Book reviews

Legacy of the Luoshu

Frank J. Swetz
A K Peters Ltd, 2008, ISBN: 978-56881-427-8

About 3000 years ago, a Chinese astrologer–mathematician whose name is unknown to us discovered the 3×3 magic square, or, as legend has it, the Sage King Yu deciphered it from the markings on the breastplate of a turtle emerging from the River Luo.

There are eight such squares, each obtained from any one of them by applying the reflections and rotations of the square. But this one was rather special. Not only were the row, column and diagonal sums equal, but also the meritorious number 9 occupied the main southern entrance hall, and the auspicious number 2 stood to the south-west. Of course there is only one magic square with these properties, and it is known as the *luoshu*, or River Luo Writings.

4	9	2
3	5	7
8	1	6

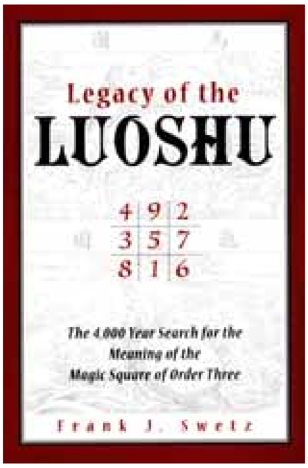
In case you are uneasy with this picture, note that until the 18th Century, Chinese cartographers oriented maps with south at the top of the page and east to the left, which demonstrates the arbitrariness of our convention. Incidentally, south is the dominant direction because it is the source in central China of warm, dry *yang*, whereas north is from where cold, wet *yin* emanates.

Not a great deal was made of the mathematical properties of the *luoshu* before the writings of the mathematician Yang Hui, about 1275. Instead it was the source of most, if not all, Chinese mysticism concerning geography, religious ritual, architecture, fortune telling, astronomy, physical exercise, classical dance moves and martial arts.

If this seems rather far-fetched, consider this: all elements of Chinese popular culture now fashionable in the West, including *yi jing* [I-Ching], *feng shui* and *tai chi* are parts of the legacy of the *luoshu*.

This book by Frank Swetz, a mathematics educator and expert on the history and development of Chinese mathematics, tells the story in great detail. He explains the importance of 'fiveness' and 'nineness' in Chinese culture from pre-historic times to the present day. For example, the nine original provinces of China were arranged very much as in the cells of the *luoshu*, with the central province, the home of the divine ruler, called *Wu*, the Mandarin word for five. South of it lies the prominent province of *Jiu*, or 9. Similarly, China itself was regarded as the 'Middle Kingdom' of nine territories forming a continent, which in turn was the centre of nine continents comprising the world.

The duality of *yin* and *yang* is represented in the *luoshu* by the interlocking cells containing even (*yin*) and odd (*yang*) numbers. The connection of the *luoshu* to building design was through the *mingtang* temple complexes which were prevalent in ancient China, and are exemplified in the Temple of Heaven Complex in Beijing. The central building in these complexes was divided into nine rooms of which the central one served a special function as the 'Chamber for Communicating with Heaven'. The surrounding halls served functions relating to the corresponding cells of the *luoshu* and were devoted to ceremonies carried out at appropriate seasons of the year. Swetz decribes in detail more than you probably want to know about the ceremonies associated with the *mingtang* temples.



The dance moves, which are manifested in both *tai chi*, spiritual exercises, and *tai chi quan*, martial arts with lance and sword, originated with the Daoist dances performed in the *mingtang* temples in moving through the halls in order from 1 north to 9 south. This became formalised into a system of shuffling dance steps emulating these moves in a restricted area, and so into a kinetic algorithm for constructing the *luoshu*. The Daoist priests owed special reverence to the constellation *bei dou*, which we know as the Plough or Great Bear, and their dance also moved in a pattern that followed the progression from one star to the next in this constellation.

We come now to divination through the casting of lots, which eventually came in the form of rectangular tiles bearing the eight trigrams, each representing a number in binary form. Combinations of these led to the 64 hexagrams. Through a rod choosing ritual, the diviner would arrive at a hexagram whose metaphorical meaning was tabulated in the *yi jing* (Book of Changes). The eight trigrams were associated with the eight cardinal directions of the compass, and hence with the *luoshu*.

There is a distinct difference between number mysticism in the West and in China. Languages which adopted an alphabet descended from Old Semitic, such as Greek, Arabic, Latin and Russian have an alphabet with an assigned order, so that the first ten letters can be used to represent the nonzero decimal digits, the next ten the multiples of ten and the next ten, including variants, the hundreds. Thus words have a numeric interpretation, and conversely. Chinese characters on the other hand have no accepted alphabetical order. Instead they have the phenomenon of homonymy, i.e. the same or related sounds represented by different characters and so having different meanings, depending on the particular dialect spoken. For example, the character for 4 is pronounced *si*, which sounds like the character for death. Consequently, 4 is considered as a number to be avoided, which explains why taxis in Singapore never have 4 on their license plates.

