



Book Reviews

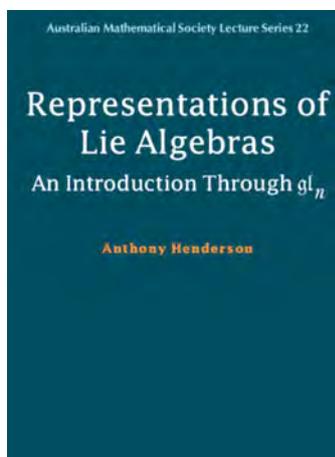
Representations of Lie Algebras: An Introduction Through \mathfrak{g}_n

Anthony Henderson
Cambridge University Press, 2012, ISBN 978-1-107-65361-0

This 156-page paperback, by Anthony Henderson of the University of Sydney, is the latest in the Australian Mathematical Society Lecture Series, a series of research monographs and textbooks suitable for graduate and undergraduate students. The book is a worthy entry in the latter category, continuing the tradition of well-written innovative texts on advanced subjects.

The structures we now call Lie algebras originate in the work of Sophus Lie in the 1890s on the symmetries of systems of differential equations, and integration techniques for them. Lie called these groups of symmetries ‘continuous groups’; the name ‘Lie group’ is due to E. Cartan in the 1930s. Lie studied n -dimensional complex representations of a continuous group G ; that is, differentiable homomorphisms of G into the group of invertible $n \times n$ complex matrices. In general, these maps are non-linear and Lie’s idea was to linearise by considering their projections into the tangent space of G at the identity. The result is a mapping of G into a subalgebra \mathfrak{g} of the full $n \times n$ matrix ring Mat_n over \mathbb{C} which preserves the operations, in the sense that if α, β in G map to a, b in \mathfrak{g} , then the product $\alpha\beta$ maps to the commutator $[a, b] = ab - ba$. Lie demonstrated the fundamental properties of this product, namely that $[a, a] = 0$ and $[a, [b, c]] + [b, [c, a]] + [c, [a, b]] = 0$. He showed that structural properties of G are reflected in algebraic properties of \mathfrak{g} , which Lie called an ‘infinitesimal group’ but which we now call a Lie algebra, this name being due to H. Weyl in 1934. Engel, Cartan, Killing, Weyl and Serre completed the structure theory of finite dimensional Lie algebras and their modules, and used the natural correspondence between representations of G and modules over its Lie algebra \mathfrak{g} to classify representations of Lie groups. More recently, Lie algebras and their generalisations have found applications in areas far removed from Lie groups, such as free groups and general relativity theory.

This history presents a dilemma to those teaching Lie algebras to undergraduates, who might be expected to have adequate backgrounds in calculus, differential equations, linear algebra and group theory, but not in differential geometry and topological groups. The definitive texts, such as N. Bourbaki (*Lie Groups and Lie Algebras*) and J.E. Humphries (*Introduction to Lie Algebras and Representation*



Theory) are certainly unsuitable for such students. One possible solution is to ignore the historical motivation altogether, and just treat Lie algebras as algebraic structures in their own right. This is the route taken for example by N. Jacobson in his book *Lie Algebras*.

Henderson takes a different approach. Building on his students' mathematical background, he concentrates on a single Lie group $G = \text{GL}(\mathbb{C}^n)$ and its associated Lie algebra $\mathfrak{gl}_n = \text{Mat}_n(\mathbb{C})$ equipped with the commutator product. In doing so, he abandons any attempt to reach the peak of the theory: the classification of simple Lie algebras and their modules. Instead, he concentrates on arguments specific to \mathfrak{gl}_n , and the classification of \mathfrak{gl}_n -modules by their highest weights.

Using matrix calculations rather than axioms, he presents all the multilinear algebra and calculus required to describe the connections between G and \mathfrak{gl}_n , the description of \mathfrak{gl}_n as the algebra of 1-parameter subgroups of G , and the relationship between group and algebra representations. Similarly, using terms familiar to students, he discusses the important subgroups of G , namely the special linear group $\text{SL}(\mathbb{C}^n)$ and the special orthogonal group $\text{SO}(\mathbb{C}^n)$ and their Lie algebras \mathfrak{sl}_n and \mathfrak{so}_n .

