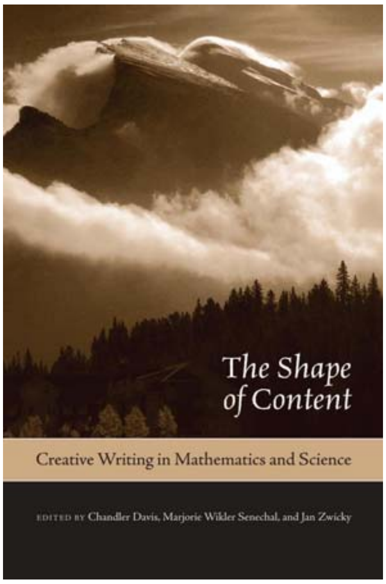
Book reviews

The Shape of Content: Creative Writing in Mathematics and Science

Chandler Davis, Marjorie Wikler Senechal and Jan Zwicky (eds)

A.K. Peters, 2008, ISBN: 978-56881-444-5

This volume, misleadingly subtitled 'Creative Writing in Mathematics and Science', is an anthology of poems, essays, plays, short stories and biographies, each of which has a mathematical or scientific theme. It arose from a series of workshops at the Banff Centre for the Arts and the Banff International Research Station for Mathematical Innovation and Discovery and so has a preponderance of Canadian authors. Some of them are professional writers and others practising mathematicians and scientists. Because of the extensive variety of content, I will confine my review to those sections of most interest to mathematicians.



The literary genre to which mathematicians are most likely to contribute is biography. The geometer Marjorie Senechal's *The Last Second Wrangler* (you have to be a mathematician to understand the title) is a fine biography of Eric Neville (1889–1961). He was the last 2nd Wrangler in 1909 because, largely due to the efforts of G.H. Hardy, the ranked listing of successful candidates in the Cambridge Mathematical Tripos was abolished in 1910. In order to be listed, Neville took the exams a year early, only two years after finishing high school. Ironically, Hardy's main argument against the competitive aspects of the Tripos was that it had no relationship to creative work in mathematics. Yet the first three Wranglers in 1909 were P.J. Daniell (of the Daniell integral), Neville and the number theorist L.J. Mordell! A principled pacifist, Neville spent the war years working in a London hospital. Apart from being a distinguished geometer, he was the first English mathematician to encourage Ramanujan and facilitated his contact with Hardy. Senechal's essay is a fragment of her forthcoming biography of the eccentric mathematician–chemist Dorothy Wrinch who had a 40-year romantic relationship with Neville. It apparently did not affect their respective marriages.

A second example in this genre is *The Birth of Celestial Mechanics* by Florin Diacu, an applied mathematician at Victoria University, British Columbia. It is a short biography of Newton, with emphasis on his life-long work on the motions of the planets. It is a delight to read, relating Newton's work to our current knowledge of celestial mechanics.

Another area of literature in which mathematicians excel is satire and parody. One thinks immediately of the humorous columns in the *Mathematical Intelligencer*, and of its resident humorist, the knot theorist Colin Adams. This volume contains a reprint of one of his best, *Robbins v. New York*, which was inspired by a real 2005 case before the U.S. Court of Appeals. A drug dealer was charged with the felony of selling drugs within 1000 feet of a school. The case hinged on the problem of how the 1000 feet was to be measured; that is, should the metric be Euclidean or taxi-cab? Adams imagines what would have happened if the case had come before the learned justices of the Supreme Court, each interested in outwitting his fellows by introducing a new metric. Naturally, relativity and quantum effects also appear.

A more problematic literary genre is 'metafiction' or fictionalised biography. In *Evariste and Heloise*, Marco Abate, a geometer from Pisa, imagines a confrontation between Galois and Auguste Dupin, the fictional detective from Poe's *The Murders in the Rue Morgue*, a thriller set in Galois' Paris of 1832. A present-day mathematics teacher Heloise, who may or not be entirely fictional, also makes an appearance. In her dreams, she is present during Galois' last hours. Incidentally, Abate's story originally appeared as an Italian comic strip, some frames of which are included.

