

What is mathematics?

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In the *AustMS Gazette* **32** (2005), 146–147, the question above was included in a letter to the Editor. Readers were asked for their responses in one or two sentences. Below is the set of responses, pseudo-randomly ordered and without comment. Respondents sometimes included extra information about themselves or their perspectives and some of these have been included. The respondents were readers of the *Gazette* who identified themselves as mathematicians or users of mathematics, and a group of people involved in the mathematics education of teachers.

Some more thoughts

Subsequent to the reader submissions, and various matters relating to the current review of mathematics in Australia, the question below has become another question of interest to me to ponder, but not to answer. The question has become pertinent for a number of reasons. This includes the fact that as our numbers become less, or we are distributed amongst other departments, then there is a trend for “non-mathematicians” to be speaking as though they are authorities on the discipline. We also lose our ability to represent our disciplines on academic boards, research committees, appointment committees and so on.

The question is “*What is a mathematician?*”

I have had discussions on this with a few in the context of the “mathematics staff” of a university. A similar question may be asked of “mathematicians” in industry. Who do we count?

- Only staff members of a mathematics department or faculty? What if we are submerged in a Department run by a person from another discipline?
- Staff actively involved in Mathematics research? Does mathematics research

include the application of known mathematics to other disciplines, or is it purely the development of new mathematics, or is it the original blending of known mathematical ideas for use in either mathematics or its application areas? Do we count only those that publish in “mathematics” journals?

- Staff with a UG degree, a masters, a PhD in mathematics? Do we count an engineer with a UG degree in mathematics who uses mathematics in engineering? Do we count the staff who are no longer doing original research, but are involved in the teaching and learning of mathematics?
- Staff of other disciplines which may have some reasonable mathematics content: physicists, chemists, engineers, computer scientists, business studies, ... In this case the total mathematics content (in terms of subjects called mathematics) may be less than the equivalent of a full-time UG year, whilst my own Bachelor of Mathematics contained the equivalent of 2 1/4 full-time years of mathematics. Other discipline staff may extend their mathematical skills by applying mathematics, whilst we might deepen our understanding by our research degrees and subsequent work within the mathematics discipline.

Can this last group of staff be expected to appropriately manage and advise on mathematics in a university department? Issues arise when there are differences in the paradigms of the disciplines. When does one get to the stage of understanding the mathematical paradigm? Have I reached that stage?

A major pragmatic example is the issue of support for attending conferences. Mathematics has very few refereed conferences. Refereed conference papers are common in

many disciplines and often represent the major formal output of staff in those disciplines. The result has been that mathematicians do not get conference funding support “as there is no DEST recognised publication attached”. This is a sorry state of affairs for the mathematicians and their research students.

Issues arise when the narrow mathematics exposure in another discipline hinders the ability of the department heads to make informed judgements about the need for mathematics or mathematicians. How does one deal with “academic leaders” from outside any discipline who know but a small part of the discipline. Consider managers of mathematics who have never heard of convergence, or an Abelian group, or managers of computer science, who have never heard of the notion of NP-hard problems or backtracking algorithms?

Some of the previous sentence is based upon real cases and some are not. The major point is that this is the way that we are tending to regard mathematics (and some other disciplines) in many Australian Universities. The mathematics discipline is often slipping into a situation in which its 2500 year old culturally rich history and evolving modern form is reduced to a set of narrow tools to satisfy increasingly narrow needs in other disciplines, and keeping students in the system!

It seems to me that this is a certain way to continue the strangulation of mathematics in Australia, and the informed structured approach to the world that is embodied in the discipline, and it makes the question “What is a mathematician (and so do we need them)?” much more pertinent.

What is mathematics?

Your replies

“Mathematics is exploring and investigating quantitative inter-relationships and developing a language to communicate findings”.

“Mathematics is a way of ‘looking’ at the world and asking questions about the world that involves, numerical, spatial (for example) concepts as frameworks for ‘looking’ in logical ways that also aim for articulating abstractions and patterns in what we are looking at”.

“Mathematics is a reasoning activity employing abstraction and generalisation to identify, describe and apply patterns and relationships”.

“Mathematics is generalisation”.

“Mathematics is a formalised language”.

“Mathematics is a truncated way of thinking, with its own patterns of procedure, language and ideas. It is a social construct — the answer to your question varies with people’s experience”.