Having established his type examples, Henderson is now free to present the axiomatic definition of Lie algebras, their algebraic properties, and a complete description of \mathfrak{g}_n in the cases $n = 1, 2$ or 3 . Before presenting the general theory of \mathfrak{g} -modules, he tackles the special case of the classification of modules over \mathfrak{sl}_2 , with the theory always rooted in explicit matrix calculations. He introduces the machinery to deal with duality, tensor products, Hom-spaces, bilinear forms and Casimir operators and finally deals with integral modules over \mathfrak{gl}_n .

The final chapter is a précis of the important results in the theory of Lie algebras which are not covered in the text, in particular the classification of simple finite dimensional complex Lie algebras and their representations. It also includes an annotated bibliography of the important monographs in the subject.

The exposition is in the form of a well-honed course of 24 lectures, suitable as a text for a unit taught in 3rd and 4th year in Australian universities and as a preparation for studying the more advanced texts cited above. Each chapter is accompanied by exercise sets, and Henderson presents complete solutions to all the exercises. This may not accord with the wishes of many teachers who would otherwise be attracted to the text, but in fact most of the problems are an essential component in understanding the theory.

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Combinatorics: Ancient and Modern

Robin Wilson and John J. Watkins (Eds)

Oxford University Press, 2013, ISBN 978-0-19-107-965659-2

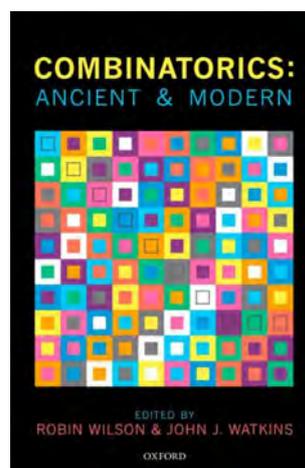
Combinatorics is one of those areas of mathematics in which the logical development closely parallels the historical development. Counting and enumerating patterns occur in the earliest historical records, preceding set and numerical partitions and later magic squares and graphical designs. Thus a book which presents the subject chronologically will also present topics of increasing complexity.

This book is a survey of the fascinating history of combinatorics from enumeration of arrangements of letters or symbols in early Indian and Chinese manuscripts to such recent developments as Redfield–Pólya pattern counting, the 4-colour theorem, Ramsey theory and algorithmic graph theory.

Individual chapters have been contributed by 16 authors, all experts in their field, as well as being noted expositors. The introduction by Donald E. Knuth is an overview of 2000 years of combinatorics, from the earliest lists of binary 6-tuples found in the Chinese *I Ching*, through the poetic metres of sacred Vedic chants and classical Greek poetry, the permutations of all sets of 5 of the 22 Hebrew consonants found in kabbalistic literature, melodic patterns of 6 notes possible in an 8-note scale in early Christian music, calculations of 5×5 determinants in 16th Century Japan, the combinations achievable by rolling several dice, the enumeration of trees by Cayley, up to the development after 1950 of electronic computers which changed our perception of what ‘difficult’ means.

The introduction is followed by seven more-detailed chapters on early combinatorics, beginning with combinations and permutations in early Sanskrit texts by Takanori Kusuba and Kim Plöfker, in Chinese divinatory practices by Andrea Bréard, in the Islamic world by Ahmed Djebbar, in rabbinical commentaries by Victor J. Katz, in Europe of the Renaissance and early modern period by Eberhard Knobloch, and on to the so-called Pascal’s triangle and its many precursors, by A.W.F. Edwards.

The later chapters trace the subsequent story, from Euler’s contributions to topics such as partitions, polyhedra and Latin squares to 20th Century advances in combinatorial set theory, block designs, enumeration and graph theory. The authors include Robin Wilson, one of the editors of the volume, Norman Biggs, E. Keith Lloyd and Lowell Beinecke. George E. Andrews’ chapter on the history of numerical partitions from Leibniz, Euler and Sylvester to Ramanujan is particularly striking. The book concludes with some combinatorial reflections by Peter J. Cameron.



