Flipping the maths tutorial: A tale of n departments

Katherine A. Seaton* Deborah M. King,** and Carolyn E. Sandison***

Abstract

The *flipped classroom* is attracting much attention recently as an innovation in tertiary teaching. In this article we describe a mathematics tutorial style which, while not new, is still novel. This style of class truly flips the chalk-and-talk responsibilities in a tutorial and replaces the 'sage on the stage' with a 'guide on the side'. Its sustained use and its adoption in a number of Australian university mathematics departments, which we discuss, speaks to its effectiveness.

Introduction

In this *Gazette*, in a 1990 report on a Survey of [Australian] Entry Level Courses, Mack commented that

> On the tutorial side, our students are given tutorial work well in advance and yet, regularly, fewer than 50% turn up to tutorials having prepared anything, thus splitting the group in two. There is merit in trying the La Trobe method of handing out problems at the beginning of each class and insisting that all work during the class.
>
> Mack [9]

The 'La Trobe method' referred to, of which active tutorials and practice classes are only one aspect, had recently been described [7] in the DEET *Discipline Review of Mathematics and Science Teacher Education* under the heading 'Examples of Good Practice'. Though the description, written by the key figure in the establishment of the La Trobe method [3], Arthur Jones, was written in 1989, in it he mentions that the system of teaching had been in use there since the 1970s.[1]

In this article, we discuss the adoption and adaptation of this tutorial style by $n \geqslant 3$ university departments, and describe the history of how this came about. We also discuss the strengths of this method under a number of headings, and give some differing perspectives. There is a rather large number of direct quotes given

*Department of Mathematics and Statistics, La Trobe University, Victoria 3086.
Email: K.Seaton@latrobe.edu.au
**Department of Mathematics and Statistics, University of Melbourne, Victoria 3010.
Email: dmking@unimelb.edu.au
***Faculty of Science, University of Wollongong, NSW 2522. Email: sandison@uow.edu.au

[1] The dates are pinned down even more precisely by Don Taylor, who worked at La Trobe only from 1972 to 1974 and recalls them clearly.

in this article, as some of the sources referred to (e.g. [7], [10], [15]) are not readily accessible.

Black-and-white boardrooms

Think for a moment of what you envisage when you hear someone mention a 'tutorial' in mathematics. You will probably find that what you experienced as a student or currently do yourself (or have done by casual or Level A staff) is what you think a normal/standard/traditional tutorial to be. Talking to colleagues from various Australian universities (and having experienced and delivered tutorials in several of them), we know there are various models in operation:

- a class in which the tutor does on a board exercises related to recent lectures, preparing students to do further exercises at home or for an upcoming assignment;
- a class for which students are expected to have already done exercises related to recent lectures (at home), and the tutor asks which of these the group wishes to see done for them on the board, possibly calling for suggestions as to how to proceed from the floor;
- a class to which students come prepared to be called on, to do [part of] a problem on the board for their peers;
- a class wherein the tutor discusses common errors in an assignment, which has already been completed and marked.

The furniture in the room may be chairs with writing arms, or chairs and tables, facing the board. The exercises may be a list of problems from a text book, or may have been set by the lecturer on a handout (or these days, available in electronic form in an LMS and read from a tablet in class).

The aspect of a 'La Trobe method' tutorial highlighted by Mack above is that its success does not depend on students arriving having already worked through problems, prepared to ask their tutor to sort out points of difficulty. Where students either have not done this preparation or encountered no problems, such a tutorial is a passive experience for them, akin to a mini-lecture [12]. Rather, the problem sheets are handed out at the start of the class, and students work on them there.

But there are certainly models of mathematics tutorials, often called practice classes, which share this feature.

- Students work on problems handed out at the class, on their own, and raise their hand if they want assistance. The tutor may either be run off their feet, or field no questions at all.
- Students work on problems handed out at the class, in groups around tables, with the tutor circulating and 'dropping in' on groups. A spokesperson (or the group) may later be called on to present their work to the whole class.

This first style of active learning class can still be confronting to weaker students, who will frequently not ask for help if no-one else seems to need it, with cultural or gender norms compounding this reluctance [12].

