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The Australian Mathematical Society

Gazette

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- Mathematical articles of general interest, particularly historical and survey articles
- Reviews of books, particularly by Australian authors, or books of wide interest
- Classroom notes on presenting mathematics in an elegant way
- Items relevant to mathematics education
- Letters on relevant topical issues
- Information on conferences, particularly those held in Australasia and the region
- Information on recent major mathematical achievements
- Reports on the business and activities of the Society
- Staff changes and visitors in mathematics departments
- News of members of the Australian Mathematical Society

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Volume 36 Number 2 2009

Editorial

Welcome to the May issue of the *Gazette*.

The lack of suitably qualified maths teachers has long been a concern, but recently the state of maths education in Australia has received wide press coverage. Nalini Joshi discusses the recent coverage in her President's column, and highlights the sometimes illogical arguments that have to be countered when dealing with national issues affecting mathematics.

But a good supply of well prepared maths students is only part of the answer, and we are all aware of the difficult situation of mathematics in universities. In Maths Matters, Larry Forbes tries to overcome 'review fatigue' to reflect on the reaction of the maths community to the Bradley Review of Higher Education. Will concentrating mathematical research in a few centres create the great mathematicians of tomorrow, or would it be better to embed mathematics in all degrees across the country?

Norman Do's Puzzle Corner once again not only provides you with opportunities for glory and a trip to the book shop (congratulations to James East, winner of Puzzle Corner 10), it has also inspired Gerry Myerson's paper on error-correcting codes. And this issue's Puzzle Corner gives you a chance to apply mathematics to your vegetable drawer, and might just inspire you to new culinary heights, as well as mathematical ones!

You can also read the first of a new series, 'Mathematical minds', where we interview Australian mathematicians about their work and their lives, and try to understand the minds behind maths. This issue's interview is with Nalini Joshi, who tells us how she went from the child of Burmese migrants to the top of the mathematical profession today.

For those of you who thought the Access Grid was only accessible from dedicated AG suites, Bill Blyth and Jason Bell give alternative ways to make use of this technology via your humble desktop or laptop computer. We also have a paper on a geometric approach to saddle points, book reviews and all the news from Australian mathematics. There are plenty of books available for review on the *Gazette* website (see http://www.austms.org.au/Gazette+-+books+available+for+review). Please contact us if you would like to review one of these books or suggest another reviewer.

Happy reading from the *Gazette* team.

President's column

Nalini Joshi[*]

Imagine a world without mathematics: there would be no mobile phones, no internet shopping, no CDs or DVDs, no DNA analysis, no climate modelling, no science. Mathematics is critical to modern life and it will be essential for formulating a response to the current economic crisis. These were the opening lines of my submission to *The Australian* on the dire state of mathematics education in Australia[1]. In addition to this article headed 'A disturbing set of numbers'[2], I have had a letter published in the *Australian Financial Review*, was interviewed for an article in the *Sunday Telegraph* and interviewed on ABC radio three times on World Mathematics Day.

Of course, my message is well known and has been stated before. Australian Year 8 students' performance in the TIMMS study has been declining since 1995. This is probably responsible for the fact that fewer Australian students attempt advanced level mathematics subjects at high school each year and consequently, fewer students attempt to complete mathematics majors at Australian universities each year. The potential pool of skilled mathematics teachers consists of the pool of mathematics majors. And so fewer teachers are available to nurture and encourage our young students to study mathematics. This vicious cycle continues and deepens each year.

Other people and groups have reiterated the same message. The latest submission to the Government entitled 'A National Strategy for the Mathematical Sciences in Australia'[4] was prepared by Professor Hyam Rubinstein, who chairs the National Committee for Mathematical Sciences, with the support of AMSI and the backing of the Australian Council of Heads of Mathematical Sciences. I was pleased to see that journalists and radio interviewers have heard the message and thought it important and interesting enough to provide the space and air-time to allow it to be heard again.