Short stories are represented by mathematician Alex Kasman's *On the Quantum Theoretic Implications of Newton's Alchemy*, in which a grotesque Dr Frankenstein figure uses quantum theory to transmute lead into gold, with unforeseen consequences. More successful is mathematician Manil Suri's *The Tolman Trick*, a rather plausible account of one-upmanship among egocentric mathematicians at an Oberwolfach Conference.

The volume contains two extracts from librettos of musical dramas. Ellen Maddow, a composer and dramatist, provides an excerpt, *Delicious Rivers* including song lyrics and dance steps, supposedly based on Penrose tilings. It may work in the theatre, but left me cold. More successful is *Star Messengers* by Paul Zimet, artistic director of a New York theatre company, based on the lives of astronomers. The excerpt includes poignant songs for actors portraying Kepler and Galileo as they approached death.

Twenty-three poems by nine authors, seven of whom are mathematicians or scientists (including Chandler Davis), complete the volume. To my mind, a successful lyric poem should be graceful, evocative and memorable. A notable example in this volume is *Dissecting Daisy*, by Madhur Anand, an ecologist at the University of Guelph. It is a meditation on the colour orange, and in 25 short lines (plus a couple of footnotes) we learn that a word for the colour, describing a common lichen, existed in Old English. However, the word 'orange', which has a Sanskrit root, appeared in European languages only with the introduction of the fruit to Europe in the 16th Century. We learn too that the Dutch masters rendered the colour by a mix containing sulphur and arsenic, and that it evokes in the author memories of sipping tea as the sun bursts through clouds.

On the whole, by my admittedly subjective criteria, the professional poets are clearly superior in this genre. Adam Dickinson invokes infinitesimals, bijections and continuous functions as metaphors for his own emotional life. Susan Emslie uses algebraic images, calling on the original Arabic meaning of Algebra as the reunion of broken parts, to appeal for reason and an equitable balance of power in human affairs. Philip Holmes appeals to the intermediate value theorem and the notion of commuting operations to make sense of his personal experiences.

A question that springs to mind is why do mathematicians seem to take less interest, either as producers or consumers, in poetry than in the other arts? After all, poetry and mathematics share common features. For example economy, that is, packing a lot of meaning into a few words, and using analogy to convey ideas. Perhaps poetry is just too difficult for us?

Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA 6009.
E-mail: schultz@maths.uwa.edu.au

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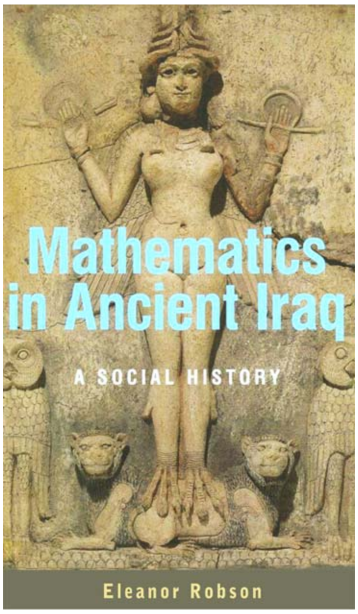
Mathematics in Ancient Iraq: A Social History

Eleanor Robson
Princeton UP, 2008, ISBN: 978-0-691-09182-2

One of the more pleasant aspects of research into the history of mathematics is that this is an area in which researchers feel freer than in most others to express how passionately they are devoted to their subject.

There is rarely enough hard evidence to enable a researcher to say, about some past development, definitively what happened, why it happened, who did what, who they were doing it for, what were their aims in doing it, et cetera. While this applies to developments over the last few centuries, it applies far more so in regards to the first recorded glimmerings of mathematics, such as those that took place in the Middle East, notably the area now called Iraq, from about 3000 BCE to the beginning of the Common Era. The book under review, which covers precisely this era and locality, is indeed argued passionately, persuasively and, I am pleased to add, enjoyably. The author's forceful, somewhat hyperbolic style leads to gems like: 'Comparisons of Euclidean or Archimedean treatises with Old Babylonian word problems do not compare like with like, for the earlier tradition was written by and for pre-pubescents, not research-active adults' (p. 286).

