Mathematics and bodysurfing

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Abstract

Bodysurfing is an art that many people can enjoy, particularly in Australia where the ocean is relatively warm and waves break regularly near a sandy shoreline. In its purest form a bodysurfer catches a wave some distance from the water's edge by swimming onto it, just as it is about to break. The bodysurfer is then propelled by the broken surf front (breaker) towards the shoreline. Rides of 50 to 100 metres are normal for experienced bodysurfers.

This paper will discuss bodysurfing in general and consider simple mathematical models for catching a wave, riding a wave and falling off the wave at the end of the ride.

1 Introduction

No one knows who the first bodysurfers were! Captain Cook noticed South Sea Islanders frolicking in the surf on one of his trips to the Sandwich Islands, now the Hawaiian Islands [3]. Australian aborigines were observed to enjoy the surf near Caves Beach, Newcastle in the 19th century. Tommy Tanna from the Marshall Islands was working in Manly, Sydney in 1889 and began introducing white Australians to the skills of bodysurfing. But such activity was against the law, which at that time said that it was illegal to bathe in waters exposed to views from any wharf, street, public place or dwelling house between the hours of 6a.m. and 7p.m.

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Enter William Gocher, a Manly newspaper editor, in 1902. He announced that he would swim in the ocean at noon on various Sundays. Although arrested, he was told that no charges would be laid as long as he wore neck-to-knee bathers. The law was rescinded in November 1903, and surfing became a popular pastime [4].

Undoubtedly the best early bodysurfers were the Hawaiians in the 18th and 19th centuries. The Australians began at the start of the 20th century followed by the Californians, New Zealanders and South Africans around 1920. My interest in the subject arose because I was taught to bodysurf by my father in 1946, and then became a research mathematician specialising in fluid dynamics. As many readers know, I still compete at the Masters' level in the Australian Surf Life Saving Championships each year, and hence I find a scientific understanding of the skills involved in bodysurfing still helpful and intriguing.

2 Waves on the ocean

Waves breaking near the shore are generated by storms and wind at sea. The waves thus generated eventually move out from the storm area and travel as "ground swell" towards a distant shoreline. The average characteristics of a wave within this ground swell depend on the action time of the storm, the fetch (or distance from the storm centre) and the intensity of the storm or wind. In deep water the water motion set up by these waves as they pass can be modelled by the equations of inviscid, irrotational fluid dynamics.

The governing equation is Laplace's equation

$$\nabla^2\phi = 0$$

which for a 2-dimensional wave travelling at speed U on the surface can be solved by separation of the variables to yield

$$\phi = (U/k)e^{ky}\sin\{k(Ut - x)\}.$$

The $X(= x - Ut)$ axis and the y axis (vertically upwards) travel with the wave, while k is the wave number. Therefore, the water particle velocity components at any point on the surface or beneath the wave are

$$\dot{X} = Ue^{ky}\cos\{k(Ut - x)\}$$

$$\dot{y} = Ue^{ky}\sin\{k(Ut - x)\}.$$

Clearly $\dot{X}$ and $\dot{y}$ decrease rapidly as points deeper and deeper are considered. Thus submarines can dive beneath a disturbed ocean surface to avoid the pitching, rolling and yawing motions caused by surface waves.

When the wind blows on the surface of the ocean at more than 20 knots (approximately 17 ms^{-1}), "white caps" appear. These are generated by the wind altering the symmetrical ground swell into an asymmetrical wave with a steepened slope on the front section of the wave form. Eventually the slope reaches a critical value and the wave breaks. But these surf fronts soon diminish, and the wave reforms as a swell and travels on with reduced energy. It is difficult for a bodysurfer to ride these "white caps", because he or she would have to position themselves way out in the ocean exactly where the "white caps" break and they would also have to accelerate to wave speed. On the other hand, ships or craft have been known to surf the bigger white caps in deep ocean.

3 Waves near the shore

Besides the wind, there is another effect which causes waves to steepen on the front side and eventually break. This is the diminishing depth beneath the wave as the shoreline is approached. The equations governing the motion are still the Navier-Stokes equations for an incompressible, inviscid and irrotational fluid but the boundary condition on the bottom fluid/solid interface must now be included. Essentially, $\partial\phi/\partial n = 0$ on the bottom and the rate at which the bottom profile changes determines the type of breaking wave that occurs.

