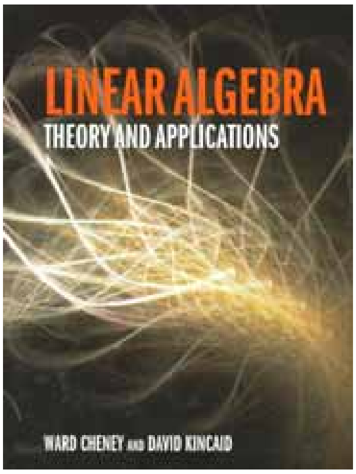
Book reviews

Linear Algebra: Theory and Applications

Ward Cheney and David Kincaid
Jones and Bartlett, 2009, ISBN: 978-0-7637-5020-6

This book is a good reference for linear algebra and is suitable for mathematics, science and engineering undergraduate students. It covers not only theories, concepts and proofs but also applications of linear algebra in our daily life with an abundance of general exercises, true–false exercises, multiple-choice exercises, and an assortment of computer exercises.



The content of this book is similar to [1] and [2], including systems of linear equations, vector spaces, vector subspaces, determinants, matrix operations, eigen systems, inner-product vector spaces, as well as some additional topics. However, this book has advantages compared to [1] and [2] in the usage of sophisticated mathematical software such as MATLAB®, Maple™ and Mathematica® to illustrate the calculations along with their codes. In [1] and [2] there are some exercises that are designed to be solved using technology such as MATLAB, Maple or Mathematica or other linear algebra software. With this approach, the authors try to encourage students to learn more about at least one of the mathematical software packages used in this book.

This book consists of eight chapters. Chapter 1 comprises sections on linear equations, systems of linear equations, Gaussian elimination, elementary row operations, reduced row echelon form, row echelon form, vectors and matrices, kernels, rank, and homogeneous equations. These are standard topics. The authors give an application of feeding bacteria and provide the code from MATLAB, Maple and Mathematica for this purpose. Other applications such as bending of a beam are also given.

Chapter 2 is devoted to vector spaces, from Euclidean to general, and linear transformation. The applications consist of elementary mechanics, network problems, electrical circuits, the predator-prey simulation, data smoothing and models in economic theory.

Chapters 3 and 4 present matrix operations and determinants. In Chapters 5, 6 and 7 the authors discuss vector subspaces, eigen systems and inner-product spaces. All the material is standard, the difference here is in the applications, which range from dynamical systems and economic models to work and forces and collision.

Chapter 8, *Additional Topics*, is devoted to students who want to learn more about the applications of linear algebra. This chapter consists of Hermitian Matrices and the Spectral Theorem, Matrix Factorization and Block Matrices, and Iterative Methods for Linear Equations with applications in internet searching, the linear least-squares problem, demographic problems, population mitigation and the Leontief open model in economics. Some topics in this chapter such as Hermitian matrices, QR and LU-decompositions and iterative methods are also found in [1] and [2].

This book has two appendices, covering deductive reasoning and proofs, and complex arithmetic. Answers or hints for general exercises are given to motivate students and give them positive enforcement.

As this book is written as an undergraduate linear algebra textbook, the title should reflect it. Perhaps *Elementary Linear Algebra: Theory and Applications* or *Introduction to Linear Algebra: Theory and Applications* would be more appropriate. However, to summarise, this is a good and well-written book and should be considered as an alternative textbook in linear algebra.

References

[1] Anton, H. and Rorres, C. (2005). *Elementary Linear Algebra Applications Version*, 9th edn. John Wiley, New Jersey.

[2] Strang, G. (2009). *Introduction to Linear Algebra*, 4th edn. Wellesley-Cambridge Press, Wellesley.

Dharma Lesmono
Department of Mathematics, Faculty of Information Technology and Sciences,
Parahyangan Catholic University, Jalan Ciumbuleuit 94, Bandung 40141, Indonesia.
E-mail: jdharma@home.unpar.ac.id

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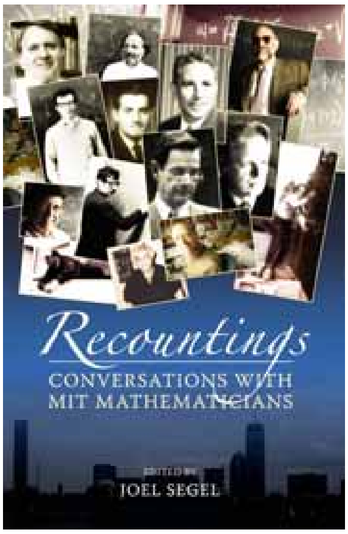
Recountings: Conversations with MIT Mathematicians