That mathematical documents survive from this early epoch, as from no other, is largely attributable to the durable nature in this arid region of the medium (clay tablets) on which it was recorded. So far about 950 tablets which can be broadly described as mathematical, have been identified and published, mostly in the early twentieth century. A large majority of these have been analysed and translated in the groundbreaking publications of Neugebauer, Thureau-Dangin and others in the 1920s and 1930s.



Jens Høyrup's 2002 book [1], [3] addressed the same texts so far published, and a reasonable question is: is there scope for another work to tread over this ground, given that virtually no new source material has emerged in the seven years since it appeared? I am pleased to answer this with a resounding 'Yes'. How this can be, if the underlying mathematics described is essentially the same in all cases, is tackled head-on by Robson. She explains that it is vital to consider the place of mathematical activities in the culture in which they occur. To analyse early mathematics only from the mostly Platonic viewpoint predominant among mathematicians of recent centuries is to blind ourselves to the culturally specific ways in which practitioners and other members of these societies viewed them. Is it even meaningful to refer to 'underlying mathematics' being the same or 'essentially the same'? To always answer 'yes' to this question is to impose a certain perspective on what mathematics is, a perspective which may not have been espoused by the original writer. Here is a concrete case from another culture where these issues arise:

> The enunciation of Euclid's *Elements* II, Proposition 1, reads: 'If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments' ([2, p. 375]). Can one say that this is 'essentially the same' as the Distributive Law?

Such questions were first vigorously raised in 1975 by Sabetai Unguru [5], and subsequently historians of mathematics have become more wary of referring to essential sameness. Høyrup [3] had tackled culturally specific aspects of Babylonian mathematics via his 'conformal translations' of the texts and 'cut and paste algebra'. Robson has carried this process further, where possible, by referring to the specific archaeological contexts (where known) in which mathematical texts have been found. Moreover, Robson specifically excludes from detailed consideration those texts, mostly of unknown provenance, which have been repeatedly considered in earlier works.

In lectures I've given in the past on the history of mathematics, I have stated that classical Greek mathematics was the oldest mathematics which can be associated with named individuals. Having read Robson's book I will no longer make such a mistake. For instance, I read on p. 94 'One Iškur-mansum, son of Sîn-iqīŝam, signed a compilation of worked solutions on problems about brick walls and right-angled triangles' probably in late 17th century BCE Sippar. This man was probably a teacher, and another nine compilations of word problems may be ascribable to him. It is particularly grounding to then read that the ten tablets are riddled with errors, some of which suggest that Iškur-mansum was copying material he did not understand! There have always been bad teachers.

What I find most grounding of all in this book is that both mathematics and literature (at least writing) seem to stem from fourth millennium accounting, so that accounting may turn out to be both the primal art and the primal science!

In the case of mathematics, Robson argues that the tablets show that, during the third millennium, concerns developed beyond mere record-keeping to quantifying needs, such as the need to have dykes repaired, and then predicting from past observations how many man-days will be needed to get the job done. But all this occurred in a welter of metrological systems, making calculations involving fractions quite difficult.

Robson considers that the development of the sexagesimal number system, indeed the evolution of a number concept, flowed from the partial standardisation of metrological units by 2000 BCE to facilitate calculations such as the above, from the use of a discrete numeral notation followed by a separate sign for the unit and from the call for complex annual balanced accounts.

As mentioned above, Robson passes over many of the texts from the 'Old Babylonian period' (roughly 2000–1600 BCE), made famous through the research of Neugebauer *et al.*, which address problems involving areas, lengths, widths and diagonals with 'cut and paste' methods, since these have been thoroughly analysed by Høyrup. She emphasises instead those recent finds which enable conclusions to be drawn about the contexts in which mathematics was learned and used. Notable among these is 'House F' in Nippur around 1700 BCE. Texts found in this small house indicate it was used to train scribes in (small) classes.

It appears that in this school, immediately after mastering the basics of cuneiform writing, students had to learn to write lists of quantities, such as sizes of boats, weights, and measures. Next followed the rote learning of lists of metrological facts such as '1 finger 0;02 cubits; 2 fingers 0;04 cubits' etc, which involve multiplication incidentally, as well as more explicit (unit free) multiplication and reciprocal tables. Finally, scribal students learned how to write legal contracts involving areas, capacities, weights et cetera. In this school some more advanced mathematics was also taught, as evidenced by a tablet which appears to indicate step by step, albeit without words, how to calculate a reciprocal.