So what is distinctive about the 'La Trobe method' tutorial?

What is *not* mentioned in [9] is that it takes place in a special style of room:

> The rooms in which tutorials are held are quite different from the usual ones in that there are no seats for the students to sit on. Instead they work at blackboards which line the walls of the room.
>
> Jones [7]

Another feature of such rooms, which perhaps is not so striking as it is an absence rather than a presence, is that there is no focal point or 'front' to the room. Rooms without rows, platform or single focus have recently received much attention in the *learning spaces* literature (see, for example, [14]).

The third feature of the tutorials is that the style is coherent within (and between) subjects [7]. Contrast this, if you will, with the alternative: that each tutor chooses how to run their tutorial and which examples to work through, so that students in the same subject can have quite different tutorial experiences. The focus on active learning is described by Jones as a thought-out joint response to disappointing student performance; again, contrast this positive pedagogical reform with the tendency to blame the students or their schools, which does not help the students that one currently has.

In the early 1990s two former La Trobe mathematicians, Sid Morris and Phil Broadbridge, arrived at the University of Wollongong within months of each other. Quite quickly the 'La Trobe method' tutorials were adopted under their influence.

A 1992 piece in the *Campus Review Weekly* [10] shows a photograph of Sid amidst students in front of a row of blackboards, and he further develops the ideas mentioned in that interview in [12].[2]

Peter Forrester moved from La Trobe to the University of Melbourne in 1994, and introduced the blackboard tutorials in one subject in 2001.[3] This had been in response to poor student results in that subject and it yielded immediate improvements. However, it was not clear that the 'La Trobe method' tutorial would spread beyond this subject. In 2002, Deborah King, who had studied at La Trobe, also took up a position at Melbourne, and championed by both Peter and Deb, the style became established, as she will describe below.

From now on, we will refer to the tutorials as 'board-tutorials', since no one institution owns them, and the boards used may be black or white.

Variations on a theme

In this section, we describe the development and application of the board-tutorial at each of three institutions and beyond.

La Trobe University

A student perspective. Deborah King writes: 'My first experience of board-tutorials was in 1985 when I arrived at La Trobe as a mature-aged student. At that time it seemed perfectly natural to me. I studied maths and history and to me the board-tutorial was the obvious analogue of a humanities tutorial; that is, you did some work prior to the class, and came ready to participate actively, sorting out any difficulties along the way.

'Observing the board-style tutorial from the perspective of an older student, I felt that the board-tutorial had real advantages over what I remembered a school class to be. For example, students were required to work in pairs. This simple activity has a variety of benefits ranging from making friends to helping students to learn by explaining to others or learning from other students, exposing students to many ways of thinking about a problem. It developed a sense of identity within the group and with the tutor; we were all on a path together.

'Another key benefit I observed as a student, was that each member of the class had the opportunity to interact with the tutor in every class. So the loudest, brightest or weakest student was never the main focus of attention. Each student was able to discuss with the tutor the particular difficulties or points of interest they had with the work.

[2] An up-dated version of [12] has appeared as [13].
[3] In this he was aided by Penny Wightwick, a tutor at Melbourne, who had been a student at La Trobe in the seventies.

'As a student it kept me on my toes. You couldn't go into a class unprepared! The tutor would generally give you enough help so you could move forward rather than show a complete solution, so there was always an element of discovery. Full solutions given at the end of class were valuable resources, since they modelled an acceptable answer including rigour and setting out, so you had a clear idea of the level of detail required for a good solution.

'As a student I loved the board-tutorials, I looked forward to them, I couldn't wait to be a tutor myself, I thought they *were* tutorials and didn't know there other ways of running tutorials. I was wrong.'

