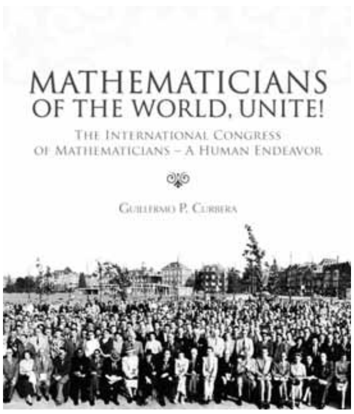
Book reviews

Mathematicians of the World, Unite!
The International Congress of Mathematicians – A Human Endeavor

Guillermo P. Curbera
A K Peters Ltd, 2009, ISBN 978-1-56881-330-1

This book describes the history of the International Congress of Mathematicians (ICM) and how mathematics has interacted with other disciplines. It also describes how political events (e.g. World Wars I and II, and the Cold War) and controversies have influenced the mathematical community and the ICMs.



The 25 congresses, from Zürich in 1897 to Madrid in 2006, are divided into five periods, according to historical events. *Early Times* (covering the congresses from Zürich (1897) to Cambridge (1912)) describes how the ICMs progressed and consolidated during that period. *Crisis in the Interwar Period* (congresses from Strasbourg (1920) to Oslo (1936)) shows how the ICMs struggled after World War I when nonscientific influences were very strong. *The Golden Era* (Cambridge, MA (1950) to Stockholm (1962)) was a period of great success for the ICMs. It was marked by the re-foundation of the International Mathematical Union (IMU), and a new era of international cooperation was built after World War II. *On the Road* (congresses from Moscow (1966) to Berkeley (1986)) was a period of increased attendance, with more than 4000 participants attending the Moscow congress. The last period, *In a Global World* (congresses from Kyoto (1990) to Madrid (2006)), saw developments such as the congress being held in the East for the first time, women as plenary speakers (apart from Emmy Noether in 1932), and the introduction of the Gauss Prize for work in the application of mathematics (the first being awarded to Kiyoshi Ito in Madrid in 2006).

At the end of each section, there are photographs of buildings and social life, details of awards and the chronology of landmarks in the history of the IMU. For each congress, there is a short history which includes details of plenary sessions and awardees of the Fields Medal, the Nevanlinna Prize and the Gauss Prize. Additionally, the histories of the first two awards are given in a separate chapter entitled 'Awards of the ICM'. The Fields Medal was first awarded in 1936, while the Nevanlinna Prize was first awarded in 1982.

I was impressed by two statements regarding the ICMs and mathematics. The first was made by Ludwig Faddeev (Russian Academy of Sciences in St Petersburg) at the closing ceremony of the Beijing congress (2002): 'The main idea of the ICM is to confirm the unity and universality of mathematics' (p. 292). The second came from Sir John Ball (the president of the IMU) at the opening ceremony in Madrid: 'Mathematicians do not own mathematics' (p. 297).

This is an excellent book, and the author is to be congratulated on his efforts in collecting references from many sources (congress proceedings, books, articles, notes and announcements in journals), especially old sources from the early times of the ICMs.

The next ICM will be held in Hyderabad, India, August 19–27, 2010.

Dharma Lesmono
Department of Mathematics, Faculty of Information Technology and Sciences,
Parahyangan Catholic University, Jalan Ciumbuleuit 94, Bandung 40141, Indonesia.
E-mail: jdharma@home.unpar.ac.id

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A Lifetime of Puzzles: Honoring Martin Gardner

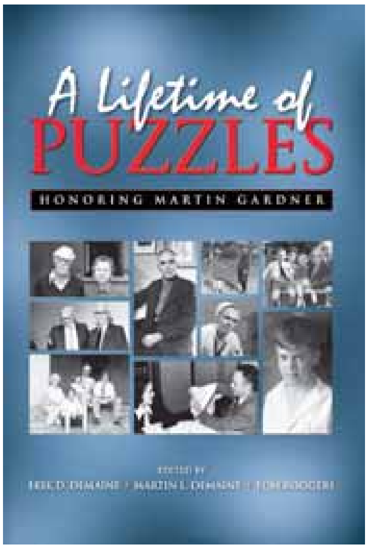
Erik Demaine, Martin Demaine and Tom Rogers (eds)
A K Peters Ltd, 2008, ISBN: 978-1-56881-245-8

