The 52nd International Mathematical Olympiad

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As the pinnacle of high school mathematical competitions, the International Mathematical Olympiad (IMO) is an annual event attracting the brightest young minds from around the globe. Over the years, many IMO medalists, such as Timothy Gowers, Grigori Perelman, and of course, our very own Terence Tao, have gone on to become renowned mathematicians. In 2011, the 52nd IMO was held in Amsterdam, The Netherlands.

Each invited country is permitted to send a team of up to six young amateur mathematicians (that is, each must be under 20 years of age and not enrolled in tertiary education), accompanied by a Leader and a Deputy Leader. This year, there were a total of 564 participating students, representing 101 countries.

Prior to the IMO, the Australian team spent about a week in Trinity College, Cambridge, for acclimatisation and final preparation. The team also competed against the UK team in a warm-up competition named the Mathematical Ashes, a recent tradition complete with its own trophy and urn. It ended in a dramatic tie, which unfortunately meant that the British retained the trophy as current holders.

The Australian and British teams competing for the Mathematical Ashes.

The Netherlands is known for its unusual geographical make-up, with about 20% of its land lying below the sea level, protected by an elaborate system of dikes.

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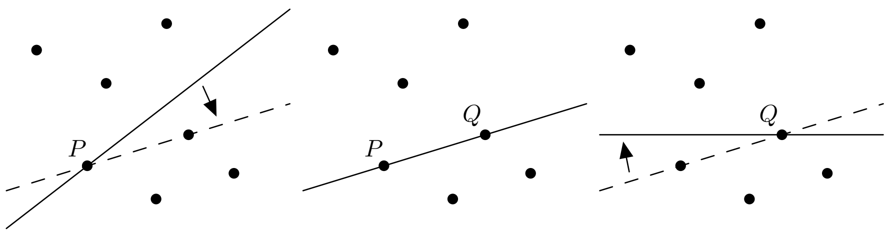
The word ‘Netherlands’ literally means ‘low-lying lands’. The capital Amsterdam is a vibrant, modern city permeated with museums, bicycles and canals. It is an ideal location to accommodate some of the best maths students in the world.

Just like the Olympic Games, the IMO began with an Opening Ceremony, which included a parade of nations. The teams marched on stage one after another, interspersed by energetic local dancers, acrobats, skaters and a freestyle BMX rider, creating a youthful and lively atmosphere. The IMO was officially opened by a banging of the gong by Robbert Dijkgraaf, Chairman of the Board of IMO 2011, and Eberhard van der Laan, Mayor of Amsterdam.

The Team Leaders arrived a few days before the IMO, for the important task of selecting the contest problems. Understandably, this process was kept secret from the students and far away from the contest location. A locally organised Problems Selection Committee had gathered problems from around the world, and narrowed them down to a shortlist of 30, spanning the areas of algebra, combinatorics, geometry and number theory. Despite being spoilt for choice, the Team Leaders eventually decided on six problems of varying flavour and difficulty. The problems were then meticulously translated into 54 languages.

The most talked-about problem this year was a beautiful combinatorial geometry question involving windmills, a Dutch cultural icon. It was initially rated as medium difficulty, but managed to baffle many strong students.

> Let $\mathcal{S}$ be a finite set of at least two points in the plane. Assume that no three points of $\mathcal{S}$ are collinear. A *windmill* is a process that starts with a line ℓ going through a single point $P \in \mathcal{S}$. The line rotates clockwise about the *pivot* P until the first time that the line meets some other point belonging to $\mathcal{S}$. This point, Q, takes over as the new pivot, and the line now rotates clockwise about Q, until it next meets a point of $\mathcal{S}$. This process continues indefinitely.
> Show that we can choose a point P in $\mathcal{S}$ and a line ℓ going through P such that the resulting windmill uses each point of $\mathcal{S}$ as a pivot infinitely many times.



To crack this problem, one has to make three important observations:

(a) The number of points on each side of the windmill stays invariant when it is not touching a second point.

(b) Call a directed line through a point a *dividing line* if the number of points on its left is one more than or equal to the number of points on its right. Then every point has a dividing line.

(c) For every possible direction, there is a unique dividing line pointing towards that direction.

Observation (a) is easily checked, while (b) and (c) are simple consequences of the 'discrete intermediate value theorem'. Now just choose any windmill starting from a dividing line. By (a), it will always be a dividing line. As the windmill rotates, it must eventually point towards every possible direction. Thus by (c), it visits every possible dividing line and, by (b), pivots about every point. The finiteness and reversibility of the system ensure this happens infinitely many times.

The contest itself consisted of two exams, held on Monday 18 July and Tuesday 19 July. Each exam had three problems and lasted for four-and-a-half hours. A maximum of seven points was possible for each problem, so the contest was out of 42. During the first half-hour of each exam, the contestants were allowed to ask, through written means, for clarifications on the problems. On the second day, nearly 190 questions were asked by the students, as the wording of a combinatorics problem turned out to be particularly problematic.

