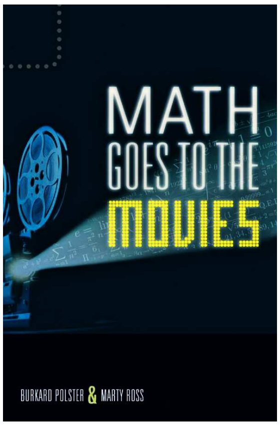
Book Reviews

Math Goes to the Movies

Burkard Polster and Marty Ross
Johns Hopkins University Press, 2012, ISBN: 978-1-4214-0483-7 (hardback)
Also available in paperback (ISBN 978-1-4214-0484-4) and for Kindle

This is an entertaining grab bag of mathematical and movie titbits that will delight mathematically minded movie buffs. The authors also have a website that includes links to relevant movie clips, and the whole project will appeal especially, perhaps, to students and teachers of mathematics. But I can do no better than to use the authors' own words to sum up the aim of *Math Goes to the Movies*:

> Our goal is to complement and significantly extend the available information about math in the movies. . . : in conjunction with our website, we have endeavored to hunt down and to describe all the "good stuff", the scenes we believe are of general appeal and usefulness. Furthermore, our emphasis is really on the math and the fun of seeing it on the big screen, not on anything else. The flipside is that our book probably offers little to experts in cinema studies and serious movie critics.



This last sentence notwithstanding, the book is designed to be 'functionally encyclopedic' in its detailed, cross-referenced lists: movies containing various levels and amounts of mathematics, famous topics and famous mathematicians who appear in movies, famous actors who have played those famous mathematicians, the mathematics consultants who have worked on these movies, and more. These lists constitute the last two chapters (or Part III) of the book. But *Math Goes to the Movies* is far more than lists of facts and figures about movie maths. Part I (Chapters 1–12) focuses on a number of films to which the authors devote more in-depth discussions — beginning with four of the most famous: *Good Will Hunting*, *A Beautiful Mind*, *Stand and Deliver*, and π — while Part II (Chapters 13–19) is organised in terms of key mathematical topics that feature in movies.

The opening chapter of the book begins with a discussion of *Good Will Hunting* from the point of view of the film's mathematics consultant, Patrick O'Donnell, whom Polster and Ross interviewed by telephone. O'Donnell, a physics professor at the University of Toronto, was initially hired as an extra — to play a drunk man in a bar! He was 'spotted' while he was having lunch at a restaurant near the university — but the movie people did their homework and tracked him to his office, where they were so impressed by the 'jottings and wave functions and things' on his board that they invited him to be their consultant. As in most 'mathematical' movies, the mathematics in *Good Will Hunting* is mostly confined to decorative 'props' — blackboards full of equations — but these are vital to the feel and look of the film.

One of the first things O'Donnell noticed was that 'none of the actors could write on a blackboard'. Among his snippets from behind-the-scenes was the fact that if the actors had to 'write and act at the same time', he had to choose material that was suitably 'dumbed down'. David Bayer made a similar discovery — he's the Columbia University mathematics professor who served as consultant to *A Beautiful Mind*, the subject of Chapter 2. As most readers will know, this movie is based on the story of mathematician John Nash, and stars Russell Crowe. Mathematics lecturers in particular will enjoy this recollection from Bayer:

> Russell had actually been very nervous the day that he did the math lecture, where he wrote the problem out and talked to Jennifer [Connelly, who played his student] at the same time in the classroom. And everybody was sitting there astounded: 'You mathematicians really talk and write at the same time?' Here we are taking like twenty takes and no-one was thinking that Russell was a yahoo and everyone is extremely impressed with it. They were basically feeling sympathetic with him that he was trying to pull that scene and they are looking at me, 'You guys do this? You gotta be kidding.'

In a related vein, Columbia mathematician Henry Pinkham, who was the consultant for *The Mirror Has Two Faces*, recalled that the movie's star, Jeff Bridges, who plays a mathematics professor, 'went to a lot of effort to learn lines that would be convincing from a mathematical point of view'.

Another appealing feature of *Math Goes to the Movies* is its photos of some of the blackboard shots in these and other movies discussed — including whiteboards created by Polster and Ross when *they* acted as consultants for an episode of the TV series *City Homicide*. There's also a great shot in which Elizabeth Hurley (playing the Devil in *Bedazzled*) is pointing to a blackboard on which Fermat's Last Theorem is written, followed by 'SHOW YOUR WORK'. As the authors remark, 'this appears to be a very funny jibe at Fermat, as if he were a negligent schoolboy for not including his proof'. More importantly (for mathematical readers), Polster and Ross often expand upon the blackboards' mathematical content.