Practitioners of *feng shui* (wind and water) determine auspicious locations for buildings, rooms within a building, and furniture within a room by examining topographical features of the location and constructing an appropriate chart consisting of three superimposed *luoshu*-derived number squares.

Apart from its official usage in temple ceremonies and other religious practices, the *luoshu* became a popular good luck charm among the people of China and surrounding countries. This volume of Swetz contains photographs of Tibetan religious artwork and amulets from the Indonesian National Museum and Malaysia featuring the *luoshu*.

Instances of 3×3 and higher-order magic squares occurred, probably independently, in other cultures, but nowhere did they attain the significance attributed to the *luoshu*. Swetz traces the spread of the *luoshu* from China first to India and the Islamic world and later its transmission, together with other Chinese mathematics and technology, by the 16th Century Jesuit missionaries to Europe, especially France. It was there that Leibniz learned of it, and was particularly excited by the hexagrams of the *yi jing*, which seemed to confirm his ideas of a universal arithmetic based on his binary system.

Perhaps with an eye on the 'New Age' market, Swetz has minimised the amount of mathematics in this book. For example, there is no explanation that every positive integer up to 63 corresponds to a unique hexagram, and there is no mention of modular arithmetic, partitions, or symmetry groups, all of which can be related to the *luoshu* and could have added some mathematical interest. There is a clumsy proof of the essential uniqueness of the *luoshu*, and an account of the higher-order squares constructed by Yang Hui. Swetz also describes recent results on the construction of magic squares with peculiar properties, but presents no systematic theory.

For those interested in light reading with a mathematical theme, Swetz has produced a well-written volume free of stylistic and typographical errors.

Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA 6009.
E-mail: schultz@maths.uwa.edu.au

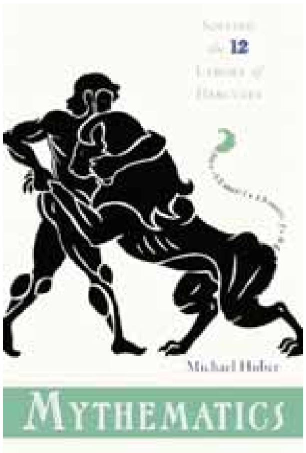
◇ ◇ ◇ ◇ ◇ ◇

Mythematics: Solving the 12 Labours of Hercules

Michael Huber
Princeton University Press, 2009, ISBN: 978-1-4008-3190-6

Most elementary calculus textbooks contain phoney applications, because real applications require too much background knowledge to fit into a standard unit. Unfortunately, these artificial examples fail to excite students or interest lecturers. Recognising this, many authors introduce fantasy examples, like determining when a dog chasing a stick thrown into a river should stop running and start swimming, or how a spider should minimise her crawling distance to a fly. Michael Huber has gone the whole hog: he has written a book of applications of elementary mathematics based entirely on the mythical 12 Labours of Hercules.

Huber is an applied mathematician at Muhlenberg College, clearly with an interest in the Classics. Of the many sources for the Hercules myths, he has chosen that of Apollodorus, in the English translation of Sir James George Frazer. Since it is written in an ironic tone appealing to modern sensibilities, this is an appropriate choice. Each chapter of the book is devoted to one of the 12 tasks and each begins with an excerpt from Apollodorus, followed by a synopsis describing the corresponding mathematical tasks, a description of the specific problems to be solved, and the mathematical techniques needed to solve them. Full solutions and additional problems follow immediately.



For example, the Fifth Labour requires Hercules to clean out, in one day, the Augean stables which have been used to shelter large herds of cattle for many years. Hercules achieves this by diverting the courses of the rivers Alphaeus and Peneus to flush out the stables. Huber invents a number of problems based on this myth. Students are asked to solve a Diophantine equation from the *Greek Anthology* to find the number of cattle; to calculate the dimensions of the stables required to accommodate all the dung, based on US Department of Agriculture research which gives an average of 14 to 18 kg/day/head; to calculate the hydrostatic pressure at the base of the two-metre high walls; to estimate the time required to fill the stables with water given the flow from the two rivers, and the time needed to empty it through a two-metre diameter bung hole, using Toricelli's Law for the rate of flow through a given hole under given pressure. Of course, all this is completely fantastic, but great fun, introducing students to ideas of mathematical modelling as well as standard mathematical techniques.

In the chapter on the Fifth Labour, the mathematical problems are clearly related to the subject of the myth. In others, the connection is more tenuous. For example, in the Seventh Labour, Hercules is required to fight and capture the Cretan bull and bring it to King Minos. This he does, but the bull escapes and terrorises the inhabitants of Marathon. The first mathematical problem asks for an estimate, with given confidence interval, of the probability of a rider staying on the Cretan bull for eight seconds. The requisite data is taken from the website of Professional Bull Riders, Inc. The second problem concerns the probability that the bull will attack someone given their distance from Marathon and a totally artificial density function of the probability $A(t)$ that the bull will attack someone within t km of Marathon.