One august and powerful body which has reiterated this message is the Productivity Commission. Its Research Report entitled 'Public Support for Science and Innovation'[4] contains the following statements:

> In the case of science and mathematics teachers, shortages have instead been accommodated by using teachers without adequate skills in these areas. This may adversely affect student performance and engagement and decrease future university enrolments in the sciences. (p. XXIV)

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[1] These lines were deleted in the published version by the sub-editor by reason of space limitations.

[2] http://www.theaustralian.news.com.au/story/0,,25069225-25192,00.html

[3] http://www.amsi.org.au/pdfs/National_Maths_Strategy.pdf

[4] Released 27 March 2007 and available from
http://www.pc.gov.au/projects/study/science/docs/finalreport.

> In regard to teachers of science and mathematics, most jurisdictions have reported ongoing shortages and difficulties in recruitment. More recent analysis indicates that these problems continue. p. 249

> According to DEST, there was a strong perception that Australia lacked sufficient suitably qualified secondary school science and mathematics teachers which had adverse impacts on student engagement in science, engineering and technology. Importantly, the lack of suitably qualified teachers in these areas limited the ability of the system to expand to increase the number of students studying mathematics and science in the senior years of high school and potential entrants to tertiary studies in science, mathematics and related courses. p. 721

It is clear from these excerpts that this issue has been brought time and time again to the attention of decision-makers in government. So my question is, what is the government waiting for?

I had a tiny glimpse of what more might be needed when I attended the Science Meets Parliament dinner, held at Parliament House in Canberra on 17 March 2009. I had the opportunity to buttonhole the Honorable Senator Kim Carr for five minutes in the mingling before dinner. I gave him the above message. Yes, he had heard it before, but there were many competing claims all stating that Australia would be in dire straits unless something was done for their area.

I moved on to a second message: that full-funding for research is essential in order to stop the erosion of overall funding to mathematics within the University system. The usual budget models adopted by Universities include a tax for 'strategic research initiatives'. For mathematics, this invariably eats into our allocation arising from our student numbers; the more students we teach, the more we get from our teaching allocation and the more tax is taken off the top. We get very little of this tax back because any claim we might make on a strategic research initiative is so cheap compared to those people who need wet labs (or telescopes) to conduct their research. If the government brought in full-funding for research then universities would not need to impose a tax in order to fund expensive research and mathematics would get the funding it is entitled to have to teach. Senator Carr heard this message also.

But my insight came when I repeated the same message to one of his departmental staff. She said she had heard the message before but she needed to convince Treasury. After all, we were saying that more funding was needed to do the same work; that is, people would do the same amount of research as before, but Treasury would need to cough up more funds for it. I was so flabbergasted for a moment that I lost the opportunity to make the logical response, that is, of course we would be more productive if more funds were available from our teaching allocation to actually allow us to hire more teaching and research staff. This would lead to lower student-staff ratios and more time to do research.

The fact that I had not heard this objection before leads me to wonder how many other illogical and political arguments are out there to counter the evidence we provide to government. As a collective group of mathematicians, we need to become more aware of these exercises in shadow-boxing that go on behind the scences. We need to be more agile and practiced at making appropriate on-the-spot responses to all such political manoeuvres. I am sorry to say that as a mathematician more used to logical deduction, I need a lot more practice!

Nalini Joshi holds a PhD and MA from Princeton University in Applied Mathematics and a BSc (Hons) from the University of Sydney. In 2002, she returned to the University of Sydney to take up the Chair of Applied Mathematics and became the first female mathematician to hold a Chair there. In 2008, she was elected a Fellow of the Australian Academy of Science. She is currently the Head of the School of Mathematics and Statistics. Her research focuses on longstanding problems concerning the asymptotic and analytic structure of solutions to non-linear integrable equations.

Maths matters

What do we need: herd immunity or a lone gold medallist?

Larry Forbes[*]

There's been quite a heated stoush in the newspapers lately, following the release of the Bradley Review of Higher Education in Australia[1]. In particular, some of the larger better-resourced universities are running an opposing position that favours 'diversification', and words like 'competition' are being tossed into the argument. There are sinister rumblings that quality research won't be possible under the Bradley reforms, and that we will all slip into mediocrity as a result.