Edited by Joel Segel
A K Peters Ltd, 2009, ISBN: 978-1-56881-449-0

This book gives a history of the Massachusetts Institute of Technology (MIT) Department of Mathematics during its formative years. The material has been edited by Joel Segel from interviews conducted with senior mathematicians including Hartley Rogers, Kenneth Hoffman, Alar Toomre, Arthur Mattuck, Michael Artin, Harvey Greenspan, Sigurdur Helgason, Steven Keiman, Daniel Kleitman, Bertram Kostant, Isadore Singer and Gilbert Strang, and with Zipporah, the widow of Norman Levinson.

Many of these interviews give accounts of research breakthroughs, novel teaching methods, or how the interviewees became professional mathematicians. Others, who were at one time head of department, talked about their leadership and administrative role. One interesting narrative was on the delicate task of keeping the disciplines of pure and applied mathematics together. Although the pure mathematics section was twice the size of the applied mathematics section, wisdom prevailed and the head was chosen alternately from the two disciplines.

Isadore Singer, who holds a prestigious Institute Professorship, mentions that he and others were constantly striving to improve the research and teaching standards of the department, by always aiming to hire better people, and as a direct result of these efforts, MIT has been consistently ranked amongst the top five departments worldwide, for at least the past few decades. He also recalls the fascinating history of the Atiyah–Singer index theorem, which is one of the greatest achievements in 20th century mathematics as well as having a significant impact on mathematical physics.



This book makes for fascinating reading, for those who are interested in the creation of an outstanding department of mathematics, and who are curious about the mathematicians who have made remarkable contributions to their respective research areas. I, for one, look forward to a follow-up book containing accounts of those in the department who were not featured here.

Mathai Varghese
School of Mathematical Sciences, The University of Adelaide, North Terrace, SA 5005.
E-mail: mathai.varghese@adelaide.edu.au

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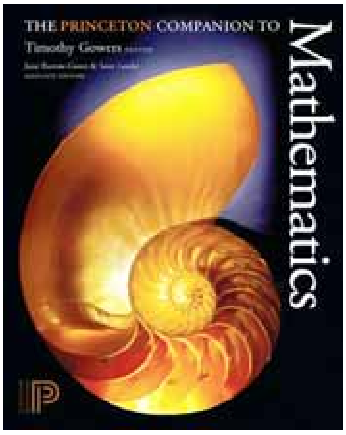
The Princeton Companion to Mathematics

Timothy Gowers (editor)
Princeton University Press, 2008, ISBN 978-0-691-11880-2

Writing a review of a book like this feels like being the blind man describing the elephant: every reviewer will look at different parts and have different opinions. It is massive, even overwhelming, and the cross-referencing encourages or even forces readers to dip into bits of it, not read it all from start to finish. The preface states that the focus of the book is modern pure mathematics, and doubtless some readers will be somewhat bemused by the areas that are featured and those that are not. In the preface, the authors express the hope that a similar compendium on applied mathematics and mathematical physics might be written, and this reviewer shares this hope.



This is a mammoth book. One cannot but admire the editor (Timothy Gowers) and associate editors (June Barrow-Green and Imre Leader) for having had the courage to take on a task of this magnitude, and for having produced such a fabulous resource. If it were not so heavy, it would be the perfect companion in cabin baggage on a plane or train, for sampling during travel, as the journey would be over almost immediately; once one starts looking into it, it is very hard to put down. One can find out about areas of mathematics that one does not know about, or find new facts about areas that one does know something about; read about the lives of the Bernouillis or the Jacobis or other famous mathematicians; be provoked into thinking about the philosophy of mathematics; or find outstanding problems to think about; one can just open a page at random and find something interesting.

There are eight parts.

1. *Introduction* (76 pages) offers general remarks on mathematics that are accessible to a wide range of readers. The titles of two of the subsections, *What is Mathematics About?* (algebra, analysis, logic, ...) and *The General Goals of Mathematical Research* (generalising, finding patterns, explaining coincidences, ...), give the flavour of this part. There is also a section much later, entitled *Why Mathematics*, which quotes a deputation of American mathematicians talking to the United States Congress: 'the main goals of the mathematical sciences are provision of fundamental tools for science and technology ...', and I found myself wondering whether this did not also belong in the introduction.