“Pure mathematics is the science of detecting patterns and proving that they hold, with the view to forming a general theory that explains these patterns. Applied mathematics is the science of applying pure mathematical theories to model, predict and utilize both natural and man-made phenomena”.

“Mathematics is ... finding pretty shortcuts”.

“Mathematics is the act of abstracting essential elements and finding rules between them”.

“Mathematics is the symbolic expression of relationships that can parallel any depth that the interpreter possesses”.

“Mathematics is the study of those properties of abstracted natural patterns that can be deduced through formal reasoning”.

based on the barest possible assumptions”.

“Mathematics is the study of pattern, form and structure using a specialised language”.

“Engineering is blessed with an excellent identity. It dates from 1824, when Thomas Tredgold used it at the foundation of British Institution of Civil Engineers. According to him, Engineering is ‘the art of directing the great Sources of Power in Nature for the use and convenience on Man.’ As engineering is ‘actively striving to do something better’ than previously done, it needs mathematics, for ‘better’ is comparative and lends itself to quantitative measures.

Unlike engineering, though, mathematics is not an action. Nor is it active of itself. It is practised, like music. So what is my one sentence definition? I am not poetic, so cannot phrase it better than this: ‘Mathematics is a body of shared thoughts that describe quantitatively the relationships between abstractions, while being conscious of their assumptions.’ It is this final qualifying clause that identifies the distinctive characteristic of mathematical thought. Applying (practising) mathematics, then — what we do as an engineer, a scientist, an accountant, an actuary or simply a parent dividing a birthday cake — is merely establishing some ‘real-world’ correspondences to the abstractions. Mathematics is characterised by rigor, precision and logic. This stern face is both its fascination and its terror”.

“I note, too, that mathematics appears to be unique as a practice, being the discipline wherein some people take pride in having no accomplishment whatsoever”.

“You may be interested, too, in a lay definition (not mine, from ‘teenage’): ‘Mathematicians do hard sums.’ Compare this with ‘accountants do money sums’.”

“Mathematics is the science of numbers and figures in which we believe in, and search for, simplicity; in particular, there is nothing to remember”.

“The first response for me is: does it matter?”

“I find so many mathematicians these days trying to defend mathematics, per se, rather than what they contribute to society, intelligence, knowledge, the world, etc..”.

“Personally, I do go with the answer: ‘The study of abstract structure’.

But then I am a logician by training :-)
Mathematics is a way of looking at the world around us — it is the tint in my glasses that colours the way that I see the world.”

“Mathematics is a way of thinking about the world focussing on pattern and order”.

“I have been disappointed by articles in the Gazette that have been narrowly inward looking. To me mathematics is part of a much wider picture. Just how large a part it plays changes over the centuries. It will certainly not disappear because it is fundamental in so much of what we do.

Over recent years mathematicians have been under threat, certainly. But a better reaction than putting up the shutters is to show how useful we are. I feel that what would benefit mathematics and mathematicians most would be for mathematicians to engage much more with the rest of the world, to help people solve their problems by bringing the tremendous power of mathematics to bear. And in this endeavour I mean all branches of mathematics, pure, applied, financial and statistics. The thought patterns that mathematicians possess far outstrip many (but not all) of those of many other scientists academics and thoughtful people”.

“Background. With a DPhil in mathematics, I am now employed in information technology and currently researching software evolution on the one hand and thirteenth century musicology (with some mathematics) on the other. In the latter, the mix of skills that our team brings to the project makes it an exciting and stimulating one. Is that really mathematics? Does it matter?”

I believe the important thing is using my brain and my skills to advance human knowledge and insight”.

“The self-contained logical and aesthetic study of number, shape, form and pattern which also enables the solution of quantitative problems of science, art and the world and universe generally”.

“Mathematics is the science of structure.”

James Franklin at UNSW has kindly sent me to include the following references for those who wish to consider a philosophical side of the question.

For the “Sydney School in the philosophy of mathematics”: <http://www.maths.unsw.edu.au/~jim/structmath.html>.

Philosophers are sceptical about the value of asking mathematicians “what is mathematics”, which is a question of philosophy, not of mathematics. See the abusive comment at the end of the article in The Australian: <http://web.maths.unsw.edu.au/~jim/highered.html>.

The reference to the following article was sent to me, but I have not seen the article.

[1] A. Reid, P. Petocz, G.H. Smith, L.N. Wood and E. Dortins, *Mathematics students’ conception of mathematics*, NZ J. Math. **32** (2003), 163–172.

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