Martin Gardner is best known to mathematicians as a highly successful populariser of mathematics, both through his 'Mathematical Games' column, which ran for 25 years in the magazine *Scientific American*, and through his many books, which are mainly collections of those columns. Some know him also as a designer of intriguing card and conjuring tricks, as well as the author of the best-seller *The Annotated Alice*, and as a persistent debunker of pseudo-scientific fads.

The volume under review is a *Festschrift* in honour of Martin Gardner's ninetieth birthday. He is now 95, and still going strong, having published his latest book in 2009. As well as his close connection with mathematicians, Gardner is equally highly regarded in the magic community. *Magician* is the American word for what we call a conjurer, but Gardner is not just a literal magician in this sense, but also a figurative one through the magic of his words, so I shall adopt the American usage.

The book contains 25 short articles, three of which are specifically about Martin Gardner and his influence on the worlds of magic and mathematics. Many of the others, including contributions from Persi Diaconis, Ron Graham, Roger Penrose, David Klarner, Solomon Golomb, Raymond Smullyan and Bill Gosper, include revealing anecdotes about Gardner and his peculiar ability to interest young people in mathematics.

There are three historical articles. Two describe the life and times of Luca Pacioli (ca. 1445–1517) who published the first work in Europe describing magic and card tricks, *De Viribus Quantitatis*, (On the Powers of Numbers, ca. 1500). The other describes the remarkable story of the world's first puzzle craze, Tangram.



Several articles describe astonishing card tricks — astonishing because they depend not on sleight-of-hand, but on combinatorial properties of permutations. Colm Mulcahy, for example, describes an apparently well mixing shuffle which returns the pack to its original order in exactly four shuffles. Other areas of recreational mathematics covered include various types of mazes, two- and three-dimensional dissections and new graph theoretic paper-and-pencil games.

The areas of mathematics that most interest Gardner are combinatorics and logic. Both topics are covered in the book. For example, there are articles on symmetric graphs, magic squares, scheduling tennis doubles competitions, continued fractions and logical surprises related to Gödel's Incompleteness Theorem.

This book contains no deep mathematical theorems, but for fans of Martin Gardner and for all concerned with communicating to the public the fascination of mathematical research, it is an ideal bedside book.

Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA 6009.
E-mail: schultz@maths.uwa.edu.au

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Topics in Graph Theory: Graphs and Their Cartesian Products

Wilfried Imrich, Sandi Klavžar and Douglas F. Rall
A K Peters Ltd, 2008, ISBN: 9-781568-814292

This book discusses graph theory in the context of a survey of Cartesian products of graphs. The intent is similar to that of Holton and Sheehan [5] who use the Petersen graph as a centrepiece for exploring graph theory.

Sabidussi [6] presented the foundations of the study of products of graphs in 1960, and gave definitions of several different products of two graphs. Products of graphs are not often discussed in contemporary texts on graph theory. Neither Diestel [3] nor Bollobás [1] mention them, and Bondy and Murty [2] mention them only briefly.

Imrich, Klavžar and Rall focus on the Cartesian product, and in 18 short chapters cover typical graph theory topics such as connectivity, planarity, crossing numbers, Hamiltonicity, colouring, domination, and independence, as well as some ideas from algebraic graph theory. Each chapter ends with a dozen or so exercises for which there are nearly 40 pages of hints and solutions. Many of the exercises fill in details required to understand the proofs of the theorems.

Let G and H be graphs with m and n vertices respectively. Informally, the Cartesian product is a graph on an $m \times n$ grid of vertices, with n copies of G in one direction superimposed on m copies of H in the other.