This is a well-designed and produced volume, superbly illustrated and, in the words of the cover blurb by Ronald Graham, ‘the first time that such a compilation has been attempted and in the opinion of this reader, it succeeds brilliantly’.

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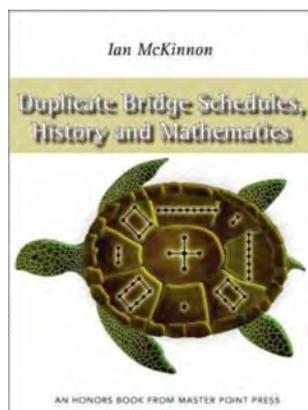
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Duplicate Bridge Schedules, History and Mathematics

Ian McKinnon

Oxford University Press, 2013, ISBN 978-0-19-107-965659-2

The author, Ian McKinnon, started a degree in mathematics in the 1960s but then discovered a passion for bridge which led to him giving up on his studies. Soon Ian also became a tournament director, and this book is his labour of love joining his interests in mathematics and bridge.



The main audience for this book will undoubtedly be tournament directors. Parts of the book are of interest to amateur bridge players who play in pair or team tournaments (this reviewer used to play in tournaments but now just plays casual rubber bridge with colleagues). For those unfamiliar with tournament bridge, it should be explained that in rubber bridge there can be a large element of luck depending on whether you are dealt good or poor hands. A well-run tournament seeks to eliminate this chance by designing the sequence of games so that the exact same hand is played over and over by different players. The hands are carried from table to table using boards. At the end of the event, the players can

then be compared fairly as (ideally) they have all played the same hands and their relative performance on the hands can be fairly assessed.

This book is very long (over 420 pages), and most of it is fairly technical as it exhaustively lists the optimal movements for any number of pairs/teams and for all kinds of tournaments.

Such information is only likely to be of interest to directors. The author has studied extensively the mathematical literature on the topic of bridge tournaments, some of which is relatively recent. He also has extensive knowledge of the history of bridge movements which is comprehensively explained. The author himself has done a lot of research on fair movements, and some of the listed sequences are ones of his own invention.

Where the maths comes into the topic of bridge tournaments is in the discussion of what ‘best’ means in the context of movements (a movement describes at what table the players sit and which boards they play, at each round). In an ideal world one would want to compare players who have played exactly the same hands, so that luck is completely eliminated and the result fair. Unfortunately, in practice this is not always possible, but a tournament should aim to be as fair (balanced) as possible. Part 6 of the book discusses this topic in detail, making use of Balanced Incomplete Block Designs (the same that are also used in statistical experiments). Latin squares (or variations thereof) and mutually orthogonal Latin squares also often show up, mostly to describe movements and a brief introduction to them is given in Part 1 (this part even covers magic squares and Sudoku, as an introduction to Latin squares).

As far as the mathematics goes, it is interesting, but the notation often caused me much head-scratching and I found the definitions/explanations often too informal (his definition of isomorphic Latin squares is one that comes to mind in this context). As a pure mathematician I found this a little unsettling! For this reason, in my opinion, mathematicians who are not bridge players are unlikely to find much in the book to be of interest.

The Appendices contain, among other things, some papers on the mathematics of bridge tournaments reproduced in their entirety, some discussions on self-orthogonal Latin squares, block designs and even on golf tournament problems!

Another element of interest for the curious bridge player is a comprehensive history of tournaments (Part 2) and how the idea of removing the luck factor emerged in the middle of the 19th century. This story actually starts with whist before moving to bridge. It is instructive to learn of the different systems used in practice (they did not think of the boards to carry cards from table to table right away!). There are a lot of pictures of old boards, which I found entertaining.

So in conclusion, this book is a must-have for tournament directors, an interesting read (skipping some sections) for the bridge player eager to know more about how tournaments are organised and about the history of their beloved game. For those unfamiliar with bridge, this book is unlikely to be of much interest.