At first reading, it all sounds noble enough, and most of us in Mathematics Departments could only agree that our beloved discipline has been savaged over the past decades. Furthermore, most of us who've been in the system for a while have 'review fatigue', and enjoy a healthy scepticism about the capacity of any government-inspired review to deliver any actual improvement.

However, I think it's worth digging below the surface of these objections to the Bradley Review, and asking how mathematics might be affected. Beneath some of the talk about diversity in the sector is the suggestion that research, and particularly the funding that goes with it, should be concentrated in only a few universities. And not surprisingly, the same universities that are proposing this concentration are recommending that they themselves should be the sole beneficiaries.

From my point of view in one of the smaller universities, this talk of research concentration by the large, to their own advantage, has all the altruism of a school of hungry piranha fish. It might, however, possibly appeal to politicians and their ilk, since it could seem to offer the prospect of easy answers to difficult questions, along with the advantage of snappy photo opportunities in the midst of a large and well-equipped research group. But I think we need to dig below even the self-interest of these suggestions, because they raise fundamental questions about what research is, and who gets to do it.

In an opinion piece in *The Australian* newspaper's 'Higher Education Supplement' (Wednesday 18 February 2009)[2], it was argued that we should get rid of our 'one-size-fits-all' notion of universities, which was described as a worn-out approach. It was argued that the teaching-research nexus is a 'myth'. By ending this myth, the way would presumably be clear for concentration of research, and its funding, in the hands of a self-appointed few.

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[1] http://www.deewr.gov.au/HigherEducation/Review/Pages/default.aspx

[2] http://www.theaustralian.news.com.au/story/0,25197,25069582-25192,00.html

I believe this argument is fundamentally and fatally flawed, and fails on its own logical criteria. It is disingenuous to suggest that all research processes in every discipline are equivalent and therefore need equal levels of concentration. The 'one-size-fits-all' description of research certainly does not work.

One of the many things I love about mathematics is its egalitarianism and its subversive impudence. Good mathematics can be done by all sorts and classes of people. This is an exciting time to be a mathematician, particularly with the relative cheapness of computing power and memory and the easy access to information afforded by the internet. Perhaps there are some disciplines that might be reliant on expensive equipment for their next research idea, but this is not a prerequisite for mathematics to thrive, although admittedly having occasional access to a group of talented people may enhance our own research creativity. At least as far as mathematics is concerned, there is just no compelling economic argument for demanding that research should be concentrated in only a few centres.

There is also the question of what, actually, constitutes mathematical research? I believe that anyone who is seriously studying mathematics at university level is engaging in research of one sort or another, though perhaps with somewhat limited horizons. After all, every student who struggles with a new mathematical technique or concept, and attempts to apply it to a new situation, is genuinely undertaking the process of research. If they are coming to terms with existing knowledge, then the fruits of their research will, of course, not be original or publishable, but I would argue that the *process* they are undertaking is not qualitatively different from what we all do when we develop new results for a research paper. I would therefore suggest that, far from being a 'myth', the teaching-research nexus is a vibrant reality in mathematics, even if it might not be in some other disciplines.

I want to argue that mathematics at all levels is so important that the benefits of having it available across the sector far outweigh any gains from concentrating it in a few locations. In fact, is it even possible to curtail research in those institutions deemed to be research-inactive, without imposing draconian teaching hours or administrative burdens on staff? I believe not. The twin activities of teaching and research in mathematics are too closely entwined for that to be an achievable outcome, much less a desirable one.

Will Australian mathematics become mediocre if we do not attempt to concentrate research? Actually, I hope so! After all, 'mediocre' literally means somewhere in the middle, and I don't think we are that far advanced yet. Too many universities still promote science-type degrees in which mathematics is seen as an optional extra, perhaps available to select clients with unusual tastes, rather than the fundamental language of technology that it really is. When students can graduate with degrees in biology, computing or environmental science and have had almost no exposure to basic calculus or statistics, then that's a major cause for concern. And at the school level, the opportunity for keen students to study mathematics is actually shrinking, as Nalini Joshi has argued in the national press. All this convinces me that mediocrity in Australian mathematics is still a worthy goal to be aspired to.