Unfortunately, some of the modelling is dubious. For example, in the Third Labour, Hercules is required to carry the Cyrenitian hind from Mt Artemisius to Mycenae. Huber claims that, assuming a mass of 125 kg and a distance of 80 km, the work done by Hercules is $125g \times 80\,000$ Newton-metres.

The mathematical topics considered include differential and integral calculus, multivariable calculus, separable differential equations, Euclidean geometry, trigonometry, difference equations, combinatorics, and probability, including simulations. There is a rudimentary appendix on solving differential equations by Laplace transforms.

The lack of a systematic development of mathematical topics, together with the format of the book, providing full solutions following each problem, makes it unsuitable for use as a course text, even a supplementary one. But this is not the author's intention. What we have instead is a reference book that is ready to be picked from the lecturer's shelf during a course to inspire appropriate homework problems. For this purpose it is ideal, provided it is used with discretion.

Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA 6009.
E-mail: schultz@maths.uwa.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Mrs Perkins's Electric Quilt

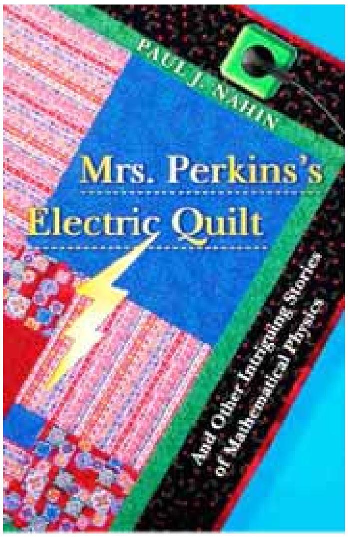
Paul J. Nahin
Princeton University Press, 2009, ISBN: 978-0-691-13540-3

Paul J. Nahin is a highly experienced author of popular mathematics books covering a wide range of topics, and this book is a well-polished addition to his work.

Over 19 largely independent chapters, Nahin discusses a number of questions mostly based on physical phenomena such as electricity, gravity and air resistance, along with a few chapters on random walks. He then shows how each of these questions is best solved with a mixture of mathematics (mostly calculus) combined with physical arguments and intuition — this 'mutual embrace' of mathematics and physics being the unifying theme of the book.

Setting the tone, an example early in the book considers the classic calculus exercise of calculating how fast the top of a ladder leaning against a wall moves down the wall if the bottom end is pulled horizontally away from the wall at a constant speed. The fairly simple mathematical solution to the exercise has the significant physical flaw that at the end of the ladder's fall, as it hits the ground, its top end is moving infinitely fast! How can the mathematical reasoning be reconciled with the physical intuition telling us that this cannot occur? (See the end of this review for the answer.)

The chapters are grouped in such a way that each group explores variations on a single topic. Thus a sequence of chapters starts by considering various properties of falling objects subject to an unspecified (but physically plausible) air drag and then extends and varies the analysis by adding the precise details of the air drag. Other thematic groups of chapters consider gravity, both above and inside the earth, and random walks, and the book is rounded off with a few chapters that stand alone. The book's title comes from one of these chapters: Mrs Perkins's Quilt is the name given by Henry Dudeney to a tiling of an integral square with smaller integral subsquares. In this chapter, Nahin explains the famous work of Brooks, Smith, Stone, and Tutte, who showed how to associate an electrical network with a squared rectangle — hence 'Mrs Perkins's Electric Quilt' — and derived various consequences from this association.



The writing is uniformly excellent and a pleasure to read. Nahin writes in a warm, conversational style, enhancing the mathematics (and physics) with liberal doses of background, historical notes, personal anecdote, and literary and cultural references, both old and new. The mathematics itself is presented carefully and completely (clearly Nahin does not subscribe to the view that every equation reduces a book's sales by half) with almost every chapter containing extended derivations, mostly calculus-based. The book claims that the assumed background is that of a student who has finished and understood 'first year in a technical major at a good American college or university', but this seems extremely optimistic to me unless this single year contains vastly more calculus than is usual in an Australian first-year syllabus.

Each of the chapters contains a few challenging problems (to which solutions are given) for the reader to work through, and finishes with comprehensive notes and academic references. This is definitely not a book for a recreational mathematician looking for some gentle mental stimulation, but more a book for a serious student with some aptitude for applied mathematics. It could also be used very effectively as a resource for lecturers looking for interesting extended examples or projects for their students. Overall this is an excellent book for those who have the necessary background, but its implicit marketing as a 'popular mathematics' book may disappoint those expecting a lighter read.

Back to the question raised earlier about the ladder: the mathematical model of the situation leading to the erroneous conclusion is that the ladder forms a triangle with the wall and the floor. It is this idealised model that is incorrect at the boundary — as the ladder is pulled away, the top end 'breaks away' from the wall at some point, and Nahin shows us how to analyse this situation. This example encapsulates the overarching theme of this book — how physics and mathematics can inform and illuminate each other.

Gordon Royle
School of Mathematics and Statistics, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009. E-mail: gordon@maths.uwa.edu.au