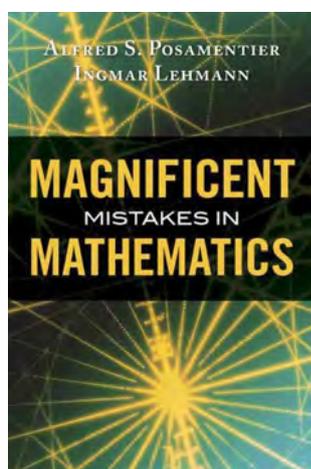
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Magnificent Mistakes in Mathematics

Alfred S. Posamentier and Ingmar Lehmann
Prometheus Books, 2013, ISBN 978-1-61614-747-1

There are three sorts of mistakes in mathematics. The first, which I call Errors, are similar to typographical errors. They are due to lapses in concentration and include thinking one digit but writing another, transposing adjacent letters or numbers, omitting a minus sign or a bracket, inadvertently skipping lines when copying a proof and so on. A famous example occurred in William Shanks' 1874 calculation of a 707-digit approximation to π . He mistakenly wrote a 5 for a 4 in the 528th digit, thus throwing out many of the subsequent digits as well. Errors are characterised by the fact that they are, or should be, picked up by the author on careful re-reading, or by a later reader, as Shanks' error was. Thus although we nearly all make Errors from time to time, they cause no lasting harm.



The second sort of mistakes I call Fallacies. These are not normally present in the works of mature mathematicians, but they occur in abundance in students' homework. They include dividing by zero, taking square roots or logs of negative numbers, misusing the symbol ∞ , geometrical reasoning based on faulty diagrams, over-reliance on calculators and misuse of algebraic operations on conditionally convergent real series. In short, Fallacies are based on careless use of algorithms whose hypotheses are not satisfied. They are usually not picked up by the perpetrators, but easily identified by more knowledgeable observers.

The final type of mistakes are called Magnificent Mistakes by the authors of the book under review. These are essentially Fallacies which occur at a much

deeper level, and are uncommonly fruitful in the sense that when finally recognised, they lead to major advances in mathematics. For example, the Pythagorean belief that any two lengths have a common measure was fallacious, but led via Book 10 of Euclid to Dedekind's axiomatisation of the real numbers. Fallacious proofs of Euclid's parallel postulate led to non-Euclidean geometry. Cauchy's proof that pointwise limits of continuous functions are continuous led to the discovery of uniform convergence as well as formal rules for manipulating quantifiers. Other examples include the famous mistake in Poincaré's Prize paper on the 3-body problem, recognised by himself, which had notable repercussions in dynamical systems theory. Fallacious beliefs about factorisation of algebraic integers in number fields had major rôles in advancing algebra and number theory.

The title of the book under review led me to anticipate a broad survey of Magnificent Mistakes and their consequences, but unfortunately I was disappointed. Although the Introduction and the first chapter, entitled 'Noteworthy Mistakes by

Famous Mathematicians', promised much, the book delivered little. The authors state that mistakes were made and claim that they were fruitful; they do not always explain what precisely were the errors, how they were discovered, or what were the consequences. In several cases, the supposed mistakes were simply conjectures that turned out to be false.

The probable reason for this disappointing lapse is the intended audience of this book. Both authors are veteran mathematics educators, A.S. Posamentier at a small New York college, and I. Lehmann, retired from Humboldt University in Berlin, but still closely engaged in mathematics enrichment activities for secondary students. Their book, which requires no more than high-school-level competence, is aimed at school students and their mentors. The only mistakes dealt with in depth are obvious Fallacies, and here there is overkill. For example, there are a dozen examples of calculations leading to absurd results because they contain camouflaged divisions by zero. Similarly, there are numerous false results obtained by manipulating inconsistent linear systems, faulty conclusions from assuming that a pattern in an initial segment of a sequence of integers extends to a general rule and misdrawn diagrams leading to fallacious geometric conclusions. There are examples of fallacious reasoning in probability, and somewhat off-target, optical illusions.

Most of the Fallacies in this book are probably derived from howlers actually committed by the authors' own pupils, and students should certainly be warned of the pitfalls attending careless use of algorithms. While this publication contains many amusing examples which serve this purpose, it hardly fulfills the promise suggested by its title.

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