Indeed, much of the book is devoted to brief, accessible amplifications of some of the mathematics that features in the various movies: graph theory in *Good Will Hunting*; game theory in *A Beautiful Mind*; the nature of pi, the golden ratio, and the Fibonacci sequence (in π, and in the classic 'teaching cartoon' *Donald in Mathmagic Land*); prime numbers (*The Cube*); calculus (*Stand and Deliver* and *The Mirror Has Two Faces*); group theory (*It's My Turn*, of which the authors say that to the best of their knowledge, this is 'the only movie with a scene dedicated to a mathematical proof'); Pythagorean triples and Fermat's Last Theorem (*Star Trek*, *The Simpsons*, and the mathematical musical *Fermat's Last Tango*); the fourth dimension and hypercubes (*Cube 2*) — and more.

The authors' expositions are at a level suitable for the interested lay reader, student, or teacher; they include some neat techniques, such as using elementary number theory to eliminate an apparent counterexample to Fermat's Last Theorem that was used in *The Simpsons* (whereas using a calculator, the numbers chosen appear to 'work', to an accuracy of nine digits). But as I implied in the previous paragraph, Polster and Ross also visit more complex territory, including 'the famous *snake lemma* from homological algebra' (*It's My Turn*). On the other hand, sometimes the authors simply mention mathematical ideas or equations used in movies — often pointing out where the film-makers got it slightly wrong; some readers may want to reach for pen and paper to fill in the gaps for themselves. And for those who like trying their hand at puzzles, there's a chapter called 'Problem Corner', which lists an amazing number of interesting mathematics problems that have appeared in movies; answers and occasional working are provided.

The emphasis in *Math Goes to the Movies* is, naturally enough, on movies, not on television shows, so fans of *Numb3rs*, *Star Trek*, and so on will have to look elsewhere; but Polster and Ross do mention a few scenes from some of these series (and they give references for further information). For instance, the following excerpt — given in Chapter 19, which lists some of the *deliberately* funniest maths scenes in movies (as opposed to the bloopers of Chapter 18) — may keep *Big Bang Theory* fans happy:

> SHELDON: There's some poor woman who's gonna pin her hopes on my sperm. What if she winds up with a toddler who doesn't know if he should use an integral or a differential to solve for the area under a curve?
> LEONARD: I'm sure she'll still love him.
> SHELDON: I wouldn't.

Or perhaps this scene will remind some readers why they *don't* like this series . . . but either way, there's no doubt something for everyone here. The authors' own favourite comic scene has Abbott and Costello 'proving' that $7 \times 13 = 28$, in the 1941 movie *In the Navy*; this scene, and its method of 'proof', is discussed in more detail in a chapter of its own (Chapter 10). To mention just two more of the many 'funny scenes' listed, there's the wonderfully wry puzzle-solving exchange between Katharine Hepburn and Spencer Tracy in *Desk Set*, and, on a purely trivial level, Woody Allen's quirky response to his drill sergeant in *Love and Death*:

> SERGEANT: One, two, one, two, one, two . . .
> BORIS (ALLEN): Three comes next, if you're having any trouble.

All in all, *Math Goes to the Movies* is a fun read for anyone interested in mathematics, and doubly fun if you're interested in movies too.

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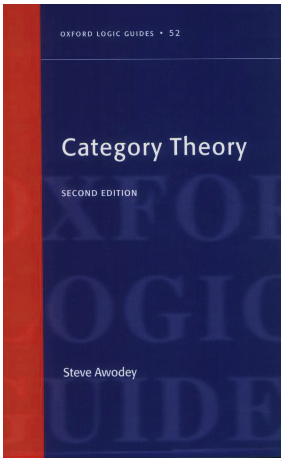
Category Theory

Steve Awodey
Oxford Science Publications, 2010, ISBN 978-0-19-958736-0 (hardback)
Also available in paperback (ISBN 978-0-19-923718-0)

Category Theory was born in 1945 when Eilenberg and Mac Lane formalised their discoveries on the connections between topology and group theory. As Eilenberg said, they defined categories in order to define functors, which they needed to define natural transformations, which they needed to define adjunction. Be that as it may, they quickly realised that category theory is ubiquitous in mathematics, serving as a uniform foundation for algebra, analysis, algebraic geometry, K-theory, number theory, logic and more recently computer science. In particular, set theory can be given a transparent categorical formulation.