2. *The Origins of Modern Mathematics* (80 pages) discusses developments in number systems, geometry, abstract algebra, rigour, proof and logic. Perhaps questions from areas such as physics might have been mentioned here, since the need to model and understand physical systems underpinned much of the development of the calculus. In any case, this is a historical panorama which might almost serve as a course in the history of mathematics — one finds the history of geometry from Euclid through Bolyai, Lobachevskii and Gauss to Klein and Hilbert, of algebra from Diophantus through Cardano and Viète to Galois, Cayley and Dedekind, of calculus from Newton and Leibniz to Riemann and Weierstrass and Cantor (though not Lebesgue), and foundations from the development of the Zermelo–Fraenkel axioms to Gödel.

3. *Mathematical Concepts* (158 pages) has a pot pourri of articles, each of a page or two, on topics such as Bayesian analysis, braid groups, cardinals, categories, the Euler and Navier–Stokes equations, expanders, Galois groups, the gamma function, irrational and transcendental numbers, the Ising model, manifolds, matroids, quadratic forms, quantum computation, the Schrödinger equation, the simplex

algorithm, varieties and vector bundles. As the sample that I have given here indicates, the topics are very wide-ranging, and tend to defy the usual classifications. Some of the topics are quite general, while others are rather specific; some topics are certainly considered to be applied mathematics (in Australia), but not all that is considered to be applied mathematics (in Australia) is featured. Some of the articles are by individuals while others are not: for instance, Terence Tao wrote the article on distributions, while the article on dimension, which treats dimension from many points of view, including topology, homology, and metric spaces, has no named author.

4. *Branches of Mathematics* (366 pages) is another interestingly idiosyncratic list of 26 areas, including logic and model theory, algebraic geometry, moduli spaces, differential topology, general relativity and the Einstein equations, harmonic analysis, partial differential equations, numerical analysis, and stochastic processes, all of which are described in some detail. This is certainly a little different from the classification of sections of mathematics that appear at the International Congresses of Mathematicians! One of the stimulating aspects of this book is the view of mathematics that it gives — the vision of the editor clearly underlies it — and the extent to which it suggests that traditional divisions, such as between analysis and algebra, are becoming irrelevant. This part will be found to be very valuable, as the articles are very well written and certainly accessible to university undergraduates. They include references for further reading that vary from undergraduate text books through graduate texts to research papers. I have set myself the goal of reading all these articles carefully over the next few years; each of them requires some time to digest.

5. *Theorems and Problems* (52 pages) sketches recent milestones such as Fermat's last theorem and the four-colour theorem, and as yet unsolved problems such as the Riemann hypothesis. This includes some less famous results (such as the Robertson–Seymour theorem) and excludes some famous problems (long-term stability properties of solutions of nonlinear partial differential equations). I would advise a bright undergraduate or postgraduate who wants to know about 'hot areas' to work in to read this part, but also to look up Hilbert's problems and to visit the Clay Mathematical Institute's website for other points of view.

6. *Mathematicians* (93 pages) gives short biographies of such luminaries as Pythagoras, Fermat, Newton, Gauss, Green, Hamilton, Hilbert, Hardy, Weyl, Skolem, Gödel, Turing, Robinson and Bourbaki. Most of them were born before 1900, but a few (now deceased) mathematicians from the twentieth century appear, including von Neumann, Turing and Weil. It is easy to think of names that surprise by their absence, but including everybody would have doubled the size of this already weighty tome. And there are lots of little gems here: I discovered that one of Abel's manuscripts on elliptic functions lay on Cauchy's desk for 15 years before it was published, which puts into perspective the delays of a year or two between submission and publication that we complain about these days. Not only that, but the manuscript was stolen and has been partly rediscovered in the last 50 years or so. And in another biography one finds: 'such was the chaos of his household that Lobachevskii's biographers have been unable to establish the number of children in it, but it may well have been fifteen or even eighteen'. I imagine that many of

us will look at this part of the book regularly to enliven our teaching with such anecdotes.