More formally, the Cartesian product $G \,\Box\, H$ is the graph whose vertex set and edge set are as follows. The vertex set $V(G \,\Box\, H)$ is simply the Cartesian product $V(G)\times V(H)$. We say that $[(g_1, h_1), (g_2, h_2)]$ is an edge if and only if either $g_1 = g_2$ and $[h_1, h_2]$ is an edge in H, or, $h_1 = h_2$ and $[g_1, g_2]$ is an edge in G.

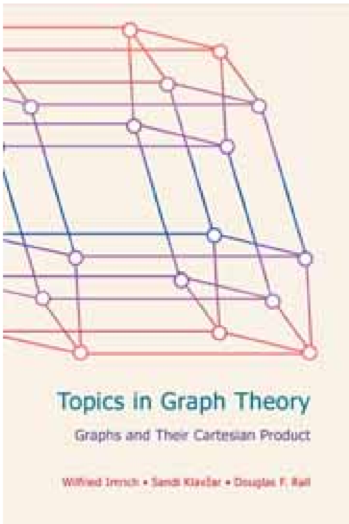
To convey the flavour of the book, we will describe a few selected chapters.

In Chapter 2, the authors show how the popular Towers of Hanoi puzzle can be modelled using Cartesian products of graphs. This puzzle has generated a considerable amount of mathematical research; see Hinz [4] for a review. The difficulties arise when there are more than three pegs and you want to find the smallest number of moves to complete the puzzle. So we go from a well-known puzzle to a mathematical model using Cartesian products of graphs, and, before you know it, you are at the frontiers of knowledge. This trip from the known to the unknown contains all the ingredients for an inspiring lecture.

In Chapter 5 the authors discuss connectivity of a Cartesian product, of which there are two types. Vertex connectivity is the minimum number of vertices whose deletion will disconnect a connected graph, and edge connectivity is the minimum number of edges whose deletion will disconnect a connected graph. They aim to show that the vertex connectivity and the edge connectivity of a Cartesian product depend only on the connectivities, the minimum degrees, and the orders of the factors. The authors use the concepts of separating sets and disconnecting sets to gain an understanding of connectivity, and these form the basis for their proof.

Tower of Hanoi (photo by Frances Mills)

The only other concept used is that of a fibre, which is the Cartesian product of a single vertex with a graph. Thus the authors accomplish their task with a minimum of mathematical machinery in a well laid-out and easy to understand style.



Prime factorisation is discussed in Chapter 15 wherein it is shown that connected graphs have unique prime factorisations. Analagous to prime integers, a graph is prime if it can not be represented as a product of nontrivial graphs. Surprisingly, factorising a connected graph can be done in linear time, and although the algorithm is not given in detail, the authors do sketch the basic idea and point the reader to a recent paper. These ideas are revisited in the final chapter with a different proof of the unique prime factorisation of connected graphs together with a very simple factorisation algorithm which does not run in linear time. In the remainder of Chapter 15, the structure of a prime factorisation is then used to describe the structure of a graph's automorphism group which is the subject of the intermediate chapters.

The format and style is ideally suited to an honours or post-graduate seminar series covering one chapter per session. *Topics in Graph Theory: Graphs and their Cartesian Products* would be a useful addition to any university library.

References

[1] Bollobás, B. (1998). *Modern Graph Theory*. Springer, New York.
[2] Bondy, J.A. and Murty, U.S.R. (2008). *Graph Theory*. Springer, New York.
[3] Diestel, R. (2005). *Graph Theory*, 3rd edn. Springer, New York.
[4] Hinz, A.M. (1989). The tower of Hanoi *Enseign. Math. (2)* **35,** 289–321.
[5] Holton, D.A. and Sheehan, J. (1993). *The Petersen Graph*. Australian Mathematical Society Lecture Series, No. 7. Cambridge University Pres, Cambridge.
[6] Sabidussi, G. (1959/1960). Graph multiplication. *Math. Z.* **72,** 446–457.

Lakoa Fitina[1], C.T. Lenard[2] and T.M. Mills[2]
[1]Department of Mathematics and Computing Science, Divine Word University, Madang, Papua New Guinea. E-mail: lfitina@dwu.ac.pg
[2]Department of Mathematics and Statistics, La Trobe University, Bendigo, VIC, Australia.

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