Following this definition, some intrinsically categorical concepts, such as duality, universal functions and limits were quickly discovered. Major categorical results appeared, such as Yoneda theory, which describes when a set valued functor F on a category $\mathfrak{C}$ is representable as $\mathrm{Hom}(C, -)$ for some $C \in \mathfrak{C}$, and Kan theory, which explains when adjoints between categories exist.



A dangerous flirtation with logical antinomies such as the Russell paradox occurred when category theorists began working with the category of Sets, and even with the category of Categories. At first they were dealt with by ad hoc methods, such as embedding everything involved in a universe. Later it was realised that it was not necessary to define categories within the confines of set theory, but that one could in fact use category theory as a new foundation for mathematics and instead define set theory within category theory. In particular, topos theory, originally developed by Grothendieck as a tool for algebraic geometry, became a categorical basis for set theory.

Textbooks on category theory aimed at researchers and graduate students in various fields appeared, notably Freyd: *Abelian Categories* in 1964; Ehresmann: *Catégories et Structures* in 1965; Mitchell: *Theory of Categories*, also in 1965; Bucur and Deleanu: *Introduction to the Theory of Categories and Functors* in 1968 and Pareigis: *Categories and Functors* in 1970. Finally the classic text that became the bible for users rather than creators of category theory, Mac Lane's *Categories for the Working Mathematician* appeared in 1971.

Other texts, not only in English, have appeared since then, but I mention these since they set the tone for subsequent category theory textbooks, and form a collective basis for comparison with the text under review, which claims descent from Mac Lane. What these texts have in common is that they are aimed at scholars who already have some familiarity with one or more of the fields to which category theory is applied, usually algebraic topology, group theory, commutative algebra or logic. Thus they assume the reader is already familiar with details of some of the applications discussed, just as a textbook on group theory assumes knowledge of elementary number theory.

Awodey's *Category Theory* is the second edition of an Oxford Logic Guide, first published in 2006. The author is a logician and his book grew from his courses at Carnegie Mellon aimed at advanced undergraduates and graduate students in computer science, mathematics and logic. Thus it assumes somewhat less prior knowledge than Mac Lane in mainstream mathematics, but rather more in mathematical linguistics including the λ-calculus. The early chapters are a careful introduction to category theory in the context of monoids and posets. Later, group theory is presented as a categorical abstraction of the notion of automorphisms of an object with or without structure. Although the mathematical prerequisites are lighter than those of Mac Lane, the standards of rigour are not compromised. A nice feature to my mind is the completeness of the proofs of the more advanced theorems and the thoughtful exercises with full solutions to many of them.

Most of the topics covered in Mac Lane are presented, together with a few more recent developments, particularly those concerned with logic, such as first order classical logic, intuitionistic logic including Heyting algebras, and λ-caculus. A notable omission is Abelian Categories, although all the tools necessary for a definition are already present.

To my mind then, Awodey's book is a worthy successor to Mac Lane, aimed at a less sophisticated audience and so well worth considering as a text for a first course in category theory, or as a preparation for reading advanced papers in the subject.

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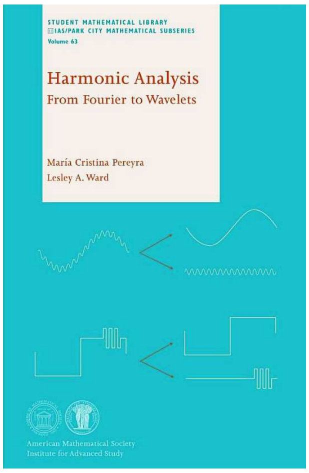
Harmonic Analysis: From Fourier to Wavelets

Maria Cristina Pereyra and Lesley A. Ward
Student Mathematical Library Vol. 63, American Mathematical Society, 2012, ISBN 978-0-8218-7566-7

This is a gentle introduction to Fourier analysis and wavelet theory that requires little background but still manages to explain some of the applications of Fourier and wavelet methods and touch on several current research topics.

The topics covered include the traditional components of a Fourier analysis course, such as Fourier series, convolution, the convergence (or nonconvergence) of a Fourier series and its Abel and Cesàro means, Fourier integrals and senses in which these converge, Plancherel and Parseval theorems, Schwartz functions and tempered distributions. However, there is also a nice discussion of the fast Fourier transform, the Haar basis and the fast Haar transform, including an explanation of why the Haar transform is faster than the Fourier transform, and then more on wavelet and wavelet-like transforms and multiresolution analyses. There is a treatment of two-dimensional image compression (illustrated with photographs of fingerprints and digital versions thereof, and JPEG), and an explanation of how wavelet methods can be used to prove the L^p boundedness of the Hilbert transform.



