Professor Bruce Berndt is 2005 Mahler Lecturer

Jim Hill

Professor Bruce Berndt from the Department of Mathematics at the University of Illinois, is the 2005 Mahler Lecturer. Professor Kurt Mahler was one of the major figures in Australian mathematics from his arrival in this country in the 1960's until his death in 1988. The Australian Mathematical Society was a beneficiary under his will, and the funds from this bequest have been used to set up a Visiting Lectureship in his honour.

The Mahler Lectureship is awarded every two years to a distinguished mathematician, who preferably works in an area of mathematics associated with the work of Professor Mahler. It is usually expected that the Lecturer will speak at one of the main Society Conferences, and will visit as many universities as is practical. The Mahler Lecturer is supported by both the Australian Mathematical Society and by the Australian Mathematical Sciences Institute.

Professor Berndt will be one of the keynote speakers at the annual winter meeting of the Society, to be held in Perth September 27th–30th (Director Professor Lyle Noakes). Professor Berndt (http://www.math.uiuc.edu/~berndt/) is very well known for his considerable efforts in establishing the veracity or otherwise of numerous formulae first derived by the Indian mathematician Srinivasa Ramanujan (1887–1920), and he has devoted the larger part of his working life to the mathematics initiated by Ramanujan. Predominantly based at the University of Illinois, he has been the recipient of many awards and Distinguished Professorships in recognition of the excellence and high standing of his work.

Professor Berndt is involved as an Editor of eight international mathematical journals, including the Journal of Mathematical Analysis and its Applications, the Ramanujan Journal and the International Journal of Number Theory. He is the author or co-author of nearly two hundred research papers, and the following nine books, predominantly dealing with the life and mathematical work of Ramanujan, are excellent examples of the high level of erudition which characterizes his work:

- *Ramanujan's Notebooks, Part I–V.*
- *Ramanujan: Letters and Commentary* (with R.A. Rankin).
- *Gauss and Jacobi Sums* (with R.J. Evans and K.S. Williams).
- *Ramanujan: Essays and Surveys* (with R.A. Rankin).
- *Ramanujan's Lost Notebook, Part I* (with G.E. Andrews).

Arrangements for seminars from the Mahler Lecturer

After the meeting in Perth, Professor Berndt will visit Adelaide, Melbourne, Canberra, Wollongong, Sydney and Brisbane, and he is willing and happy to present seminars to individual universities as required. Those interested should contact the appropriate person:

- Perth: Lyle Noakes
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- Adelaide: Bob Clarke
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- Melbourne: Ole Warnaar
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- Wollongong: Jim Hill
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- Sydney: Mike Hirschhorn
  mikeh@unsw.edu.au
- Brisbane: Elizabeth Billington
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Please find below seven possible talks, which Professor Berndt is willing to give. The first talk is entirely suitable for a general audience, and almost all of the lecture would be understood by laypeople. The next three are on the lost notebook, while the fourth deals with Ramanujan's forty identities for the Rogers-Ramanujan functions. Professor Berndt has been working on this with students and former students for over three years, and their goal was to find proofs for all the identities in the spirit of Ramanujan. The last two lectures are more specialized, and are more suitable as seminars rather than general colloquium talks.

Talk 1: Ramanujan's Life and Notebooks

Ramanujan was born in southern India in 1887 and died there in 1920 at the age of 32. He had only one year of college, but his mathematical discoveries, made mostly in isolation, have made him one of this century's most influential mathematicians. An account of Ramanujan's life will be presented. Most of Ramanujan's mathematical discoveries were recorded without proofs in notebooks, and a description and history of these notebooks will be provided. The lecture will be accompanied by overhead transparencies depicting Ramanujan, his home, his school, his notebooks, and those influential in his life, including his mother and wife.

