Logarithmic superconformal minimal models

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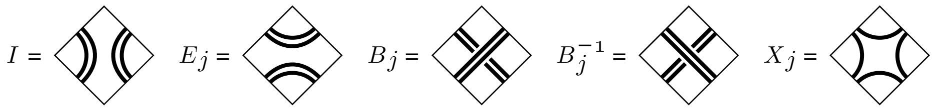
Polymers and percolation in two dimensions can be solved exactly [1] at criticality by studying lattice loop models in the form of *logarithmic minimal models* [7] specified by a primitive root of unity x. Algebraically, these models are described by the planar Temperley–Lieb algebra [8], [4] with degrees of freedom consisting of polymer segments or connectivities which are self-avoiding.

To allow for crossings and knotting and to construct generalised loop models called *logarithmic superconformal minimal models* [6], we move to the fused (doubled) Temperley-Lieb algebra (which is a one-parameter specialisation of the Birman-Wenzl-Murakami braid monoid algebra [5, 2]). The generators of this diagrammatic algebra,



are not all independent since they satisfy

$$B_j = B_j^{-1} + (x^2 - x^{-2})(E_j - I) \tag{1}$$

$$2(x + x^{-1})X_j = (x^2 + x^{-2})(I + E_j) - (B_j + B_j^{-1}) \tag{2}$$

as well as the usual braid-monoid relations and a cubic in the braids B_j[9].

Configurations of our models are generated by tiling the square lattice with the generators I, E_j, B_j and B_j^{-1} or equivalently I, E_j and X_j. Typical configurations are shown in Figure 1. Each closed loop in a configuration is assigned the weight or *loop fugacity*

$$\beta_2 = x^2 + 1 + \frac{1}{x^2} \tag{3}$$

A single closed loop with weight $\beta_2 = 1$ is shown in Figure 1(b).

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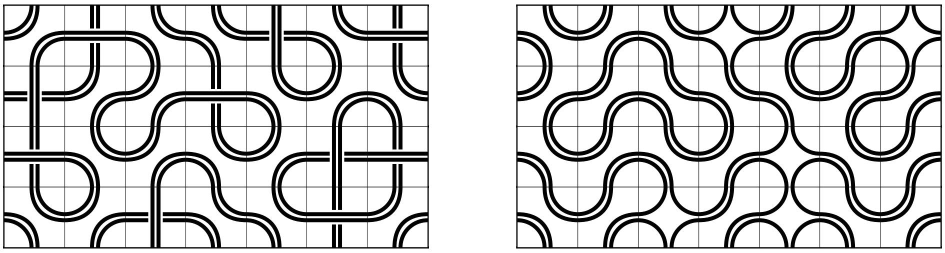

(a) Superconformal polymers (b) Superconformal percolation

Figure 1. Typical lattice configurations of (a) superconformal polymers ($x = e^{\pi i/2}$, $\beta_2 = 0$) with generators I, E_j, B_j, B_j^{-1} and (b) superconformal percolation ($x = e^{2\pi i/3}$, $\beta_2 = 1$) with generators I, E_j and X_j. The superconformal polymer segments can cross and knot but not form closed loops. The inter-connecting connectivities of superconformal percolation form clusters which can percolate from one side of the lattice to the other.

The critical behaviour of lattice statistical models is encapsulated in a set of numbers $\{\Delta\}$ called *conformal dimensions* which succinctly encode the power-law scaling behaviours of the thermodynamic functions and physical quantities. For simple *rational* theories, this set of numbers is *finite*. In stark contrast, for *logarithmic* theories, this set of numbers is *infinite*. Using specialised Yang–Baxter techniques [1] and the methods of Conformal Field Theory [3], we obtain the complete set of conformal dimensions for any root of unity x including

$$\text{Superconformal Polymers: } \Delta \in \big\{ -\tfrac{1}{16}, -\tfrac{1}{6}, 0, \tfrac{13}{48}, \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{15}{16}, 1, \ldots \big\} \tag{4}$$

$$\text{Superconformal Percolation: } \Delta \in \big\{ -\tfrac{1}{16}, 0, \tfrac{1}{16}, \tfrac{3}{16}, \tfrac{1}{4}, \tfrac{1}{2}, \tfrac{9}{16}, \tfrac{11}{16}, \ldots \big\} \tag{5}$$

A negative conformal dimension indicates that the theory is *nonunitary* and that the associated physical quantity diverges at criticality. These numbers completely characterise the *universality class* of critical behaviour for these new theories.

References

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Elena completed a Master of Science (Mathematics & Statistics) in 2012 with supervisors Paul Pearce and Jørgen Rasmussen at the University of Melbourne. She is currently studying for a PhD on superconformal logarithmic minimal models and their generalisations under the supervision of Paul Pearce.