn has collected errors in statements about probability in the works of Cardano, Leibniz, Pascal, Bernouilli, D'Alembert and Laplace. Many of our students are indeed in distinguished company!

Philosophical entries address the contribution of mathematics to the 'good life', the inexorable rise of Big Data, non-linear scaling in human perception, and speculation concerning the shape of the universe.

A useful feature of both volumes is Pitici's annotated bibliography of recently published works of the same genre. His work fills a gap between expository mathematics and popular explanation. It is a welcome contribution to improving public perception of our discipline.

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Introduction to Probability and Statistics for Engineers and Scientists (5th edn)

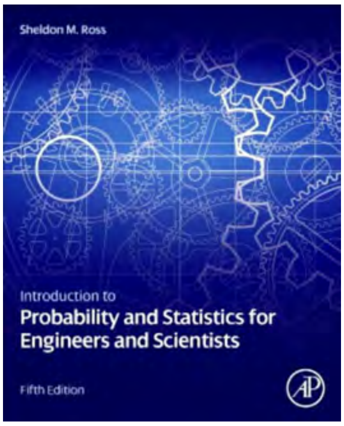
Sheldon M. Ross
Academic Press, 2014
ISBN 978-0-123-94811-3

I jumped at the opportunity to review this textbook, with a motive. I'm not a statistician, but I have been teaching a one-semester second-year subject *Probability Models* to a class of both computer systems engineers and scientists for several years now. In the first year, we referred to a text chosen by a previous lecturer, but few students bought it, having been forewarned by the class of the year before. Second time around, there was no specified reference text, but I would consider setting one in future, should I find one that I thought would be acceptable. While I am not an expert on probability and statistics, I am a very experienced educator, and I know what I am looking for in a textbook.

Although textbooks are written by expert authors, and chosen by lecturers, it is the students who need to get the most from them. So *acceptable* should not mean 'a book written exactly the way I would' or 'a book that I want to refer to' (even though as a non-expert I have needed several of those!). The students have to find it approachable, get what they need from it, and not be made too unhappy. Choosing a textbook is somewhat akin to the decision a parent makes when serving peas (but not peppermint ice-cream) rather than Brussels sprouts as a green, and it is from this perspective of being conscious of student needs and reaction that I approach this book review.

But what tempts students to open the covers of a book? And what do they dislike?

My students complain about cost, particularly if we cover only a small portion of a massive tome with a correspondingly large price. They are intolerant, not unreasonably, of errors. They like answers in the back of the book. And they need to be able to find the information and key facts clearly set out. They also like relevance, a book that seems to be for them. They would not like this book.



This is the fifth edition of a book first published in 1987. The first edition ran to around 500 pages, in 12 chapters, and included a disk which had to be used in a PC to solve some of the examples and exercises. A review of this first edition[1] commented that this feature distinguished it at that time from similar texts. The fifth edition, which adds only four further subsections to the fourth, has 15 chapters and some 670 pages, though none of these are devoted to answers to exercises. This is the first edition *not* to have an accompanying disk; in one sentence in the preface, it is mentioned that the software useful for solving problems is accessible at the publisher's website. However, as a result of non-existent proof-reading, the chapters and exercises themselves constantly refer students to the non-existent 'text disk'.

In fact, solutions to certain examples say things like 'by running Program M.N.n we obtain ...'. The software is not just 'useful'; it is the only way offered to tackle these problems. To enhance the relevance for students, standard twenty-first century software, such as they will use in employment, could have been adopted.

The preface states that the book is intended for an introductory course, and assumes elementary calculus. The publisher's blurb more appropriately describes it as being for upper-undergraduate level; it requires multi-variable calculus and a fluency in reading symbolically-dense expressions. However, the book was intended for a year-long course when first published, and is now about 25% longer, so unless one is able to agree with others in one's department to use this book across a number of subjects, students would be paying for material they would not cover in a one-semester unit should one adopt it. (In my current subject, we would use about four chapters.)

This book has an overwhelmingly north American flavo(u)r to it. For example, the exercises refer to heights and weights of adults in inches and pounds, gallons of petrol consumed per mile, the amount of salt available for snow-covered roads, and NFL statistics. One exercise starts: 'Use the results of a Sunday's worth of major league baseball scores to test the hypothesis that ...'. On the plus side so far as I am concerned, there are no examples that assume knowledge of a 'standard deck of playing cards' and not too many where one is required to draw imaginary coloured balls from an urn. The tables of data provided in many of the exercises would seem foreign to my students, the past being as distant a country as the

[1]Review by Marion R. Reynolds Jr., *Technometrics* 1988, Vol. 30, No. 4, accessed via JSTOR

US. For other tables of supposedly real-data, no source is given, which gives the impression that they are constructed for the purpose of the question.

