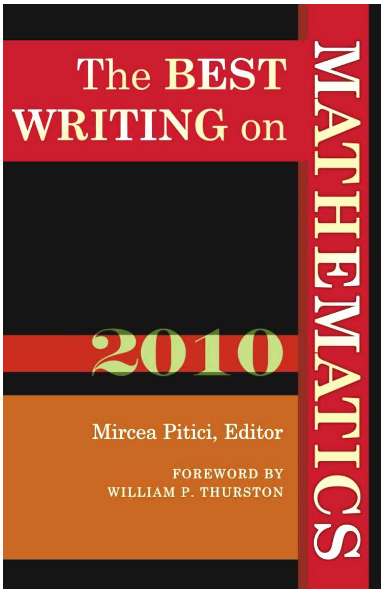
Book Reviews

The Best Writing on Mathematics 2010

Mircea Pitici (editor)
Princeton University Press, 2011, ISBN: 978-0-691-14841-0

Mircea Pitici is a mathematics education specialist at Cornell University. He also teaches writing courses and has edited this anthology as a way of presenting non-technical expositions of mathematical topics to a general audience. The articles are organised somewhat arbitrarily into six broad categories: ‘Mathematics Alive’, illustrating the versatility of mathematical writing; ‘Mathematicians and the Practice of Mathematics’, challenging stereotypes of mathematicians; ‘Mathematics and Its Applications’, including networks, probability and homology; ‘Mathematics Education’, ranging from high school to post-graduate level; ‘History and Philosophy of Mathematics’; and ‘Mathematics in the Media’.



The sources vary from popular mathematical journals such as *The American Mathematical Monthly* and *The Mathematical Intelligencer*, journals of record like *The Bulletin* and *Notices of the American Mathematical Society*, to blogs and columns from *The New Yorker*, *The Guardian* and *The New York Times*. The articles are presented without critical commentary and on the whole are well chosen and written to appeal to a wide mathematically literate audience. A few, however, especially those on education, are of limited relevance outside of North America.

There are far too many articles to review individually, so I will just comment on ones that I found particularly appealing. In ‘Mathematics Alive’, there is an amusing but serious discussion by the geometer Branko Grünbaum, called ‘An enduring error’, concerning the enumeration of the Archimedean Polyhedra: are there 13 or 14? The ambiguity dates right back to Kepler, who in different places claimed both. It persists in research papers and textbooks right up to the present, and Grünbaum gleefully traces its passage. Of course, he also discusses the reason: there are two incompatible definitions of an Archimedean Polyhedron, that is, a convex polyhedron having regular polygons for faces. One definition states that it has isomorphic vertex figures (this is the source of the 14), and the other that its automorphism group acts transitively on the vertex figures (this eliminates one of the 14). Grünbaum carefully describes the two possible rhombicuboctahedra, one of which is transitive on the vertices and the other not.

The section on 'Mathematicians and the Practice of Mathematics' contains an essay by Phillip J. Davis, published in a popular Swedish mathematical journal, on the mathematical jottings in the notebooks of the Symbolist poet and philosopher Paul Valéry (1871–1945). Valéry read widely in mathematics and physics, and was acquainted with both E. Borel and Hadamard. He was *au fait* with contemporary developments in relativity and quantum physics. While Davis finds nothing mathematically noteworthy in his notes, he points out that Valéry was obsessed with clarity of language and saw mathematics as a model for defining concepts precisely and relating them in a coherent way.

This section also contains the curious history, by Alicia Dickenstein, of the lost cover page in the English 1920 translation of Einstein's main article on the *Theory of General Relativity*, published in 1916. This page, which Dickenstein found in the online Einstein archives from the Hebrew University of Jerusalem, contains unstinting praise of the mathematicians who developed the absolute differential calculus on which the theory is based.

There are several novel results in the 'Mathematics and Its Applications' section. In 'Mathematics and the Internet', Willinger, Anderson and Doyle debunk the popular 'scale-free' or power law model of the Internet, pointing out that it is based on easily obtained but unreliable data. Brian Hayes, in 'The Higher Arithmetic', shows how to use different number scales to make sense of the very large numbers that pop up in computing, finance and astronomy. In 'Knowing When to Stop', Theodore P. Hill discusses the mathematical justification of optimal stopping rules in various practical situations.