The analysis of a breaking wave has been carried out by many researchers. An excellent summary is given by Peregrine [8]. When the depth of the water beneath the wave reaches less than half the wavelength, the wave starts to change with its height increasing, its wave speed and wavelength decreasing, and its period remaining the same. Also, the wave crest at the surface gradually assumes a higher speed than the wave trough in front of it. The front or forward slope between this crest and trough becomes increasingly steeper, the crest eventually becomes unstable, and it spills over forming a breaker. Because this occurs near the shoreline, and within a regularly defined region, humans can take advantage of this physical phenomenon and ride the broken surf front (the breaker) to the shore.

The mathematical analysis for the formation of the breaker has only been obtained through an approximation to the Navier-Stokes equations, known as the Boussinesq shallow-water approximation [10, Section 13.11]. The relevant equations are

$$u_t + uu_x + g\eta_x = 0$$

within the wave, while the free-surface (water/air) boundary condition is

$$\eta_t + u\eta_x + \eta u_x = 0.$$

Here u denotes the horizontal velocity component in the x direction, t denotes the time, and $y = \eta(x, t)$ is the unknown wave profile. It was noticed by Stoker [9] that these are the same equations that determine the behaviour of a one-dimensional compressible gas flow with η as the variable density. The breaking of the wave is equivalent to the unstable behaviour of a compressible gas forming shock waves. These shallow-water equations are hyperbolic and solvable by the method of characteristics. One solution is the solitary wave [1]. Although the Boussinesq approximation breaks down as soon as the wave is about to break, this shallow-water theory has nevertheless proved useful in describing the behaviour of breaking ocean waves.

4 Catching a wave

Galvin (1972) [5] describes four types of breakers: spilling, plunging, collapsing and surging. Only two of these are useful for bodysurfing, namely spilling and plunging. For spilling waves, the bottom profile changes gradually and only the neighbourhood of the crest becomes unstable initially. The surf front of foam, bubbles and water starts to tumble down the front face of the sloping wave front. These rollers or gently breaking waves are ideal for bodysurfing, and they frequently occur near the high point of each tide cycle.

Plunging waves (or dumpers) overturn along the whole front of the wave with a jet of water plunging to the toe of the wave and trapping air inside the overturning tube. They occur if the depth changes quickly, particularly near the time for low tide. If refraction of the wave front occurs at the same time (for example, around headlands or along the edge of sandbars), the wave can be ridden obliquely. For surfboard riders, these are the “barrels” or “tubes” that they love to ride, but for bodysurfers, the sideways breaking of the wave along the wave front is usually too fast to enable the bodysurfer to stay level with the break, although swim fins (or flippers) can assist sometimes.

In order to ride the surf front, a bodysurfer has to accelerate up to wave speed, and float within the travelling turbulent surf front of air and water. The surfer’s motion is clearly governed by Newton’s laws. The vertical components have buoyancy balancing the surfer’s weight, while the horizontal motion shorewards has the force generated within the turbulent front and accompanying wave balanced almost by the drag on the surfer. There is a slight decrease in the surf front speed as the depth becomes shallower within the surf zone, but essentially, the surfer is carried along within the surf front at almost constant wave speed.

Beginning bodysurfers find that riding the surf front is not one of the main difficulties in bodysurfing, but catching a wave at the start or staying on the wave near its finish are the more difficult skills to acquire.

The problem in catching a wave is that most rideable waves travel at speeds faster than a person can swim, even faster than the best Olympic swimmer. When the wave has already broken and the surfer can stand in leg-deep water ahead of the wave, he or she can launch themselves up to wave speed by a huge thrust of their legs against the sandy bottom, at the same time propelling themselves into a horizontal position. Timing is important as the window of opportunity for catching the front of the wave is small. Too early, and the speed developed by the leg thrust is lost before the surf front reaches the surfer; too late, and the surfer falls off the back of the wave as it passes.

When the water depth is such that the surfer cannot stand on the bottom, it is almost impossible to catch a broken wave, although a few experienced and strong swimmers have sometimes been able to do this.

More frequently experienced bodysurfers swim out to where the waves are consistently breaking, and attempt to swim onto an unbroken wave just as it is about to break. Mathematically, this position can be determined by stipulating a limiting value on the steepness of the wave front for a train of periodic waves. However, studies have now indicated that it is better to consider each wave crest as an independent entity like a solitary or cnoidal wave rather than as part of a periodic wave train. Grimshaw [6] has provided a theoretical framework for a more general study of these waves.