A mathematically and quantitatively literate population bestows a kind of ‘herd immunity’ on the community in the sense that everyone benefits, including people who themselves may have no interest in mathematics. This situation is certainly what we need, if we are to maintain our first-world living standards in the long term. For this to occur, we need a wide dissemination of mathematics to a reasonable standard, rather than a narrow concentration in a few centres to the neglect of everywhere else. Furthermore, this is precisely what is needed in order to foster the next generation of gold-medal-winning research stars that the system claims it wants to produce.

Larry Forbes is the professor of applied mathematics at the University of Tasmania. He was privileged to have done his PhD in 1981 at the University of Adelaide under the supervision of Len Schwartz and Ernie Tuck. There, he was introduced to the delights of free-surface hydrodynamics and integral-equation methods. He subsequently held assistant professorships at the Hydraulics Lab at the University of Iowa and in the Mathematics Department at Kansas State University. He then obtained a position at the University of Queensland in the Mathematics Department, and was there from 1985 until 1999. In 2000 he took up the professorship at the University of Tasmania. He was head of the School of Mathematics and Physics from 2001 until 2008, which is a chic occupation for someone with a mild aversion to administration, and he also served briefly on the ARC mathematics panel. He has research interests in fluid mechanics, dynamical systems and the design of medical imaging equipment, and has been in the fortunate position of enjoying long research collaborations with a number of colleagues and friends who work in these areas.

Puzzle corner

Norman Do[*]

Welcome to the Australian Mathematical Society *Gazette*'s Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of $50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judge's decision — that is, my decision — is absolutely final. Please e-mail solutions to N.Do@ms.unimelb.edu.au or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 12 is 1 July 2009. The solutions to Puzzle Corner 12 will appear in Puzzle Corner 14 in the September 2009 issue of the *Gazette*.

Soccer stats

At some point during the soccer season, a player had scored in less than 80% of the matches. At some later point during the season, they had scored in more than 80% of the matches. Prove that there must have been some point when the player had scored in precisely 80% of the matches.

Photo: Andrzej Skwarczyński

Loopy potatoes

You are given two potatoes. Show that it is possible to draw a non-trivial loop on each so that the two loops are congruent when considered as subsets of space.

Island tour

A circular island is divided into states by a number of chords of the circle. Consider a tour that starts and ends in the same state without passing through the

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intersection of any two borders. Prove that the tour must involve an even number of border crossings.

Table trouble

One thousand chairs, numbered from 1 to 1000, are equally spaced around a circular table. There is a plate on the table in front of each chair and these are also numbered from 1 to 1000. Is it always possible to rotate the table so that no chair has a number which matches the number on the corresponding plate?

Height differences

Twenty boys and twenty girls are paired off to form twenty couples. The difference in height between the boy and girl in each couple is no more than 10 centimetres. The boys and girls are then rearranged, so that the tallest boy is paired with the tallest girl, the second-tallest boy with the second-tallest girl, and so on. Prove that the difference in height between the boy and girl in each couple is still no more than 10 centimetres.

Prisoner perplexity

One hundred prisoners will each have a random integer from 1 to 100 written on their forehead. They will then be taken to a room where they can see the other prisoners' numbers, but not their own. Afterwards, they will be individually asked to guess the number on their own forehead. If at least one prisoner supplies the correct answer, then they will all be released; otherwise, they will all be executed. Given that the prisoners are allowed to discuss their strategy beforehand, show that they can devise a scheme which guarantees their freedom.

Photo: Robin Davis

Weighing coins

(1) You are given 68 coins, all of whose weights are distinct. Determine the lightest and heaviest coins by using a balance scale 100 times.