That accuracy in measurement ranked very highly — indeed more highly than literacy — in the Old Babylonian period is further attested to, according to the author, by various sculptures and texts which commemorate kings being endorsed by deities. This endorsement consists in them being presented with a measuring rod and a rope, a far cry from the crown, orb and sceptre still used in royal iconography.

Robson stresses the importance of 'metrological justice', fairness and equality based on accurate measurement. Just how this might have worked out in practice is another matter. For instance was it considered unjust to allocate to one heir a square, and thus larger, block with the same perimeter as an elongated rectangular block assigned to another heir, when the latter block has a small frontage on an irrigation channel and the former none?

Robson next turns her attention to the far less documented mathematics in Assyria some 300 km north of Babylonia. Few texts survive from the Old Assyrian period but those that do suggest they were written by traders, both male and female, rather than scribes. Just one mathematical text survives from the Middle Assyrian period (1350–1000 BCE) and five later ones. Part of the reason for this is that, as a relief indicates, some scribes now wrote on waxed (and hence erasable) wooden boards, and on perishable media such as parchment and papyrus. An inscription on behalf of King Aššurbanipal ca. 640 BCE has him claiming, 'I solve difficult reciprocals and multiplication lacking clear solution . . . ' as well as mind-boggling skills such as 'interpret[ing] the series, "If the liver is the mirror of heaven".' (p. 147) Oh, that royalty would still possess such skills!

Finally coming to the last period covered, from about 485 BCE to 75 CE after which no more cuneiform tablets appear to have been written, the author proposes to define as a mathematical document anything 'written for the purpose of communicating or recording a mathematical technique or aiding a mathematical procedure to be carried out' (p. 218) and then notes that a large collection of astronomical procedure texts fall within this category. Further, attempts to find patterns in other natural phenomena resulted in diaries of atmospheric and terrestrial observations also being kept. Accuracy in prediction was certainly appreciated: apparently (p. 260) a scribal family had predicted a lunar eclipse in 531 BCE which failed to happen. This led to the entire family becoming the subject of an official inquiry. These days, inaccurate economic forecasts are not so diligently pursued.

Robson draws attention to the distinction made, from Old Babylonian times until this late period, between the vocabulary of physical addition and subtraction on the one hand, and their numerical counterparts on the other. But, in the later period, besides the cut and paste completion of the square technique for solving problems which involved areas and sides, a new technique was also used. This new technique does not involve any cut-and-paste procedure. It might be worth noting here that tablets using this new technique are broadly contemporaneous with Euclid and the other famous Greek mathematicians.

Some other new techniques have emerged as well, such as calculating reciprocals of regular numbers by repeated factorisation, again a technique that avoids cut and paste methods.

A couple of corrections. One minor, but frequent idiosyncrasy is Robson's use of the term 'coprime to 60' when she refers to numbers which have a prime factor coprime to 60. This occurs on pp. 135, 145, 147, 179 and 265 and possibly others.

Usually historians of mathematics refer to such numbers as (sexagesimally) irregular. She gets it completely wrong on p. 132 where she refers to '7, 9 and 11 all [being] coprime to 60' and she appears to consider 81 as irregular on p. 145.

Further errata: p. 130 has '981' instead of '891'; p. 145 second last paragraph has a spare '13'; p. 279 has $l = (l - b) + b$ instead of $(l - w) + w$.

I also found some parts of the second last chapter rather turgid, with its long lists of texts written by members of various scribal families and summaries of their content. However, all in all I can heartily recommend this book, in addition to Hoyrup's, to anyone who wants to dig a bit below the surface into this early chapter in mathematics. And as a final reflection I note that the 2500-year span of cuneiform mathematics was as long as the entire period from Pythagoras to the present.

References

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Bob Berghout
School of Mathematical and Physical Sciences, University of Newcastle, NSW 2308.
E-mail: Bob.Berghout@newcastle.edu.au