

# NOTES ON MATHEMATICAL TYPING

MICHAEL COWLING

Having written many mathematical papers, refereed hundreds of them, and been an editor of a journal, I have seen many reports from many referees. This is a summary of some of the more common errors of mathematical typing which editors and referees comment on, and hence an attempt to raise the quality of mathematical writing. It should be read in parallel with the London Mathematical Society's guide on writing mathematics. An important point is that typing a mathematical paper is not the same as writing lecture notes, and different conventions prevail.

## Notation and definitions

1. Notation is singular in English; it is used as if it meant the “system of notation”.
2. Standard mathematical conventions should be adhered to: e.g., if  $\mu$  is a measure, then  $d\mu$  is a volume element, while if  $f$  is a function, then  $f(x)$  is its value at  $x$ .
3. It is good practice to use the same font for the same kind of object. Thus  $\mathbb{Z}$  (the integers) should be set in the same font as  $\mathbb{R}$  and  $\mathbb{C}$  (the real and complex numbers).
4. Except in standard notation, such as  $\mathbb{R}$  and  $\mathbb{C}$ , mathematical objects of the same type should be labelled by letters from the same general area of the same alphabet. So variables might be  $w, x, y$  and  $z$ , and functions  $f, g, h$ . When objects of one kind are associated with objects of another kind, it helps if the order of the objects corresponds. So if there are three functions and three variables, it is better to write  $f(x), g(y)$  and  $h(z)$  than  $f(y), g(x)$  and  $h(z)$ .
5. The same symbol should always be used for the same object, both in text and in formulae. For instance, if  $G$  stands for a group, always write  $G$  (typed `$G$` in the `TEX` file) and not just `G` (typed `G`) in text.
6. Definitions should not appear as part of the statement of a theorem, but before the theorem. Key definitions should appear in the introduction.

## Numbering

7. When numbering theorems, propositions, corollaries, etc., use a common numbering system for all of them. It is hard to find Theorem 3.1 when it follows Corollary 3.2.

## Writing mathematics

8. Display very long formulae (more than 3 centimetres in length), particularly any which appear on more than one line.
9. Insert a small space (type `\,`) before  $dx$  in integrals. Some would like us to use a roman letter `d` in integrals, and a roman `e` in exponentials. So they write

$$\int e^x dx = e^x + C$$

(from the source `\int {\rm e}^x \, {\rm d}x = {\rm e}^x + C`).

10. Users of  $\text{\LaTeX}$  should use  $\text{\LaTeX}2\epsilon$ , which has eliminated a number of problems in the earlier version (just start the file with the `\documentclass` command rather than `\documentstyle`). In particular, the old environment `eqnarray`, which puts unnecessary spaces, should be replaced by `align` in the `amsmath` package, as this puts spaces of the correct sizes.
11. If possible, use an editor which matches left and right parentheses.
12. Mathematical abbreviations such as `max` (for maximum), `dim` (for dimension) or `deg` (for degree) should be set in roman type, to distinguish them from products of letters (for example, to differentiate the identity `id` and the Gaussian integer `id`). Some of these are already defined in  $\text{\TeX}$ : just type `\max`, `\dim`, or `\deg`. If the abbreviation is not already defined, then write, for instance,

`\newcommand{\vol}{\operatorname{vol}}` in  $\mathcal{AMS}\text{\LaTeX}$ , and  
`\def\Ad{\mathop{\rm Ad}}` in  $\text{\TeX}$ .

Do not use `\hbox`, as this does not shrink when needed and comes out in the wrong font in proclamations; `\text` has the latter problem.

13. Use “log” for logarithm, not “ln”; mathematicians don’t use logarithms to base 10 anymore. Expressions such as `SL`, `Sp` and `SU` for groups are abbreviations, and should be set in roman font. Use `C` or other letters for constants, not *const*.
14. In set notation, for “such that”, use a colon (type `:`) or a vertical line (type `\mid` to get correct spacing), not a solidus (`/`) (which looks like a division) or a semicolon (`;`) (which does not space correctly). Those who use `|` for absolute values or for the measure of a set, should use a colon—it is hard to read  $\{|x||f(x)| > \lambda\}$  (meaning the measure of the set of  $x$  such that the absolute value of  $f(x)$  is greater than  $\lambda$ ).
15. It is problematic to use `]` to open and `[` to close intervals. In  $\text{\TeX}$ , write `\left]` and `\right[` to get the right spacing. Alternately, one can simply use `(` and `)`, which are more standard in English, and more elegant.
16. A related error in  $\text{\TeX}$  is the use of `<` and `>` as brackets, e.g., for inner products.  $\text{\TeX}$  provides the commands `\langle` and `\rangle` for these; their shape is a little different. Compare the spacings when the right (on the left) and wrong (on the right) forms are used:

