**Question**: Prove by Mathematical Induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \geq 1$.

**Inaccurate Student Solution:**

Let $P(n)$ be the statement that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

*Step 1:* Prove $P(n)$ is true for $n = 1$.

Excellent logical communication of ideas

$$\sum_{i=1}^{1} i^2 = 1 = \frac{1(1+1)(2\times 1+1)}{6}.$$ ✓✓

Hence $P(1)$ is true.

*Step 2:* Assume $P(k)$ is true. So assume

Great you have clearly defined assumptions

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}.$$

*Step 3:* Prove $P(k+1)$ is true. So prove

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}.$$

Note $\sum_{i=1}^{k+1} i^2 = 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$

Proof:

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^2 \\
&= \sum_{i=1}^{k} i^2 + (k+1) \\
&= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\
&= \frac{k(k+1)(2k+1) + 6(k+1)}{6} \\
&= \frac{(k+1)(k(2k+1)+6)}{6} \\
&= \frac{(k+1)(2k^2+k+6)}{6} \\
&= \frac{(k+1)(k+2)(2k+3)}{6}
\end{aligned}$$

There is an error here

Great you have used the inductive hypothesis

note the squared term. This term arises from substituting k+1 into $i^2$

Expanding numerators shows these two lines are not equal. Hence you know something is wrong