The current situation. In the Department of Mathematics and Statistics at La Trobe, the commitment to students spending 50% or more of their class time in active learning activities has been maintained, though the teaching model described by Jones [7] in the eighties and recognised as good practice within the wider University context [15] in the nineties, has seen modifications in the twenty-first century, in response to resource issues and a push to reduce face-to-face teaching hours. One result of University-wide curriculum renewal (*Design for Learning*) has been the heightened awareness that the Department should take primary responsibility for developing graduate capabilities, such as spoken and verbal communication and teamwork, in the students who are taking a mathematics or statistics major. Whereas board-tutorials were previously utilised across all first and second year mathematics subjects, including service-taught subjects, they are currently a unique and key feature of the second year subjects which lead to the capstone third year subjects in the major. Recently refurbished rooms within the mathematics building (computer laboratories and a small lecture room) include walls of whiteboards; the largest original blackboard room remains. (To misquote Mark Twain, the report of their demise [2] was greatly exaggerated!)

One of the points emphasised in the report [15] is the regular involvement of lecturers in the active learning classes, be they practice classes or board-tutorials for their subject, and in the production and refinement of well-crafted question sheets. Lecturers can reflect on the clarity and effectiveness of their lectures and teaching materials [7], and stay in touch with the attitudes, backgrounds and abilities of the current generation of students; that is, learning happens in both directions. Using blackboards (or whiteboards) invites the possibility for students to accept changes suggested either by their partners or by the tutor, fixing errors and false starts without

the crossing out that comes when work is 'corrected' on paper, hence producing a polished final product.

University of Melbourne

Deborah King writes again, now from the perspective of a teacher: 'I spent a number of years at LaTrobe as a casual tutor while I was studying and later during my PhD. So by the time I came to teach at The University of Melbourne, I was well experienced in the art of the board. Being on the other side of the chalk had probably increased my enthusiasm for this interactive style of tutorial. As a teacher it held enormous advantages for me. First up, I could see what everyone was doing; who was working, who was not, who was flying, who was stuck. I could tailor my explanations to where each student was at, going very slowly and simply for some, throwing out challenges and extension opportunities to others, allowing students to develop their communication skills by explaining ideas to their peers and to me. I could work with a student to further their approach to a solution, rather than giving my own and expecting them to regurgitate it. It is amazing how many approaches to a problem you see ... many you wouldn't have thought of yourself! If the whole class just didn't get something, I could feed this back to the lecturer, a really important link in the feedback chain.

'When I arrived at Melbourne in 2002, I realised that not all tutorials were board-style. Though Peter Forrester had introduced them in one subject, they had not spread. A shortage of appropriately equipped rooms was one difficulty, and it appeared that the benefits of this style were not apparent to all staff, so there was a reluctance to take it further.

'I was lucky to be the coordinator of tutorials in a second year operations research subject, so of course, I was quick to run these classes as board-style, which meant that I needed to ensure that the classes were timetabled in appropriate spaces. As the popularity of these classes caught on, rooms were refurbished with boards where possible. Over time we have equipped more and more rooms with whiteboards and now most of our tutorials run as board-style. When spaces are not available, the tutorials run along similar lines (that is the problems are given to students when they arrive, they work in pairs or groups at tables and get solutions when they leave) but it really isn't quite the same, unfortunately.

'The change I noticed in moving from traditional tutorials to board-style was significant. The traditional classes I taught here tended to be very flat. Students often came to class ill-prepared. They may have missed lectures, or not have attempted any problems themselves, so didn't have an idea of what their problems were or what they needed to ask. Tutes ran by students nominating questions they had trouble with, or the tutor could nominate questions, to be worked through at the front of the class and students would copy down solutions. This is very hit-and-miss for students, and tends to aim at the middle band. Students who really struggle don't ask questions in this setting, and students who have mastered the basic material are bored. This tended to result in poor attendance and a lack of engagement. However, in board-tutorials, the atmosphere is different. Students

generally come to class ready to work, attendance is higher, students start to form friendships quickly, it gets really noisy, the engagement and enthusiasm is palpable. But perhaps the most important outcome is that every student in the class gets something out of it; no matter what their level, by the end of the class they have progressed.

'I am painting a rosy picture here, it's true, so by way of balance let me point out some difficulties. Not all students feel immediately comfortable with board-tutorials. We have some very shy students, students whose first language is not English, and students who don't like to make mistakes in public. For these reasons, it takes some sensitivity on the part of the tutor, to make students feel 'safe'; that is, to feel there is nothing wrong with making a mistake. Some students complain that "they are doing all the work", a comment I have always found amusing, since that's the whole point. Initially some staff (including casual tutors and graduate students who had been undergraduates pre-board-tutorials) thought that board-style tutorials would be easy for them: "I won't have to do anything". After a few weeks they told me how exhausted they were . . . it is hard work as a tutor bouncing backwards and forwards through different levels of material. Tutors do not have the same control, since all the questions are driven by students, not pre-prepared by the tutor.