A controversial article in the 'History and Philosophy of Mathematics' section is 'Kronecker's algorithmic mathematics' by Harold M. Edwards. He demolishes popular legends concerning Kronecker's non-belief in the existence of objects like non-constuctible irrational numbers. He points out that Kronecker's algorithms (in the absence of computers) are intended to be theoretically, not practically, computable. Finally, as a strong adherent of Kronecker's version of constructive mathematics, Edwards pours scorn on Brouwer's lawless sequences as well as Hilbert's proof of the existence of integral bases of algebraic number fields. One may feel the desire to argue with Edwards, but there is no doubt that he presents interesting ideas in an intriguing way.

In the 'Mathematics and the Media' section, the classical and jazz musician Vijay Iyer, in a column from *The Guardian* newspaper called 'Strength in numbers', describes the way in which mathematical ideas have explicitly influenced musicians. For example, he traces harmonic rhythms based on initial segments of the Fibonacci sequence in the music of Karnatak India, West Africa, Bartok and even Michael Jackson's 'Billie Jean'!

In summary, I recommend this book to *Gazette* readers as enjoyable bedside reading.

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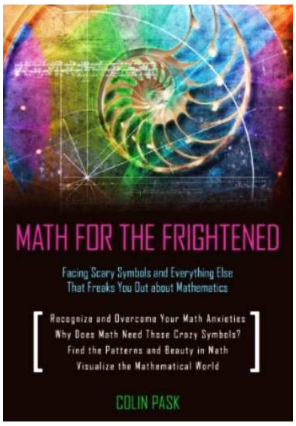
Math for the Frightened

Colin Pask
Prometheus Press, 2011 (distributed in Australia by Footprint Books),
ISBN: 978-1-61614-421-0 (paperback), 978-1-61614-422-7 (ebook)

Like many readers of this review, I have spent a large part of my professional life teaching calculus, linear algebra and discrete mathematics to students who had little affection for mathematics before their course, and even less after it. In accordance with the perceived wisdom, and indeed with all the usual textbooks for service courses, I tried to present the subjects in the context of the students' professed interests, usually engineering, business or biological science. By dint of marking to the curve, I usually managed to get the mandated percentage of students into the next stage of their education.



Colin Pask suggests another approach. Basing his ideas on recent research in cognitive science, he recommends a radically different way of handling the difficult transition of students from secondary to tertiary mathematics. Pask is Emeritus Professor of Mathematics at The University of New South Wales, Canberra. He had a distinguished career in applied mathematics, and has also published in education and cognitive science. In this book, he presents his views on mathematics and its teaching. The book is also notable for using Australian examples, ranging from Australian Bureau of Statistics population age distributions at 25-year intervals starting in 1925, to a Ginger Meggs comic strip illustrating a negative image of mathematics.

This not a textbook, but rather a polemic on how to teach mathematical techniques to students who need them as tools for their careers, but who hate or fear the subject. For example, whereas most texts start linear algebra either with systems of real linear equations or with the coordinate geometry of two- and three-dimensional Euclidean space, before presenting vector spaces in all their axiomatic glory, Pask recommends that you start by explaining expressions as simple as $2+3 = 3+2$ and gradually introduce concepts like variables and parameters when needed, explaining why they are needed. Next introduce and explain identities, linear equations in one and then in several variables, systems of equations and so on.

Pask points out that we often fail to realise that while symbolism is, for us, a way of simplifying logical arguments, for many students it is a way of obfuscating them.

As the philosopher Thomas Hobbes, criticising John Wallis, said:

> [your work] is so covered over with the scab of symbols that I had not the patience to examine whether it be well or ill demonstrated.

For example, the following statements mean the same thing to mathematicians, but not to struggling first-year students:

1. Let n be a positive integer. The sum of the first n odd numbers is the nth square.
2. $\forall n \in \mathbb{N}$, $\sum_{i=1}^{n}(2n-1) = n^2$.

Pask's basic message is: don't introduce symbols or quantifiers until you really need to; and when you do, explain exactly why they are needed.

Following this principle he shows how to deal with elementary number theory, logic, geometry, dynamical systems with applications to population, particle physics and relativity. While he does not delve deeply into any of these topics, they are introduced in a transparent and engaging way.

Apart from teachers of early undergraduate mathematics, others who might benefit from this book include students who struggled through their courses but who in the end were still not sure of what they were really doing, or why they were doing it; and politicians, journalists and business people who sense that mathematics is important but who are unable to say just why or how.

It should be noted that although Pask's common-sense approach is a distillation of his many years' experience, he presents no evidence of a controlled experiment to show that it actually works.

A different approach is recommended by Tim Gowers on his blog http://gowers.wordpress.com/2012/06/08/how-should-mathematics-be-taught-to-non-mathematicians/.

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