$$\alpha = \langle \beta, \gamma \rangle = \delta \quad \text{and} \quad \alpha = < \beta, \gamma > = \delta.$$

If you really must type `<` and `>`, you should later edit the file, changing `<` and `>` to `\<` and `\>` or `\left\<` and `\right\>` (where appropriate). The latter form grows when needed, but if you use it, you must match each `\left\<` with a `\right\>` on every line of output.

17. Include quantifiers in formulae. Tell your reader whether you mean “for all” or “for some”. The word “for” by itself is ambiguous, so not a good quantifier.

### Mathematics in the text

18. It is better to avoid fractions using the `\frac` or `\over` commands in text. Thus  $\pi/2$  is easier to read than  $\frac{\pi}{2}$ . Similar comments apply to indices (both in text and displayed), so, for example,  $\mathcal{H}_t^{\delta/\epsilon}$  is more legible than  $\mathcal{H}_t^{\frac{\delta}{\epsilon}}$ .
19. Small numbers in the text should be written in words, e.g., write “three-dimensional” or “four vector fields”.
20. Write  $(2n + 1)$ -dimensional, not  $2n + 1$ -dimensional.

### Writing text

21. Check the punctuation (the full stops, also known as periods (.), commas (,) and so forth). In English, there is always space after punctuation, but not before (except for the em-dash, which need not concern us here). After all punctuation marks except the full stop, exclamation mark and question mark, we normally start the next word with a lower case letter. It is not usual to start a new line after a full stop unless there is a change of topic. It is important to add full stops at the end of displayed formulae which are the ends of sentences, and commas where appropriate, that is, where there is a natural pause in reading the text.
22. Accents in  $\text{\TeX}$  are different for text and for formulae. Type  $\text{\H"ormander}$  and not  $\text{\H$\ddot{o}rmander}$ , for instance, to get Hörmander and not Hörmänder.
23. Do not use “implies” signs to replace the word “implies” in text. Write, for instance, “To show that (i) implies (ii), we first ...”. The symbol “ $>$ ” is read “is greater than”. It is therefore ungrammatical to write “let  $\epsilon > 0$  be small enough”, and better to write “let  $\epsilon$  be a sufficiently small positive quantity”. It is legitimate to write “suppose that  $\epsilon > 0$ ”.

### Hyphens and en-dashes

24. It is better to write “nondegenerate” without a hyphen (it is standard to write nonsense, rather than non-sense). Similar remarks apply to other words starting with “non-”. The underlying rule is that hyphens are used to join two words, but when the two parts are not words in their own right, then one of them is a prefix or suffix and a hyphen is not needed. It is not really clear whether one should write overruled or over-ruled, and expressions like non-Hausdorff are often used. If you write “three-dimensional”, then logically you should also write “one-parameter” and “finite-dimensional”. But you may omit all these hyphens if you wish, provided you are consistent. Cambridge University Press, which has been around much longer than I or  $\text{\TeX}$ , traditionally did not use hyphens in expressions like “ $x$  axis”.
25. When linking names with a hyphen, use an en-dash to join names of different people and a hyphen for just one author with a double name: so one should write Hahn–Banach (type  $\text{\Hahn--Banach}$ ) and Harish-Chandra. This means that the reader can deduce that the Birch–Swinnerton-Dyer conjectures have two authors, not one or three, even if one knows no mathematics or mathematicians at all.

### Abbreviations

26. Expand out all abbreviations such as iff (if and only if), iid (independent, identically distributed), wot (weak operator topology), wrt (with respect to) and masa (maximal abelian subalgebra). This makes it much easier for the reader, and for the translator when works are being translated. Abbreviations also change with time. The same holds for “i.e.”, “e.g.”, and “cf.”, which are better as “that is”, “for instance” or “for example”, and “see” (or “contrast with”).
27. Do not use the abbreviation “resp.” for “respectively”. And only use “respectively” in a format like “A set is said to be open (closed) if ... (respectively, ...)”. The word “respectively” is much overworked, and almost always redundant.
28. Do not use “let’s”. Use either “let us” or, more simply, “we”.

