Kinetic theory of rarefied gas flows with modern applications

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The Navier–Stokes equations and the no-slip boundary condition provide a rigorous mathematical description of the dynamics of many important flow phenomena. The validity of these equations is contingent on the continuum approximation, which assumes that the collective (average) motion of an ensemble of particles can be tracked. This assumption is violated when operating in miniaturised systems or at low gas densities, a concept that is formalised by the definition of the Knudsen number

$$\mathrm{Kn} = \frac{\lambda}{L}, \tag{1}$$

where λ is the mean free path of the gas and L is chosen to be the characteristic length scale of the flow. Classical or continuum flows are characterised by an infinitesimal Knudsen number $\mathrm{Kn} \to 0$, while a flow is regarded as rarefied when $\mathrm{Kn} \gtrsim 0.01$; see Hadjiconstantinou (2006).

Attempts to model rarefied flows at low densities began with Maxwell (1879), where he derived a slip boundary condition for a dilute gas using kinetic theory. Based upon particle conservation principles, a kinetic theory approach remains valid across the full range of Knudsen number. The general boundary value problem for the steady flow of a slightly rarefied gas, $\mathrm{Kn} \ll 1$, across a solid wall of smooth, arbitrary shape was later solved by Sone (1969, 1974). The gas was assumed to obey the Boltzmann–BGK equation, where inter particle collisions are modelled via a relaxation process to qualitatively explore the underlying physics (Bhatnagar *et al.* (1954); Welander (1954)). Small perturbations about the equilibrium state of the gas were assumed, and the equation was linearised. Differential equations were derived via an asymptotic analysis; these *hydrodynamic equations* related the nth order terms in the asymptotic expansions of the density, mean velocity and temperature of the gas, away from the wall. Interestingly, the Stokes equations of creeping flow were recovered to all orders in Knudsen number. To account for rarefaction effects, a matched asymptotic expansion was performed; the Knudsen boundary layer near the wall was studied in detail up to $O(\mathrm{Kn}^2)$. In the limit of infinitesimal Knudsen number, the no-slip boundary condition was recovered. Slip models were also derived at first and second order in the Knudsen number.

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The steady flow assumption underlying this approach is brought into question for modern nano electromechanical systems (NEMS), which exhibit Knudsen numbers of order unity and flows that are distinctly oscillatory (time-varying). In Nassios and Sader (2012), we generalise the existing theory to the oscillatory (time-varying) case. The characteristic oscillation frequency is assumed to be much smaller than the relaxation frequency of the gas. Importantly, the first-order steady boundary conditions for the velocity and temperature are found to be unaffected by oscillatory flow. In contrast, the second-order boundary conditions are modified relative to the steady case, except for the velocity component tangential to the solid wall. Interestingly, the leading-order effect of oscillatory motion was shown to arise through a modification to the bulk flow (outer solution) hydrodynamic equations at $O(\mathrm{Kn})$, for non isothermal flows. The additional body force terms at this order contrasts with the steady case, where rarefaction effects were localised near the walls.

To demonstrate the utility of this approach, we also examine the flow induced by oscillatory (time-varying) temperature gradients applied along two parallel plane walls, separated by a small distance L, i.e. thermal creep. For certain degrees of inertia, the new body force terms dominate pressure gradients in the gas, and drive the bulk flow.

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Jason completed his undergraduate degrees in Science (Hons) and Commerce at The University of Melbourne in 2009, whereupon he commenced his PhD in Applied Mathematics under the supervision of Professor John E. Sader. He recently commenced work at Mercer Australia, as a Senior Analyst in the Investment Consulting team. In his spare time, he enjoys tennis, gardening and spending time with his wife and pet dog Oscar (not in that order!).