'Some years down the track my feeling is that the board-tutorials are well established now. The feedback from students is overwhelmingly positive and has helped to establish and spread these tutorials [at Melbourne University]; that is, they would often comment when going on to a subject without board-tutorials, that the classes had been less effective and that they wished that subject had board-tutorials. The next generation of graduate students, who have been students of board-tutorials, take to it like ducks to water. It will never please all students or all staff, but as I walk through the department and see the happy and enthusiastic students engaged in mathematics, I can't help but smile.'

University of Wollongong

Caz Sandison writes: 'It was 1991–1992 when the then Department of Mathematics at the University of Wollongong was introduced to board-tutorials. Professor Sid Morris was recently appointed as the foundation Dean of the newly formed Faculty of Informatics — to which the Department of Mathematics belonged — and Phil Broadbridge became Professor in Applied Mathematics. Both had come from La Trobe and brought with them the concept of board-tutorials. The concept met with a little resistance from the University as it raised the contentious issue of 'space'. However, as Dean of Informatics, Sid was able to arrange the installation of the first board-tutorial room at Wollongong. Granted, it was at the far end of campus, a distance from most teaching spaces, in a building normally rented to external companies, but we had our room. It was air-conditioned (a rare commodity at that time), a necessity with the design of a room with blackboards on all four walls and no windows, and the only other furniture in the room was a collection of tall stools. The lack of tables was noticeable.

'There was some skepticism amongst academic staff about the board-tutorials — although it was hard to tell whether this was towards the board-tutorial per se or as a result of the concurrent introduction of the policy that lecturers were now required to take a tutorial in their subject. One colleague who was around at the time of introduction comments now, "At the beginning, I thought that it was a fuss. I think now the whiteboard rooms are good for tutorials of larger first and second year classes."

'Board-tutorials were first used in 1992 for two large first year classes. In both subjects, upon arrival to tutorials, students were given a sheet of problems to attempt at the boards — either on their own or in pairs — and at the end, students were given a solution sheet, because it was recognised that students could not take their worked solutions home with them. Students were still encouraged to work on problems and exercises outside class time but the tutorial classes took on more structure. Instructions to tutors were to answer any questions students had on arrival to class, then simply ensure that students worked on the tutorial problems at the boards, putting their name at the top, and not erasing any working until it had been checked. Tutors could choose to ask students to work in pairs or on their own. Further advice to tutors was to only do a 'class presentation' if it seemed the majority of those in the room were having difficulty with a particular concept or question.

'What a difference! Where tutorials had once been quiet, passive affairs, they were now full of animated, engaged learners and teachers. Previously, students mostly worked individually (if they worked) and the tutor responded to questions (if there were any); some tutorials were simply the time to submit and return assignments. With the introduction of board-tutorials, students interacted with the subject material, collaborated with other students, and interacted with the tutor; tutors could identify weaknesses and clarify misconceptions with ease, without the student having to take the first step. I remember coming out of my first board-tutorial exhausted and hoarse, yet hyped and with a sense of having genuinely helped students in my class.

'In those early days, not all tutorial classes were board-tutorials as we only had the one room. However, the use of board-tutorials influenced how other tutorials operated as all tutors had at least one tutorial in the board room and they took what they could from that experience to a standard classroom. With a worksheet for each class, students had direction in what they were to complete in the class. Tutors asked students to work in pairs and in some cases, asked students to take turns writing on the one board in the room.

'Within one semester, board-tutorials became a permanent fixture of first year Maths teaching at Wollongong. By 1994, we had negotiated to transform a second room on campus into a board-tutorial room — with the compromise that the standard tables and chairs remain so the room could be used by other groups. This allowed for other maths subjects to adopt them as well. Over the years, the rooms have been relocated around campus, whiteboards have replaced blackboards, and while we believe a square room with no tables equipped with boards on all walls is

the best configuration, we would rather a rectangular room with tables and chairs but with boards on all walls than a regular classroom.

