# Lesson Plan

1. *Select topic and determine the goal of the lesson*

   Topic: Differentiation from first principles; Goal: students will understand the theory behind differentiate from first principles and how to implement this process.

2. *Determine prior learning and skills*

   Students will know the definition of and how to sketch simple functions. They will have worked with linear functions, graphing them and finding their gradient. They will know how to relate the gradient of a straight line to the concept of the change in the value of the function over, say, time.

3. *Decide on student learning outcomes* ***and*** *indicators of student progress*

   At the end of this teaching session student will be able to

   - explain the concept of instantaneous rate of change of a function of one variable,
   - relate this concept to the gradient of a tangent line,
   - derive a formula for the calculation of the instantaneous rate of change of these functions, understanding the importance of the limit that appears there,
   - evaluate the derivative of a given (simple) function from first principles,
   - generalise results to obtain formulae for derivatives of polynomial functions,
   - reason how knowledge of the properties of trigonometric and other functions can be used to obtain their derivatives,
   - apply knowledge and skills to analyse and solve applied problems involving differentiation,
   - understand that every differentiable function is continuous but that the reverse doesn't necessarily hold.

4. *Select and organise resources*

   Resources available include

   - Course outline
   - First year calculus textbook from the library
   - Notes from previous introductory mathematics course
   - Lecture notes from colleague who took course last year
   - Symbolic manipulation packages such as Maple, MATLAB and Scientific Notebook
   - On-line resources
   - Tutors and tutorial sessions
   - First year learning centre

5. *Determine a sequencing for the development of knowledge and skills*

   - Revise notation and basic knowledge of: algebra, a function, the graph of simple functions, the gradient of straight line, the concept of a limit.
     Function notation, if $f(x) = 4x^2 + 2x$ then $f(x+h) = 4(x+h)^2 + 2(x+h)$
     Gradient of a straight line $m = \frac{y_1 - y_2}{x_1 - x_2}$
     Absolute value function
     $$f(x) = |x| = \begin{cases} x & if \quad x \geq 0, \\ -x & if \quad x < 0 \end{cases}$$

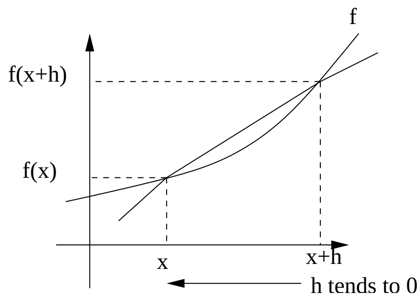
Multiplying by a conjugate

$$\frac{\sqrt{(x+h)+2}-\sqrt{x+2}}{h} = \frac{\sqrt{(x+h)+2}-\sqrt{x+2}}{h} \times \frac{\sqrt{(x+h)+2}+\sqrt{x+2}}{\sqrt{(x+h)+2}+\sqrt{x+2}}$$

$$= \frac{(x+h)+2-(x+2)}{h\sqrt{(x+h)+2}+\sqrt{x+2}}$$

$$= \frac{1}{\sqrt{(x+h)+2}+\sqrt{x+2}}$$

- Calculate rates of change, both average and instantaneous




- Formula for the instantaneous rate of change of a function *ie* the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

- Its relationship to the gradient of the tangent line, so the equation of a tangent line at $x = a$ is given by $y = f'(x)(x-a) + f(a)$
- Implement formula for a range of functions and emphasise the development of skills and problem solving techniques when working through these examples

$$f(x) = 3x^2 + 4$$

$$g(x) = (x+1)(x-2)(x+3)$$

$$f(x) = \sqrt{x}+2$$

$$g(\alpha) = \sin\alpha$$

- Give examples of where the derivative does not exist and explain why. (Diagrams need fixing)

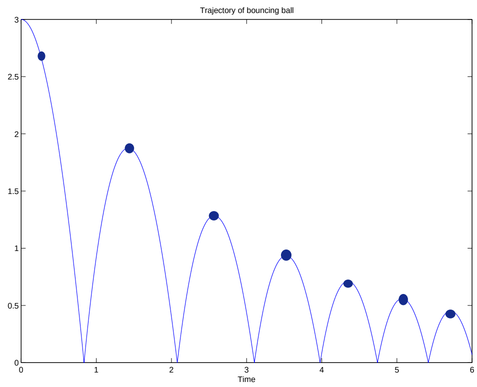
$$f(x) = |x|$$

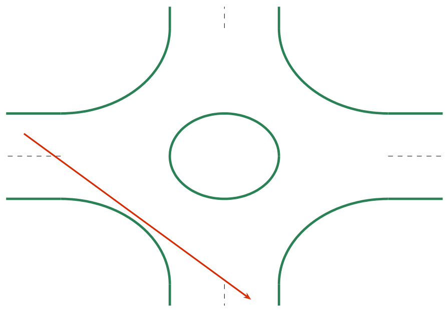
$$f(x) = \begin{cases} x^2+1 & if \quad x \geq 0, \\ x^2-1 & if \quad x < 0 \end{cases}$$

- Applied problems to motivate and contextualise, for instance introducing rates of change of a bouncing a ball, with a discuss the rate of change of the distance against the time, ie the speed. Use graphic showing the height of the ball at any given time. Measuring the slope of

the tangent at a given point provides the rate of change of the height at that point, that is, the speed of the ball. So using the graph we can find instantaneous velocities of the ball.

Alternatively, the arrow in the following diagram shows for a car entering on the left the line of sight to cars on the right at a roundabout. This line of sight is a tangent to the outer curve. Think about those roundabouts you travel through and how hard at times it is to see traffic approaching on your right.



6. *Select appropriate teaching strategies and assessment tasks*

   Develop a framework for the interweaving of student-based activities with lecturer-focused activities to engage the students in the exploration of this topic. Choose exercises for the students so they may practice and consolidate these ideas. Choose insightful examples from these exercises to assess students' understanding of this work.

7. *Reflect on and evaluate the lesson*

   Seek out feedback from students and support staff on the level of student understanding as a result of this activity. Refer to students' written work to gauge their level of understanding.

   For areas of misunderstanding talk to students and support staff about possible approaches to overcome these difficulties.

   Where necessary return to the above activities or provide supplementary activities to help students overcome these difficulties.