For a bodysurfer to catch a wave in deep water, it is clear that he or she must be competent enough to accelerate quickly to wave speed. Timing is important once again, so that this must be accomplished just as the wave is passing. Some surfers can do this with one or two strokes only, if they position themselves correctly in the wave-breaking area.

A mathematical quantitative examination of this skill is of interest. For a person swimming in a pool or lake, the forces acting in the vertical direction are just gravity and buoyancy which balance each other, and hence physically explain the swimmer's ability to remain at the surface interface. In the horizontal direction the forces are the propulsive forward force P due to the swimmer's technique and a quadratic drag force.

The governing equation is therefore

$$m\ddot{x} = P - k\dot{x}^2$$

where m is the swimmer's mass, and the resistance coefficient is

$$k = \frac{1}{2}\rho A C_{\mathrm{D}}$$

with ρ as the water density, A the swimmer's area of cross-section normal to the direction of motion, and C_{D} as the drag coefficient.

The maximum speed that can be attained by the swimmer is therefore

$$\dot{x} = \sqrt{P/k}$$

occurring when $\ddot{x} = 0$. This value can only be improved by increasing P (technique training), decreasing A (weight loss) or decreasing C_{D} (special swimming suits).

Consider now a similar one-dimensional model of a wave travelling with speed $U(> \sqrt{P/k})$ towards a surfer. As the wave approaches its breaking point there is an increase in the water particle speeds near the crest and a strong shoreward surging force is experienced within the wave as it passes.

This force rises sharply from zero as the wave approaches, reaches a maximum at the crest, and dies quickly as the back of the wave passes. A simple model for it could be $F\cos[W(x - Ut)]$ where F is the maximum force at the crest on breaking and W is a wave property related to the field of influence $-\frac{\pi}{2} < W(x - Ut) < \frac{\pi}{2}$ of this force about the crest. Outside this region the generated force is zero until the next wave arrives. It is this force which enables the surfer to accelerate up to wave speed.

The dynamics of catching a wave within this limited window of opportunity are based on the forces shown in Figure 1.

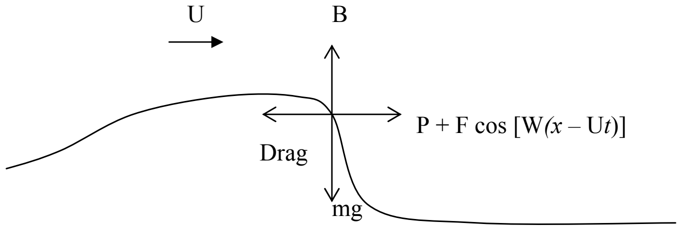

Figure 1

The horizontal forces for this simple model yield

$$m\ddot{x} = F\cos[W(x - Ut)] + P - k(\dot{x} - U)^2 \tag{1}$$

where $(\dot{x} - U)$ is the speed of the swimmer relative to the passing travelling wave. Using the co-ordinate transformation $X = x - Ut$, as before, equation (1) becomes

$$m\ddot{X} = F\cos(WX) + P - k\dot{X}^2.$$

Since $\ddot{X} = \frac{\mathrm{d}}{\mathrm{d}X}(\frac{1}{2}\dot{X}^2)$, this differential equation can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}X}(\dot{X}^2) + \frac{2k}{m}\dot{X}^2 = \frac{2F}{m}\cos(WX) + \frac{2P}{m}. \tag{2}$$

If the surfer starts to swim at the beginning of the field of influence of the wave force then

$$\dot{x} = 0, \quad \dot{X} = -U, \quad \text{when } t = 0, \quad WX = \frac{\pi}{2}. \tag{3}$$

Using the integrating factor $\exp(2kX/m)$, the solution of equation (2) with conditions (3) is

$$\dot{X}^2 = \frac{P}{k} + \frac{2F}{4k^2 + m^2W^2}\, 2k\cos(WX) + mW\sin(WX) \\ + U^2 - \frac{P}{k} - \frac{2FWm}{4k^2 + m^2W^2}\exp\left(\frac{k}{m}\left(\frac{\pi}{W} - 2X\right)\right). \tag{4}$$

Now the surfer will catch the wave when $\dot{X} = 0$ (i.e., $\dot{x} = U$), and this is only possible if there is a solution of the right hand side of equation (4) equal to zero in the range $0 < X < \pi/(2W)$, the front face of the wave.

Typical values for a surfer are $m = 75$ (kg), $k = 15$ (kg m^{-1}), $P = 15$ (N), while, for a wave about to break, typical values are $U = 3$ (ms^{-1}), $W = 2$ (m^{-1}), $F = 1000$ (N).

