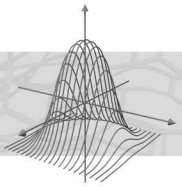

Threshold concepts

ALTC project: A national discipline-specific professional development program for lecturers and tutors in the mathematical sciences

You will hear the word *threshold* a lot in the next year. The government is looking at *Threshold learning outcomes* for a graduate of a science degree in mathematics. These are the minimum outcomes you would expect of a mathematics graduate. More on this in a later edition. Here we want to look at the idea of threshold concepts.

Threshold concepts in mathematics are aspects that transform and integrate ideas. Many people would consider *linearity* as a threshold concept that threads through mathematics learning at all levels. Once linearity is truly understood then common errors such as

$$\sin\alpha \pm \sin\beta = \sin(\alpha \pm \beta)$$

are less likely to occur. Linearity is key to many areas of mathematics such as differential equations; it is met time and time again throughout a mathematics degree.

Another example is that of a complex number. It is conceptually difficult to grasp and almost seems absurd, yet an understanding of complex numbers underpins the solutions to many mathematical problems, even those that, on the surface, appear to involve only real numbers, and has applications in both the pure and applied sciences [2]. Limit is another threshold concept. For instance, the fact that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

may be counter-intuitive when first encountered, but being able to calculate such limits 'is the gateway to mathematical analysis and constitutes a fundamental basis for understanding some of the foundations and application of other branches of mathematics such as differential and integral calculus' [1, p. 2].

Threshold concepts are:

- transformative (they trigger a shift in perception)
- irreversible (they usually cannot be easily unlearned or discarded)
- integrative (they expose previously hidden or unrecognised connections and interrelations)
- bounded (often bordering thresholds into new conceptual spaces—in fact may demarcate disciplines)
- troublesome (appearing complex, alien, counter-intuitive or incoherent) [2].

Think about the threshold concepts in your area of mathematics. What are the key concepts or procedures needed to move to a higher level of mathematical thought? Do you think colleagues in your area would agree to these key ideas?

Next, how do you ensure students learn these key ideas and move to higher levels of mathematics? How do you communicate to students about these threshold concepts? How do you design learning and assessment strategies to develop and test these ideas? We suggest that it is valuable when planning your unit and learning activities to spend some time considering the threshold concepts in your discipline. We suggest that teachers dedicate adequate time and focus to developing students' understanding of threshold concepts. Possible strategies include extra time dedicated to instruction (including provision of applied examples), time for students to experiment with problems, and peer explanation tasks.

There are several projects under way looking at threshold concepts. One is investigating what lecturers state as the threshold concepts in their subject area and then interviewing students to see whether the students articulate the same threshold concepts as the lecturer. Other projects are looking at problems and examination questions that students who understand a particular concept get correct and those who don't understand it are unable to do.

The idea of threshold concepts can assist lecturers and departments make sure that the really important ideas in mathematics are taught and assessed, and that students are ready to move to higher levels of mathematical understanding.

References

[1] Meyer, J. and Land, R. (2003). Threshold concepts and troublesome knowledge: Linkages to ways of thinking and practicing within the disciplines (Occasional Report 4). Edinburgh: Enhancing Teaching-Learning Environments in Undergraduate Course Project.

[2] Meyer, J. and Land, R. (2005). Threshold concepts and troublesome knowledge (2): epistemological considerations and a conceptual framework for teaching and learning. *Higher Education* **49,** 373–388.

This article is adapted from a professional development unit for lecturers and tutors in the mathematical sciences. Threshold concepts are addressed in the learning module on models of mathematics learning. Other topics in the unit include planning and conducting mathematics lessons, teaching in service units, assessing students in classes and units, planning and managing mathematics units, evaluating mathematics teaching and developing mathematics learning communities.

We are currently trialling this unit before launching it publicly on the Australian Mathematical Society website later in 2011. We hope that all new lecturers and tutors in the mathematical and other quantitative sciences will complete this unit as part of their professional development in teaching. We will run a professional development workshop focused on learning and teaching in mathematics on 29–30 September 2011 in conjunction with the University of Wollongong immediately following the annual meeting of the Australian Mathematical Society.*

*www.austms.org.au/ALTC

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