The authors have taken care to be accessible to undergraduate mathematicians. Functional analysis is largely avoided, and the Lebesgue integral is mentioned but almost all integration is reduced to Riemann integration of continuous functions. Further, by largely avoiding the 'Theorem-Proof' structure, they ensure that students with holes in their mathematical backgrounds should still be able to get to grips with the main issues of Fourier and wavelet analysis. There is a summary of some essential results from analysis in Chapter 2, and an Appendix with more of these, to which reference is often made.

The authors communicate their enthusiasm for harmonic analysis in different ways. Compared to standard texts, this book is characterised by more personal and historical information, including footnotes explaining the *dramatis personae* and references to comparatively recent important work such as Carleson's convergence theorem, without proof but with an explanation of its significance, and mention is made of several papers that are less than ten years old, such as the Green–Tao theorem and results of Hytönen and of Petermichl on the averaging of dyadic operators to get translation-invariant operators such as the Hilbert transform. It comes with many projects for interested students, and lists a number of open-ended problems that suggest further developments and should engage interested students. The reader is given the impression that theory and applications in Harmonic Analysis are feeding off each other in a way that is beneficial for both. At the same time, chapter and section titles such as 'The Fourier transform in paradise', 'A bowl of kernels', 'Monsters, Take 1' and 'Interpolation and a festival of inequalities' illustrate the lively writing style that make the book a pleasure to read.

The authors have taken considerable care with their writing. I noticed only one typo (a wrong sign in the definition of the Hilbert transform in a footnote on page 329). There are one or two places where it might be argued that a little bit more would have been nice (for instance, a comment that Lebesgue measurable functions behave better than Riemann integrable functions as far as pointwise limits are concerned), or that a little bit less would have sufficed (for instance, the proof of the Riemann–Lebesgue lemma for Schwartz functions is arguably overkill). But these are matters of personal taste, and one can always understand the authors' choices, even when one does not entirely agree with them.

In summary, this is a well-written and lively introduction to harmonic analysis that is accessible and stimulating for undergraduates and instructive and amusing for the more sophisticated reader. It could also be argued that the material herein should be part of the knowledge of most undergraduates in mathematics, given that the modern world relies more and more on data compression. It is therefore timely as well. It has certainly earned my enthusiastic recommendation.

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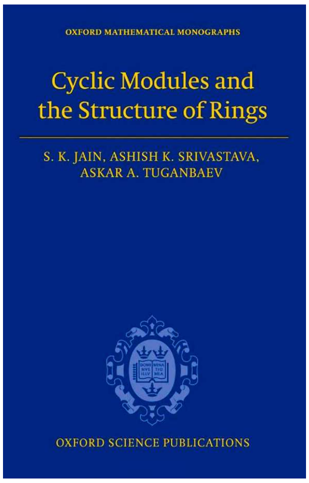
Cyclic Modules and the Structure of Rings

S.K. Jain, Ashish K. Srivastava and Askar A. Tuganbaev
Oxford Science Publications, 2012, ISBN 978-0-19-966451-1

What properties of a ring R are determined by its cyclic right modules? For example, division rings including fields are completely determined by the structure of their cyclic modules: the non-zero cyclic modules are all isomorphic and their endomorphism rings are isomorphic to the ring itself.



This monograph is a survey of 50 years work on the problem of determining the structure of rings over which cyclic modules satisfy various module theoretic properties such as finiteness or homological conditions. The prototype is a classical 1964 result of Osofsky that R is semi-simple Artinian if and only if all cyclic R–modules are injective. For commutative rings, similar results were already proved in 1950 by Cohen: every proper factor ring of R is Artinian if and only if R is Noetherian and non-zero prime ideals are maximal.

In this book, the authors classify rings each of whose proper cyclic modules satisfy ascending or descending chain conditions, homological properties such as injectivity or projectivity, have cyclic or projective essential hull or satisfy several combinations and generalisations of these properties. To capture the flavour of the book, here are two typical results. Each proper cyclic module of R is perfect (i.e. it has a projective cover satisfying some technical conditions) if and only if R is either right perfect, prime or a local domain.

Each cyclic module M is quasi-injective (i.e. every homomorhism of a submodule of M into M extends to an endomorphism of M) if and only if R is a direct sum of a semi-simple Artinian ring and a finite direct sum of self-injective, rank zero, uniserial duo rings with Jacobson radical zero.

Several chapters are devoted to related properties with cyclic replaced by simple or modules replaced by left or right ideals or proper factor modules of R.

The coverage is encyclopedic but the exposition is rather dry. So this volume is important if you work in the area, but is not light reading for the novice.

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