'Today at UOW, board-tutorials are still used comprehensively across our suite of standard first year calculus subjects. Essentially, the format introduced more than twenty years ago continues, with the only noticeable difference being that worked solutions are no longer handed to students, rather posted on the subject website. However, we are now beginning to see the effect of personal digital devices such as iPads and iPhones: many students now take photos of their board work before leaving class.

'The general opinion about board-tutorials among staff at UOW is a positive one, with comments such as "I would be very disappointed if tutorials were in rooms without whiteboards (unless they had blackboards!)", "[The format] gets students up and about, encourages social learning, discourages anti-learning activity", "[T]he first time I saw them in action was here in 2007. The feeling was one of 'where have you been all my life?!' ", "It provides a good environment for the tutor to address problems as necessary", "It was a new idea to me when I arrived in 2003... I don't think I'll ever leave Wollongong... but if I did then I would enthusiastically take the idea of whiteboard rooms with me!" '

The second degree of separation

So far, we have told human interest stories of how former La Trobe staff have spread the board-tutorials to two other universities. But they continue to spread, and we see them now rippling out from UOW and Melbourne to other places, one step removed from La Trobe.

Judy-anne Osborn, who was a graduate student and casual tutor at the University of Melbourne, has been at the University of Newcastle since 2011. Board-tutorials started there in 2012.[4] The previous tutorials had been based on text-book problems, so the writing of customised question sheets was required, in addition to the equipping of rooms with whiteboards. This was possible due to money being available (and needing to be spent) at just the right time. One refinement practised at Newcastle is that each member of a group must use a different colour marker, so their respective contributions are evident; free-loading or 'passenger' behaviour is one thing students and staff distrust in team-work situations.

Most recently, a UOW Master of Mathematics student, who is originally from the UK but works in the Caribbean, is doing some tutoring for a first year subject which uses the board-tutorials. He is so impressed with the style of classes that he intends on introducing the style into his teaching (at the A-level/first year level) when he returns. Sometime Head of Maths, he wants to roll it out as their standard

[4]Interview with Judy-anne Osborn by KS in December 2013.

way of teaching maths (and other generic skills). In a similar vein, school teachers who teach the University of Melbourne extension program in their schools are inducted into using the board-tutorials; some describe it as 'the best PD I have ever done'.

Finally, we have been able to spread the word formally and informally through workshops for OLT projects, and conferences such as ACSME and Delta 2013. We were amused by the email from one colleague, asking how many linear metres of board space each pair of students would need.[5] Another practicality is to consider the vertical height of the boards; having them lower on some walls than others can be useful for the taller and shorter students. With the introduction of board-tutorials, in the past there may have been an increase in printing costs and some complaints about chalk dust or the smell of markers (yes, really). However, these issues are outweighed in spectacular fashion by the incredible learning advantages of having board-tutorials and costs ameliorated by providing sheets electronically.

Benefits

Despite the variations outlined above, there are underlying, common pedagogical benefits from the board tutorials. While we intend to explore some of them further in future publications, we describe them briefly under a number of headings.

Active learning. Halmos, in the article from which the first line only is often quoted,[6] uses the analogy:

> For a student of mathematics to hear someone talk about mathematics does hardly any more good than for a student of swimming to hear someone talk about swimming.
>
> Halmos, Moise and Piranian [5]

The board-tutorials require students to be actively engaged in discussing, solving and writing solutions to mathematical problems, learning in Halmos's best way, with support and guidance. The term 'built pedagogy' has been used [11] to refer to the way in which educational philosophy is embodied in architecture. The board-rooms support [socio]constructivism both through their decentredness [4] and their shared writable surfaces [18]. They are 'shaped by learning rather than instruction' [14]. On the other hand, the traditional classroom configuration and the unflipped tutor-led tutorial suggest, perhaps unintentionally, an instructivist or transmission model of teaching.

[5] It's not a silly question; 1.2–1.4 m per group is cosy but not too squishy.
[6] 'The best way to learn is to do; the worst way to teach is to talk.'