Student reviews of the fourth edition (for example on Amazon[2]) were scathing about the number of errors in the exercises, not unreasonably expecting that a fourth edition should be largely error free. Correction of such errors is not explicitly mentioned as one of the amendments made in the fifth edition, but as the publisher maintained an errata list for the previous edition, one might expect this has been dealt with. Student users commented that the important facts and definitions were hard to locate in the fourth edition, often being swamped with text or indistinguishable from examples, and comparing this new edition with sample pages of the fourth on-line[3], that has not changed. I am inclined to place responsibility for the layout of pages and the typesetting at the feet of the editorial staff, rather than the author. In modern typesetting, changing the relative prominence of elements such as headings, subheadings, definitions etc. and incorporating highlighting such as spacing or coloured boxes should not be insurmountably difficult. The book would look less dated if the fonts used in the figures matched the font in the text body (e.g. x doesn't match x).

The book has made it to its fifth edition and been around for nearly 30 years, and I have no doubt that it covers adequately its theoretical content. But, while my students would probably be briefly amused if I chose a textbook with an author called Sheldon from California, my search for a text for my Australian students will go on.

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Zombies & Calculus

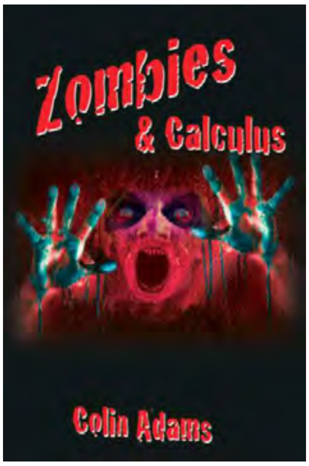
Colin Adams
Princeton University Press, 2014, ISBN 978-0-6911-6190-7

Shaun of the Dead, World War Z, Zombieland, The Walking Dead . . . the zombie has crossed the road, not to eat the chicken, from being a staple of the B movie to a staple of popular culture.

[2]http://www.amazon.com/Introduction-Probability-Statistics-Engineers-Scientists/dp/0123704839

[3]http://store.elsevier.com/Introduction-to-Probability-and-Statistics-for-Engineers-and-Scientists/Sheldon-Ross/isbn-9780080919379/

Evidentially most characters that appear in zombie movies have not watched zombie movies. If they had, they would have a greater chance of surviving by avoiding the 'traps' that catch the unwary. However, it is easy to be wise from the comfort of the sofa. You may believe that your pre-existing knowledge of zombies will give you a cutting edge. You may also believe that your intellect will aid you in surviving. Let's be honest. The average academic is going to become zombie fodder pretty quickly. Whilst it may be appealing to fantasize about which of your colleagues will be eaten first, the sad reality is that soon afterwards they will be coming for you. Yes, your intellect, your braaaiiinnnsss will be in demand.



Colin Adams has constructed an entertaining story of a zombie outbreak at a small liberal arts college in New England. The story starts at 'hour 6', when the first zombies arrive on campus, and finishes at 'hour 24', with a small band of survivors heading for the Canadian border. An epilogue summarises developments over the next three months. Against expectations, not only does Professor Craig Williams survive the zombie apocalypse but he finds that his knowledge of calculus aids in his survival.

During the course of those first eighteen hours Professor Williams finds a number of opportunities to explain to his companions how calculus can explain what's happening. The index contains 51 scientific terms, mostly drawn from calculus with a small amount of statistics and a few scientific terms, mostly connected to viruses. However, whilst he explains why the number of zombies is initially going to grow exponentially the secretary shows more sense and discovers that the security guard has been bitten! Professor, save the exposition for later. The ease with which academics can be distracted from the task at hand by providing them with an opportunity to discuss the importance of their subject, this is one reason why I anticipate a high mortality rate amongst them. Still, it suggests that in the case of a zombie outbreak a good survival strategy will be to include a brace of academics in your survival party — better them than you should the need arise . . . .

Anyone interested in zombies or the combination of mathematics and fiction will enjoy this book. The book illustrates how some standard calculus examples can be recast using zombie cladding. Three examples that I particularly enjoyed concern the classic 'pursuit problem', taking into account that zombies head towards where the target is now and not where they are heading towards. The first time we are introduced to this problem, the Dean of the Faculty is heading for Sleason Hall. The 'real-life' solution to this problem illustrates that when applying a mathematical solution we must always consider possible limitations due to assumptions made. It is a good approximation to model the Dean and the zombie as points. However, the Dean is caught by the zombie not because their points coincide but because the Dean becomes within the reach of the zombie's arm. (The Dean escapes, but there were zombies in Sleason Hall!) A few pages later, one of the survivors is on a bicycle pedalling around a walkway that encircles the interior of a quad. In this case the pursuing zombies settle into a circular path on a slightly smaller radius than the one being cycled. The third time we encounter the pursuit problem the survivors are at the top of an auditorium while the zombies are at the bottom. Instead of following the survivors' path, the zombies can only move in a straight line towards where the survivors are: climbing straight over the seats.

Does it work as a piece of fiction? To use mathematics as a plot device slows down the pacing. Furthermore, most readers will either already know the mathematics or have no interest in the mathematics. Thus the readership that will benefit from the exposition, except perhaps for lecturers wanting to use specific examples in their teaching, is rather limited.

Calculus, good against zombies in a fictional work. That is one thing. Good against the living dead in the next zombie apocalypse? That's something else.

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