7. *The Influence of Mathematics* (128 pages) discusses the applications of mathematics in diverse areas, including biology, chemistry, information theory, economics and finance, and statistics (but not physics). This contains a fascinating reflection on the relationship between mathematics and statistics, due to David Cox, addressing the Royal Statistical Society in 1981:

> Lord Rayleigh defined applied mathematics as being concerned with quantitative investigation of the real world "neither seeking nor evading mathematical difficulties". This describes rather precisely the delicate relation that ideally should hold between mathematics and statistics. Much fine work in statistics involves minimal mathematics; some bad work in statistics gets by because of its apparent mathematical content. Yet it would be harmful for the development of the subject for there to be widespread an anti-mathematical attitude, a fear of powerful mathematics appropriately deployed.

8. *Final Perspectives* (60 pages) offers thoughts on numeracy and problem solving as well as a discussion of what mathematics is for and advice to young mathematicians. The advice is also thought-provoking: from Michael Atiyah, one learns that 'it is a mistake to identify research in mathematics with the process of providing proofs'; Bela Bollobás tells us to work on a range of problems from the 'dream' problem that we cannot reasonably expect to solve to problems that should be below our dignity, but which we can deal with quickly, while Alain Connes invites us to take walks and try to do computations in our head, as an occasional source of inspiration and a good mental exercise. Dusa McDuff asks the question 'how *any* young person can build a satisfying personal life while still managing to be a creative mathematician'. Peter Sarnak reminds us that we should attend departmental colloquia regularly (advice that is as relevant to a head of school as it is to an aspiring youngster).

All in all, this book will be a very useful resource to young and old. One can criticise it for focussing on pure mathematics, and omitting much of classical applied mathematics. I have speculated quite a lot about what this says about the British view of the roles of applied and pure mathematics. I cannot help but think that while the quotation from Cox applies to other fields than applied mathematics and statistics, it can also be turned around: it would be harmful for the development of mathematics for there to be widespread an anti-applications attitude. On the other hand, this volume is already too heavy for most of us to hold comfortably and read, and if it included applied mathematics and mathematical physics it would be impossibly large. So in the end, I just hope that there will be a companion *Companion* soon, focussing on these areas, and as well written and interesting as this tome, and I hope that the *Gazette* editors, when this comes out, will be willing to let me review it too.

Michael Cowling
School of Mathematics, University of Birmingham, Edgbaston B15 2TT, UK.
E-mail: m.g.cowling@bham.ac.uk

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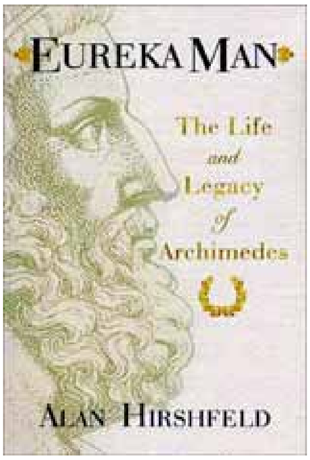
Eureka Man: The Life and Legacy of Archimedes

Alan Hirshfeld
Walker & Company, 2009, ISBN 978-0-8027-1618-7

Over the last 10 years, Archimedes has been the flavour of the month from among ancient scientists and mathematicians. While his genius and contributions have long been recognised among mathematical scientists, the new attention derives in large measure from the 1998 sale and subsequent decipherment of the so-called 'Archimedes Codex' [1], [2]. Most readers will be aware of this document with its chequered history. Briefly, besides other ancient writings, it contains Greek texts of Archimedes' *Method*, *Stomachion* and *On Floating Bodies*, themselves copied from older manuscripts in the tenth century, largely erased in the thirteenth (so that the parchment could be reused for a prayer book) passed from library to library, identified and largely transcribed by J.L. Heiberg in 1906, subsequently lost and passed into private hands who largely destroyed it, bought by its present owner in 1998 and subsequently largely deciphered. I will refer to this text as 'The Palimpsest'.

The book under review rides on this bandwagon, and a good question is: Is any such book needed, given the ready availability of [1], written by Netz and Noel, who are coordinating decipherment of the Palimpsest? My answer is, probably not. I should refine this answer somewhat, given that there are at least three categories of potential readers.



The first, and smallest, is the category of specialist historians of mathematics. For them the classic text of Dijksterhuis [3], complemented by the more literal translations of Archimedes' works into French [4], [5] and by the bits of the Palimpsest that have been published, fill out the picture quite well. Reviel Netz's so far partly published English translation [6], with its extra focus on the specifics of the diagrams and the Palimpsest, will make life a little easier for English speakers and add some new perspectives. So the category of specialists is quite well catered for without needing the book under review. Indeed specialists might even feel somewhat irritated by such extraneous matters as speculations about what Archimedes might have done during the siege of Syracuse.