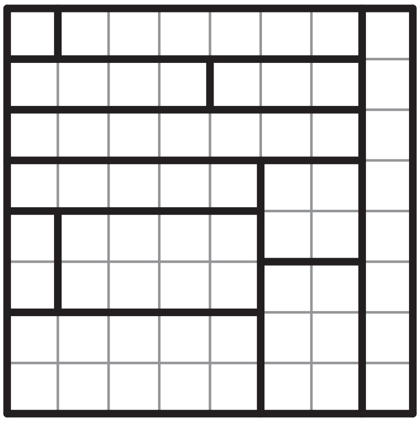
(2) You are given 61 coins. Two of them, whose weights are equal, are counterfeit and are either lighter or heavier than a genuine coin. Determine whether the counterfeit coins are lighter or heavier than a genuine coin by using a balance scale three times. (It is not necessary to identify the counterfeit coins.)

(3) You are given 12 coins. One of them is counterfeit and is either lighter or heavier than a genuine coin. Determine the counterfeit coin and whether it is lighter or heavier than a genuine coin by using a balance scale three times.

Solutions to Puzzle Corner 10

The \$50 book voucher for the best submission to Puzzle Corner 10 is awarded to James East.

Chopping a chessboard



Solution by James East: There are precisely 13 rectangles with integer side lengths and area at most 9: 1×1, 1×2, 1×3, 1×4, 2×2, 1×5, 1×6, 2×3, 1×7, 1×8, 2×4, 1×9 and 3×3. Therefore, 13 rectangles, no two of which are congruent, must tile an area greater than or equal to the sum of their areas, namely 72. So it is impossible to cut a regular 8×8 chessboard into 13 rectangles along its gridlines so that no two of the rectangles are congruent. However, the diagram shows that it is possible to cut it into 12 rectangles in such a manner.

How many numbers?

Solution by Simon Tyler: Suppose that the real numbers are $x_1 \leq x_2 \leq \ldots \leq x_n$. The problem states that $x_1 + x_2 + x_3 = 5$, $x_{n-2} + x_{n-1} + x_n = 7$, and $x_1 + x_2 + \cdots + x_n = 20$. It follows that $3x_3 \geq 5$, $3x_{n-2} \leq 7$, and $x_4 + x_5 + \cdots + x_{n-3} = 8$. These combine to give us

$$(n-6)x_3 \leq 8 \leq (n-6)x_{n-2} \Rightarrow \frac{5}{3}(n-6) \leq 8 \leq \frac{7}{3}(n-6) \Rightarrow \frac{66}{7} \leq n \leq \frac{54}{5}.$$

The fact that n is an integer gives the result $n = 10$.

Penny in a corner

Solution by John Graham: The locus is the part of the sphere $\{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 2\}$ lying in the cube $\{(x,y,z) \in \mathbb{R}^3 \mid -1 \leq x,y,z \leq 1\}$. The crux of the solution is the following result, whose proof is an exercise in analytic geometry.

> Let $(u_1, u_2, u_3) \in \mathbb{R}^3$ be a unit vector. Consider the disk with centre (q_1, q_2, q_3) which lies in a plane perpendicular to the unit vector (u_1, u_2, u_3). Then the disk is tangent to the yz-plane if and only if $q_1^2 + u_1^2 = 1$. Similarly, the disk is tangent to the zx-plane if and only if $q_2^2 + u_2^2 = 1$ and to the xy-plane if and only if $q_3^2 + u_3^2 = 1$.

It remains to determine for which $(q_1, q_2, q_3) \in \mathbb{R}^3$ we can simultaneously solve

$$q_1^2 + u_1^2 = 1, \qquad q_2^2 + u_2^2 = 1, \qquad q_3^2 + u_3^2 = 1, \qquad \text{and} \qquad u_1^2 + u_2^2 + u_3^2 = 1.$$

Observe that $q_1^2 + q_2^2 + q_3^2 = (1 - u_1^2) + (1 - u_2^2) + (1 - u_3^2) = 3 - u_1^2 - u_2^2 - u_3^2 = 2$. Hence, the centre of the disk must lie on the sphere $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 2\}$. Also, $q_1^2 = 1 - u_1^2 \leq 1$ implies that $-1 \leq q_1 \leq 1$ and, similarly, we have $-1 \leq q_2 \leq 1$ and $-1 \leq q_3 \leq 1$. Hence, the centre of the disk must also lie in the cube $\{(x, y, z) \in \mathbb{R}^3 \mid -1 \leq x, y, z \leq 1\}$. Conversely, given (q_1, q_2, q_3) on the prescribed locus, we can find $u_1 = \pm\sqrt{1 - q_1^2}$, $u_2 = \pm\sqrt{1 - q_2^2}$, $u_3 = \pm\sqrt{1 - q_3^2}$, so these two necessary conditions are also sufficient.