Talk 2: Ramanujan's Lost Notebook

Srinivasa Ramanujan, generally regarded as the greatest mathematician in Indian history, was born in 1887 and died in 1920 at the age of 32. Most of his work was recorded without proofs in notebooks. In the spring of 1976, while searching through papers of the late G. N. Watson at Trinity College, Cambridge, George Andrews found a sheaf of 138 pages of Ramanujan's work. In view of the fame of Ramanujan's "ordinary" notebooks, Andrews naturally called this collection of sheets Ramanujan's "lost notebook". This work, comprising about 650 results with no proofs, arises from the last year of Ramanujan's life and represents some of his deepest work. After a brief history of Ramanujan's life and notebooks, the history and origin of the lost notebook will be given. The remainder of the lecture will be devoted to a survey of some of the most interesting entries in the lost notebook. These include claims in q-series, theta functions, continued fractions, integrals, partitions, and other infinite series.

Talk 3: Ramanujan's Lost Notebook with particular attention to the Rogers–Ramanujan and Enigmatic Continued Fractions

In the spring of 1976, George Andrews visited the library at Trinity College, Cambridge, and found a sheaf of 138 pages containing approximately 650 unproved claims of Ramanujan. In view of the fame of Ramanujan's notebooks, Andrews called his finding "Ramanujan's Lost Notebook". I will provide a history and description of the lost notebook. I will then give a survey on several entries of the lost notebook pertaining to the Rogers-Ramanujan and "enigmatic" continued fractions.

Talk 4: Ramanujan's Contributions to Eisenstein Series Especially in his Lost Notebook

Eisenstein series are the building blocks of modular forms; in particular, every analytic modular form on the full modular group can be represented as a polynomial in two particular Eisenstein series. For Ramanujan, the primary Eisenstein series were, in his notation, $P(q)$, $Q(q)$, and $R(q)$. We provide a survey of many of Ramanujan's discoveries about Eisenstein series; most of the theorems are found in his lost notebook. Some

of the topics examined are formulas for the power series coefficients of certain quotients of Eisenstein series, the role of Eisenstein series in proving congruences for the partition function $p(n)$, representations of Eisenstein series as sums of quotients of Dedekind eta-functions, a family of infinite series represented by polynomials in P, Q, and R, and approximations and exact formulas for π arising from Eisenstein series.

Talk 5: Ramanujan's Forty Identities for the Rogers-Ramanujan Functions

The Rogers-Ramanujan identities are among the most famous identities in combinatorics. Late in his stay in England, Ramanujan derived forty further identities relating the two Rogers-Ramanujan functions at different arguments. Although almost all of the forty identities have now been proved, principally by L.J. Rogers, G.N. Watson, D. Bressoud, and A.J.F. Biagioli, an impenetrable fog still lies over the ideas which led Ramanujan to derive these identities. In the past four years, the speaker and several of his former doctoral students have been attempting to find proofs in the spirit of Ramanujan's mathematics. At this moment, we have proofs of thirty-five of the identities that could have been given by Ramanujan. In this mostly expository talk, the various methods that have been used to prove the forty identities are discussed. It is possible that Ramanujan used asymptotic analysis of the Rogers-Ramanujan functions to discover, but not to prove, the identities. We also describe the ideas behind this approach.

Talk 6: An Unpublished Manuscript of Ramanujan on Infinite Series

Published with Ramanujan's lost notebook is a four-page, previously unpublished, handwritten fragment on infinite series. Partial fraction expansions, the Riemann zeta-function, alternating sums over the odd integers, divisor sums, Bernoulli numbers, and Euler numbers are featured in the formulas in this manuscript. Ramanujan's formula for $\zeta(2n+1)$ is perhaps the most famous formula in the manuscript. The manuscript appears to have been originally intended as part of a published paper. Each proof is sketched in one line. However, not all the results are correct, and not all the proofs, even of correct results, are rigorous. We discuss the formulas and what led Ramanujan to go astray.

Talk 7: Finite Trigonometric Sums and Class Numbers

In a paper coauthored with Liang-Cheng Zhang in 1991, two evaluations of trigonometric sums arose as corollaries of two of Ramanujan's theta function identities. In 2003, Zhi-Guo Liu found some more examples. In joint work with Alexandru Zaharescu, we evaluate several large classes of finite trigonometric sums without any appeal to theta functions. These sums can be regarded as generalizations of Gauss sums. Complex analysis and the classical Gauss sums are the tools used. The previous results mentioned above are special cases. The evaluations are effected in terms of class numbers of imaginary quadratic fields. All the evaluations appear to be new.

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