The second category of potential readers is that of the mathematically literate who are not specialist historians but may have read outlines of Archimedes' approach to questions about the circumference and area of a circle, and his quadrature of parabolic segments as done, for instance, in *The Method*. In short, the category of most of the readers of this *Gazette*. What might such people gain from reading *Eureka Man*? First, they will gain a highly readable account — though neither particularly better nor worse than that found in [1] — of the travails of the Palimpsest. However, Hirshfeld does not add anything of interest to the mathematical–historical content of the Palimpsest beyond what has been readily available for all of the last century. I recommend to anyone wanting to learn a bit more about the newly revealed text and its accompanying diagrams that they should go to Chapters 8 and 10 of [1].

The third category of potential readers of *Eureka Man* consists of everyone else who is interested in Archimedes or has had their appetite whetted by press reports and TV documentaries. Such people are likely to be drawn in by Hirshfeld's engaging style and, in the process, learn some of the mathematical results obtained by Archimedes as well as what little there is to know about the man himself. The mathematical results in Hirschfeld's book are all presented by modern mathematical equivalents. For instance *Measurement of the Circle* is presented as approximating π. I add that Hirshfeld is just plain wrong when he asserts that Archimedes' starting polygons in his approximation are inscribed and circumscribed triangles and squares. Instead, Archimedes starts with the easiest figures to inscribe, circumscribe or discuss, namely hexagons.

Does *Eureka Man* live up to its subtitle 'The Life and Legacy of Archimedes'? Not really. For starters, as noted above, very little can be reliably said about his life. About his legacy, the spread of Archimedean science across the Middle East and then Europe, a lot more can be said even at the level of popular mathematics. Unfortunately this book does not do so. A reader might conclude that the Palimpsest represents the summit of Archimedes' legacy. In fact, while the Palimpsest fills out somewhat further our picture of Archimedes' approaches and achievements, it is not a legacy in the sense of something which has influenced the development of modern mathematics or is likely to stimulate new research. History is littered with such ideas which, if they had fallen on the right ears at the right time, might have been hugely influential but which, in fact, died shortly after the author, except for some traces in documents found centuries later. Nicolas Chuquet's *Triparty* is just one example. If any readers of the *Gazette* want to gauge the huge mathematical legacy of Archimedes I suggest that, rather than reading *Eureka Man*, they start by flipping through some of Marshall Clagett's many books.

The nonmathematical potential readership at whom the book is probably aimed may well enjoy Hirshfeld's discussion of writing materials over the years, their durability and their relative suitability for being in scroll form or being bound into codices and, being aimed at the wider market, the book can probably get away with nonreferenced hyperbolic passages such as

> Archimedes could see it hovering before his mind's eye: a circle — free of any distortions, perfect in every sense, scribed into his consciousness by the radius of pure imagination.
>
> (p. 46)

I must admit my lectures were never up to this standard of eloquence!

Because he is not bound by any confidentiality agreement, Hirshfeld, unlike Netz and Noel, is free to speculate who the anonymous current owner of the Archimedes Palimpsest, referred to as 'Mr B', might be. Hirshfeld's candidate can be found on p. 187 of his book. I won't steal his thunder by giving it away.

References

[1] Netz, R. and Noel, W. (2007). *The Archimedes Codex*. Da Capo Press, Philadelphia.
[2] Palimpsest Project (undated). The Archimedes Palimpsest Project. At: www.archimedespalimpsest.org/google-book.html (accessed 15 January 2010).
[3] Dijksterhuis, E.J. (1987). *Archimedes*. 2nd edn. Princeton University Press, Princeton NJ.
[4] Mugler, C. (1970–1972). *Les Oeuvres d'Archimède* (4 volumes). Budé, Paris.
[5] Ver Eecke, P. (1960). *Les Oeuvres Complètes d'Archimède* (2 volumes). 2nd edn. Albert Blanchard, Paris.
[6] Netz, R. (2004). *The Works of Archimedes*. Volume 1. Cambridge University Press, Cambridge.

Bob Berghout
School of Mathematical and Physical Sciences, University of Newcastle, NSW 2308.
E-mail: Bob.Berghout@newcastle.edu.au