Equation (4) then particularises to

$$0 = 1 + [2.56\cos 2X + 12.8\sin 2X - 4.82]\exp(0.31 - 0.4X)$$

which has a solution 0.11 lying in the range $0 < X < 0.79$. In this case, the surfer will catch the wave.

Stronger swimmers have a higher value for P than weaker swimmers enabling them to catch some waves that others can't. Swim fins enable all swimmers to raise their P value.

5 Riding in the surf zone

Once a wave has been caught, the surfer is transported through the surf zone. The dynamics of this region have been surveyed by Battjes [2]. Basically, the surf front appears to be quasi-steady shortly after the initial breaking of the wave, with the turbulent surf front dissipating slowly as the wave travels towards the shoreline. The detail of the turbulent front is not well known yet, nor the dissipating mechanisms, but the turbulence seems independent of whether the wave is of the spilling or plunging type.

Of interest to the bodysurfer is how he or she falls off the wave as the surf front diminishes. In particular surfers can stay longer on the wave by kicking their legs, reducing their drag by streamlining the body with arms in a diving position, and by stroking with one arm. As the horizontal depth of the turbulent front decreases, there comes a point where the surfer's legs are no longer being carried along within the wave front even though the upper torso still is. The drag starts to increase dramatically and the wave force $F\cos[W(x - Ut)]$ is diminishing through a reduction in its maximum value F. Hence, the surfer decelerates and falls off the back of the wave.

6 Bodysurfing skills

There are many aspects of bodysurfing that have not yet been researched scientifically.

If an inert floating object, such as a log or floating surf craft, enters the surf zone, it moves with the surf front in a transverse orientation. Humans cannot do this, and they ride waves in the longitudinal orientation. A new toy has been developed which is a small-scale plastic model of a human on a surf mat with a keel at the back. This also rides waves in the longitudinal orientation. But humans do not have a built-in keel, so it is intriguing why they can ride waves in the longitudinal orientation and not turn sideways as all inert floating objects do.

Riding a wave is generally accomplished by bodysurfers with either their arms by their sides and their heads up so that they can see where they are going, or with their arms held in the diving position and their heads down. Variations include hydroplaning, where the arms are straight ahead and the palms of the hand form hydrofoils on the front slope of the wave. Hand boards have been developed to assist bodysurfers to do this.

The legs are usually held in the stiff prone position to enhance the streamline shape of the surfer, but some surfers in earlier days bent one leg at the knee, so that the foot was vertically above the knee. It is thought that this may have assisted the surfer on large turbulent surf fronts, but no evidence is available yet concerning this, and the practice seems to have died out. Additionally, no one yet seems to have investigated bodysurfing with both legs bent at the knees.

Another fascinating aspect of bodysurfing is that waves can be ridden by humans in the longitudinal orientation with their hands at the front (either face down or face up), but not with their feet first. Practical experiments conducted so far indicate that it is difficult to get up to wave speed in this orientation and, of course, the surfer finds it difficult to breathe because his or her face is deep within the turbulent surf front.

A useful skill is to practise riding obliquely across the face of a wave about to break. The lateral speed of the surfer can be increased by using swim fins, and these are used in bodysurfing events in Hawaii and California. The advantage of learning how to slide across the face of a wave is put to practical use when inadvertently catching a dumper (plunging wave). Surfers who try to catch these waves normal to the wave front may suffer severe back or neck injuries when they hit the shallow water in front of the dumper. Sliding across the face of these dumpers shoots the surfer out the back of the wave as it dumps its crest onto the shallow water below. The surfer is trapped inside the tube and suffers no injuries at all.

More advanced bodysurfing skills include "cork-screwing" down the face of a wave, "porpoising" through the face of a wave just before it breaks, and "piggy-backing" with two people on the same wave, usually with the lighter person on top.

There is much still to investigate about the scientific aspects of bodysurfing. The forces within a nearly breaking wave and within a broken surf front need further analysis. The length and depth of the turbulence within the moving surf front also needs closer scrutiny, although Longuet-Higgins and Turner [7] considered a model for this for a spilling breaker.

Of course, bodysurfers don't need to understand the mathematics and physics of bodysurfing to enjoy the thrill of riding a wave shorewards. However, a more detailed knowledge of what is happening can enhance one's ability by developing new skills, becoming more efficient in the various techniques, and perhaps finding new ways to enjoy the surf.

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