Afterwards, the Leaders and Deputies spent about two days assessing the work of their students, while a team of local mathematicians called Coordinators assessed the papers independently. The two groups then met to decide the final score of individual scripts. Occasional disagreements can occur, since it's often difficult to judge partial mathematical progress on a linear scale and prepare exhaustive marking schemes for every possible creative approach. But due to the excellent preparation of the Coordinators this year, the entire process went by without any major disputes.

Based on individual scores, a total of 281 (49.8%) medals were awarded to students, including 137 (24.3%) Bronze, 90 (16%) Silver and 54 (9.6%) Gold. The medal cuts were set at 28 points for Gold, 22 for Silver and 16 for Bronze. Most Gold medalists solved about four problems, most Silver medalists solved about three, and most Bronze medalists solved two and a bit. Of those who did not get a medal, a further 121 contestants received an Honourable Mention for solving at least one problem perfectly.

There were a couple of outstanding performers worth mentioning. The first is Lisa Sauermann from Germany, who was the only contestant to achieve a perfect score of 42. This caps off an illustrious IMO career for Lisa. With four Gold and one Silver, she is now the most decorated contestant in the history of the competition, at the top of the IMO Hall of Fame. The other is the 13-year-old Peruvian Raúl Arturo Chávez Sarmiento, who solved five questions for a score of 35, placing sixth overall. With one Gold, one Silver and one Bronze so far, the young Raúl has a bright future in front of him, including possibly overtaking Lisa if he chooses to keep coming back.

The Australian IMO team finished equal 25th in the unofficial country rankings, tying with European powerhouses Hungary and Serbia. This is a fantastic result despite the team's inexperience. Three Silver medals and three Bronze medals were gained. With a total of 116 points, the team was also extremely efficient, as a minimum of 114 points are required to achieve this medal haul.

The Australians and their medals at the closing ceremony.
From left: Nancy Fu, Colin Lu, Timothy Large, Declan Gorey, Angel Yu and Yanning Xu.

Silver medals were awarded to: Declan Gorey in Year 12 of Sydney Boys High School, NSW; Timothy Large in Year 12 of Sydney Grammar School, NSW; and Yanning Xu in Year 11 of St Peter's College, SA. Declan, Tim and Yanning all solved three questions perfectly, and achieved 22 points with identical score breakdowns for their Silver medals.

Bronze medals were awarded to: Colin Lu in Year 12 of Melbourne Grammar School, VIC; Angel Yu in Year 12 of Perth Modern School, WA; and Nancy Fu in Year 11 of James Ruse Agricultural High School, NSW. Colin, Angel and Nancy all solved two questions each, and obtained enough part marks to achieve totals of 17, 17 and 16 respectively for their Bronze medals.

The awards were presented at the Closing Ceremony. Various media personnel, sponsor representatives and the Mayor of Amsterdam were there to congratulate the medal winners for their accomplishments. Special recognition was given to Lisa Sauermann for her extraordinary IMO record. She has certainly become a role model for other aspiring girls in their mathematical studies.

Many thanks to members of IMO 2011 Organising Committee, guides, coordinators and many behind-the-scenes staff, crew members and volunteers for putting together a truly wonderful IMO. Also thanks to the staff and volunteers of the Australian Mathematics Trust for their mathematical enrichment activities across all levels which, amongst other things, form the basis to the selection and training of the Australian IMO team.

Visit www.imo2011.nl and www.imo-official.org for detailed results, problems, solutions and other information.

Australian IMO team scores								
Name	Q1	Q2	Q3	Q4	Q5	Q6	Score	Award
Nancy Fu	7	0	0	7	2	0	16	Bronze
Declan Gorey	7	1	0	7	7	0	22	Silver
Timothy Large	7	1	0	7	7	0	22	Silver
Colin Lu	3	0	0	6	7	1	17	Bronze
Yanning Xu	7	1	0	7	7	0	22	Silver
Angel Yu	7	1	1	6	2	0	17	Bronze
Totals	38	4	1	40	32	1	116	
Australian Average	6.3	0.7	0.2	6.7	5.3	0.2	19.3	
IMO Average	5.4	0.7	1.1	4.1	3.3	0.3	14.7	

The medal cuts were set at 28 for Gold, 22 for Silver and 16 for Bronze.

Some unofficial country rankings					
Rank	Country	Score	Rank	Country	Score
1	China	189	15	Ukraine	136
2	USA	184	17	Canada	132
3	Singapore	179	17	U.K.	132
4	Russia	161	19	Italy	129
5	Thailand	160	20	Bulgaria	121
6	Turkey	159	20	Brazil	121
7	North Korea	157	22	Mexico	120
8	Romania	154	23	India	119
8	Taiwan	154	23	Israel	119
10	Iran	151	25	Australia	116
11	Germany	150	25	Hungary	116
12	Japan	147	25	Serbia	116
13	South Korea	144	28	Netherlands	115
14	Hong Kong	138	29	Indonesia	114
15	Poland	136	29	New Zealand	114

Ivan is a PhD student in the School of Mathematics and Statistics at The University of Sydney. His current research involves a mixture of multi-person game theory and option pricing. Ivan spends much of his spare time playing with puzzles of all flavours, as well as Olympiad Mathematics.