Complete the table

Solution by Peter Nickolas: Let A_n, B_n, C_n and D_n denote the top-left, top-right, bottom-left and bottom-right quarters of the $2^{n+1} \times 2^{n+1}$ block of numbers in the top-left corner of the table. We will prove by induction that

- each row and column of A_n contains all of the numbers $0, 1, 2, \ldots, 2^n - 1$;
- the blocks A_n and D_n are identical; and
- the blocks B_n and C_n are identical and obtained by adding 2^n to each entry in A_n.

It can be checked that the statements hold for small values of n. In order to proceed by induction, we now suppose that they hold for some particular value of n, and consider the block A_{n+1}. Note that this is simply the amalgamation of the blocks A_n, B_n, C_n and D_n. By the inductive hypothesis, each of its rows and columns contain all of the numbers $0, 1, 2, \ldots, 2^{n+1} - 1$. The occurrence of $0, 1, 2, \ldots, 2^{n+1} - 1$ in B_{n+1} is then ruled out, but there are no other constraints. So the pattern of entries in B_{n+1} will follow the pattern of entries in A_{n+1}, but with each entry larger by 2^{n+1}. This reasoning applies similarly to C_{n+1}. Each column above and each row to the left of D_{n+1} contains all of the numbers $2^{n+1}, 2^{n+1} + 1, \ldots, 2^{n+2} - 1$, so the pattern of entries in D_{n+1} will follow the pattern for A_{n+1}, and the induction is complete.

This result allows the calculation of any entry of the table to be reduced to the calculation of an entry which is closer to the top-left corner. Let us index the entries of the table as $a_{i,j}$, for $i, j = 0, 1, 2, \ldots$. Then we have

$$a_{455,122} = a_{199,122} + 256 = a_{71,122} + 128 + 256 = a_{7,58} + 128 + 256 = \cdots,$$

and continuing in a similar manner, we find that $a_{455,122} = 445$.

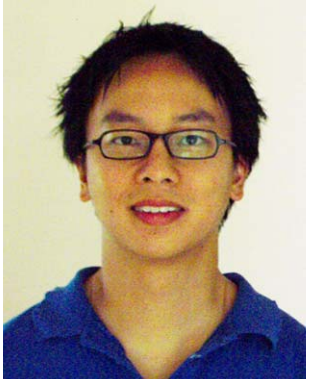
It also follows that the value of $a_{i,j}$ can be obtained by adding all the powers of 2 that correspond to the places in which the binary expansions of i and j differ. For example, $455_{10} = 111000111_2$ and $122_{10} = 1111010_2$, so $a_{455,122} = 256 + 128 + 32 + 16 + 8 + 4 + 1 = 445$.

The way to Heaven

Solution by Dave Johnson:

(1) Ask one of the people, ‘Would your companion say that the left path is the way to Heaven?’ If the answer is YES, take the right path; if the answer is NO, take the left path.

(2) Ask one of the people, ‘If I asked your companion whether the left path is the way to Heaven, would the answer be GEB?’. If the answer is GEB, take the right path; if the answer is MIU, take the left path.
(3) Ask person A, ‘Is person B more likely to tell the truth than person C?’ If the answer is YES, you deduce that either A or B is the random answerer; if the answer is NO, then you deduce that either A or C is the random answerer. In either case, you identify a person who is not the random answerer. You may now ask that person, ‘If I asked you whether the left path is the way to Heaven, would the answer be YES?’ If the answer is YES, take the left path; if the answer is NO, take the right path.



Norman is a PhD student in the Department of Mathematics and Statistics at The University of Melbourne. His research is in geometry and topology, with a particular emphasis on the study of moduli spaces of algebraic curves.