### For those whose first language is not English

29. Ask a native speaker or near-native speaker to check your writing. And use a spell-checker. Most computers and most word processors come equipped with spell-checkers, e.g., `spell` or `ispell` in UNIX (`ispell` is good for checking  $\text{\TeX}$ , as it ignores commands), or the standard checker in MS-Word.
30. In English, we use paragraphs to gather together discrete parts of an argument or discussion.
31. Articles (“the” and “a” or “an”) are a source of confusion. The definite article “the” is used with a specified thing, while the indefinite article is used when the thing is not specified. So we write “by a theorem of Banach” (when we don’t know which theorem is meant) but “by the Banach–Tarski theorem” (as there is only one of these). “The Banach–Tarski theorem” is the same as “the theorem of Banach and Tarski”.
32. Another source of confusion is that we sometimes write, for instance, “by Plancherel’s formula” and at other times we write “by the Plancherel formula”. In the first case, Plancherel is a noun, and if we were to put “the” in front, it would go with Plancherel. We do not use articles with proper nouns (that is, the names of people, places, . . .). In the second case, Plancherel is being used as an adjective—in this case, “the” goes with “formula”; this is appropriate as the formula is specified. In short, if you use the possessive form of the name (with the “apostrophe s”), then do not use “the”, and if you use the adjectival form (without the “apostrophe s”) then use “the”. It is not correct to use the possessive form when there are two or more names: “the Atiyah–Singer Index Theorem” should not be written as “Atiyah–Singer’s Index theorem”.
33. Do not write “by Cauchy–Schwarz” but “by the Cauchy–Schwarz inequality”. And, by the way, this Schwarz has no “t”.
34. Another problem is when one should use upper case or lower case. In English, we usually write “Section 2” or “Theorem 5” or “the Law of the Iterated Logarithm”, but “the next section” or “the next theorem” or “the following corollary”.
35. “Above” and “below” are not adjectives. For instance, you should say, “the property below”, not “the below property”.
36. In English, one writes “neither A nor B”, not “nor A nor B”.
37. “The sequel” refers to another paper, yet to come, not the rest of the present paper.

### References and citations

38. Ideally, initials are used at the first mention of a name, and thereafter only where needed to distinguish between authors with the same surname. If you use initials, type, for example, `N.~Bourbaki` to get a nonbreaking space between the initial and the name. You should also do this with page numbers and references to theorems, etc., for example, `p.~11` or `Corollary~1`. Write, for example, “[1, p. 11]”, not “[1], p. 11”. In  $\text{\LaTeX}$ , this can be achieved by writing `\cite[p.~11]{ref1}`, where `ref1` is the abbreviation for the first reference.
39. Use standard abbreviations (see, e.g., *Math. Reviews*) for journals. This means that your work can be used in searchable databases. It is not clear whether the abbreviation “H. M. J.” means the *Houston Math. Journal*, the *Hiroshima Math. Journal* or the *Hokkaido Math. Journal*. And to get correct spacing, type a backslash after the full stops which indicate abbreviations, thus: `Hokkaido Math.\ Journal`.

40. In many systems for presenting references, book titles and journal names should be in italic font. In standard English, book titles use upper case letters for nouns, verbs, adjectives, and adverbs. Thus, we write *Differential Geometry, Lie Groups, and Symmetric Spaces*. For titles in other languages, follow the usage for that language. For books, give the details, including the publisher, and place and year of publication.
41. The ranges of pages in the papers cited should be specified with en-dashes (type, e.g., 409--419).
42. Quotations should be enclosed in quotation marks, or “inverted commas” (as here). These also come in ‘single versions’ (as here). The “open quotation” mark is different from the “close quotation” mark: the former uses the “grave accent” key (often located at the top left of an English keyboard) and the latter uses the regular apostrophe key (often located near the “Enter” key on English keyboards). These are ASCII symbols 96 and 39 respectively, and can also be typeset using the T<sub>E</sub>X commands `\lq` and `\rq`.