Help when and where it is needed.

> The advantage of [board-tutorials] is that the tutor can see at a glance what each student is doing, and can correct mistakes *as they are being made*. Students who need help receive it, while others are not disturbed.
>
> Jones [7]

Two important outcomes of this advantage are early detection of students at risk, and identifying misconceptions the students themselves are unaware they have. The tutor is able to direct help and correction towards the students who require it. In this way, students get immediate feedback which is more likely to be retained and there is little time for incorrect ideas to become established. Students who have prepared can check their understanding and gain clarification; if there are no misconceptions, the student has confirmation of their understanding. Compare this to assignment feedback that more often than not comes at least a week after completion of the task. Knight [8] refers to conversational, informal feedback as background assessment, and rates its authenticity as high (provided the tasks are well-designed), since it takes place during normal activity.

We have also noticed that board-tutorials are good for transitioning from High School where the teacher is 'looking over the shoulder' and guiding much more closely. Finally, if there is a problem common to more than one person or group, the tutor can run a demonstration/discussion for those students on their board — sometimes, this might be the entire tutorial class.

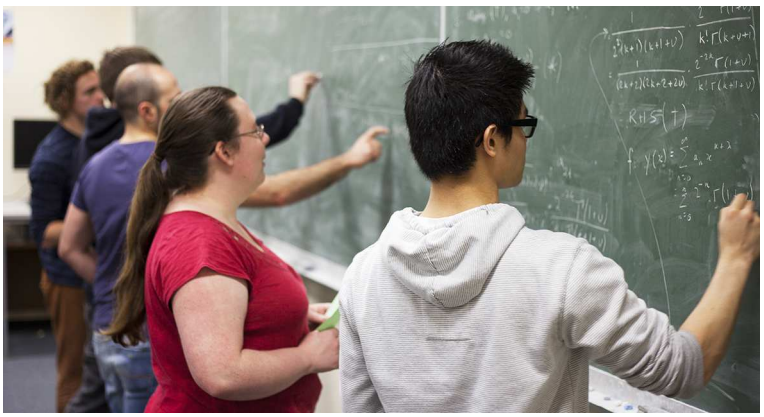
Peer learning. In board-tutorials, students learn from one another, as well as from the tutor. Jones remarked:

> The use of blackboards encourages students to work in small groups and to discuss their work with other students. More able students can be asked to explain points to other students.
>
> Jones [7]

Morris noted that this extends beyond the students they are working with:

> [Students] can move around the room looking at what other students are doing and interact with each other.
>
> Morris and Hudson [12]

Top students can hone their understanding through explanation to others; less able students receive the help they need and those in the middle who need a tip or two can receive it without any fuss by looking across the room, each group's work being visible to other students, as well as to the tutor.



Breaking down barriers: student to student. Making connections to peers has been identified as a strong predictor of persistence and retention at university [6]. The board-tutorials promote such connections:

> In the first few tutorials students are asked to write their names on the top of their blackboards so as to encourage them to get to know each other and to break down their sense of isolation. The intimacy of working in a smaller group ... provides some variety in the teaching program.
>
> Jones [7]

Students observe that others need help, too, and this boosts their feeling of belonging.

Breaking down barriers: student to teacher. Similarly, board-tutorials break down the formality between tutor and student inherent in the 'sage on the stage' model of teaching. Quite literally students and teacher are on the same footing; they are standing shoulder-to-shoulder, the same writing implement in hand. Shyer students can approach a tutor to ask their questions without the whole group having to hear them, or find that the tutor comes to them, if they are observed as making slow progress. Student contact with staff who 'know their name' is another factor related to engagement by the report [6].

Generic skills: group work and communication. We have already mentioned in our various accounts of board-tutorials at our institutions the not-directly-mathematical skills that are used and developed in a board-tutorial. Students can work together, the tutor can hold all members of a group accountable for what is on the board in front of them, they communicate to one another and the tutor about what they are thinking, and they write up solutions or proofs and refine them together. Rather than having to devise ways to incorporate generic skills into our subjects, they grow naturally alongside mathematical proficiency.

Authentic preparation. The board-tutorials provide an insight to the work of the research mathematician: collaboration, experimentation, visualisation. Think of how you use the whiteboard in your office or tea-room:

> Mathematicians don't just use boards for teaching. Ideas are written up to consider privately and with colleagues. The contents of these boards are essentially ephemeral. Things get amended, refined and perfected then rubbed out.
>
> Shepherd [16]

But they also provide an important example to future teachers, as noted by Weissglass [19]; we can model that maths learning should be active, participatory and engaging.

Benefits to the teachers. Board-tutorials have been described as 'exhausting ... but more satisfying' [10]. You never have to peer over a student's shoulder to try and see what lack of progress they are trying to conceal from you or ask for a show of hands—it is all visible on the board, and you can tell where everyone is up to with a glance around the walls. By writing the question sheets and then using them with one's own group, skills in task design and the right level of explanation and clarity are developed. This can feed back (or rather forward) both into next year's iteration of the sheets, and into one's (next) lectures, especially when other tutors also provide feedback to their colleague. In this way, pedagogical content knowledge (that is, not knowledge *of* one's discipline, but knowledge *of how to teach* one's discipline [17]) is enhanced.

Conclusion

The *flipped classroom* is a currently prominent idea in tertiary teaching; for a recent review, see [1]. Technology is generally invoked, so that content is delivered as down-loadable out-of-class readings or using video recordings; we are not describing such a scenario. But at its core, in the flipped classroom pedagogy, face-to-face time is reserved for active engagement with the material, not as homework problems, but by discussion and interaction with peers and teachers. The intimidating 'sage on the stage' becomes the 'guide on the side', and responsibilities shift from being focussed on the teacher to being shared with the learner. It is in this sense that we suggest the board-tutorial is a tried-and-true, discipline-appropriate example of flipped teaching. Indeed, one post-doc being inducted into tutorials at La Trobe commented 'This is exactly the opposite of what I did before'.

Active participation in mathematics is the key to learning. As Sid Morris remarked:

> Maths is not a game for spectators. It's not like watching a game of football.
>
> McIvor [10]

The board-tutorial model demands such participation from students. Moving the responsibility of driving the class from the tutor to the students opens up opportunities for students to discuss mathematics and engage with the content in an environment that is non-threatening and inclusive. Everyone is compelled to be active in class: the low-ability student, the top student, and the ones in the middle; the students who prepare and the students who do not prepare. Furthermore, this model of tutorial makes the perfect companion to the more generally accepted concept of the flipped classroom — it is the ideal environment for discussion and practice of mathematical concepts and ideas. And the benefits extend to the tutor and the lecturer as they expand their pedagogical content knowledge while assisting a variety of students from different backgrounds.

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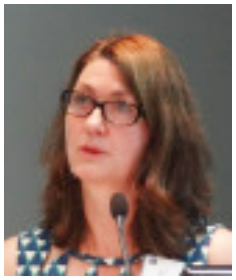
Associate Professor Katherine Seaton has been at La Trobe since 1993; she has been Department and School Director of Teaching and Learning as well as lecturing at all levels. She completed her undergraduate studies (1987) and her PhD (1990) at the University of Melbourne, and also completed a Graduate Certificate in Higher Education Curriculum, Teaching and Learning at La Trobe in 2011. From 1990–1992, Katherine was a post-doc in statistical mechanics at the University of Amsterdam, where she met the European custom of cleaning blackboards with a damp sponge, rather than smearing them with a dusty duster.



Deborah King is Director of the Mathematics and Statistics Learning Centre in the Department of Mathematics and Statistics and Assistant Dean (Undergraduate Programs) in the Faculty of Science at the University of Melbourne. She is currently involved in numerous projects which seek to enhance teaching and learning in tertiary mathematics including leading the OLT funded project, Building Leadership Capacity in University First Year Learning and Teaching in the Mathematical Sciences. Her research interests also include topological dynamics.

Carolyn (Caz) Sandison is Senior Lecturer of mathematics at the University of Wollongong. She has a keen interest in mathematics education, spending lots of time with students and working out more effective ways of teaching and assessing, while thinking about Topological Groups in her spare time.