Mathematical minds

Nalini Joshi[*]

Nalini Joshi is an applied mathematician researching nonlinear systems. In addition to her role as President of the AustMS, Joshi is also Head of the School of Mathematics and Statistics at the University of Sydney. For this, the first of a series of interviews with Australian mathematicians, the *Gazette* spoke to Joshi during a break at the 7th Australia–New Zealand Mathematics Convention (December 2008), to talk about her life and mathematical career.

Early life

Gazette: Were you born in Australia?

Joshi: No, I was born in Burma.

Gazette: When did you come to Australia?

Joshi: In 1971, when I was twelve. My family migrated to Australia. We came to Sydney — I remember landing at the airport and being amazed. My father had a total of $4 in his pockets for a family of four because you are not allowed to take foreign currency beyond a very small amount out from Burma. But it was really good that a family member of a patient of my father's in Burma knew that we were coming. He came to the airport every day to check which flight we were on. He took us to his home until my father started his job and we could afford to pay rent.

Gazette: What did your parents do?

Joshi: My father was a doctor and my mother was a housewife.

Gazette: Did your father get accreditation to work as a doctor in Australia?

Joshi: Yes. It was just after the white Australia policy ended and doctors were very much wanted. He went through a system that was considered good enough, because it was the colonial British system in Burma.

I remember he told us the story of how he was hired. He was conscripted into the army in Burma, because they didn't want professionals of ethnically different origin from the Burmese to prosper, so he knew quite a lot about how to treat people in a medical emergency and in war settings. And he said when he came to a district hospital and he was interviewed the first thing they asked him to do was a tracheotomy, where they cut a hole in the throat to let somebody breathe. He knew how to do that because he'd done this hundreds of times on the battle field — he got a job straight away. My parents were born in Burma, but my grandparents were ethnically Indian. The Burmese Government, which came in through a coup

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in 1962, wanted to make Burma predominantly Burmese, they wanted to get rid of other ethnic groups. They conscripted professionals in the army, paid them very little, and didn't promote them, among other things. That was the major reason why my father left Burma, because he felt discriminated against.

However while he was in the army, I had the best time of my life because we lived in the countryside in various far flung outposts near the Golden Triangle Region which is bounded by China, Laos and Thailand. I had a very adventurous and free childhood there.

Gazette: Any brothers and sisters?

Joshi: I have a younger brother who was trained as a classical musician. He now works in arts administration for the Australia Council.

Education

Gazette: Were you encouraged to do mathematics at high school?

Joshi: I get bored very quickly. I remember that at school I was always a chapter behind the others in the maths class, and when I needed to I would catch up. I don't know if anyone encouraged me, or just let me go at my own pace. Those days, I never knew anything about mathematics competitions or maths camps. It was only when I got to university that I found out about them. So in a sense, I guess it was benign neglect. It was fine because I could go at my own pace.

Also, the first three years of high school I attended a girls' school, then it merged with a boys' school. In the first three years I didn't have that factor that they say is so important in influencing girls to turn away from maths. We didn't have the boys creating that peer pressure to turn you away.

I just enjoyed maths so much at school, and I could tell that I could do things faster than some of the other people seemed to be able to. And I loved explaining to the other students in class. So I knew that this was something I could do and wanted to do. But in fact, when I went to uni I wanted to do astrophysics. I was in love with astronomy and astrophysics and I actually wanted to become an astronaut. That's what I told my career advisor at school, and she said 'That's a bit unrealistic, dear'.

When I went to uni, I did first year physics and that's when I found out that I couldn't be a physicist; I have an empty set for physical intuition. So I went off into applied maths instead. There I could understand everything. I really wanted to work on problems that would have an impact on understanding the world.

As I said, I was in love with astronomy and astrophysics, and there was a lecturer, Ted Fackerell, who was a relativist and he had just started to become interested in integrable systems theory, who became my honours supervisor. He had this romantic idea that you could think about the interaction of two black holes in the universe like the interaction of two solitons in integrable systems. This captured my imagination and I started learning more and more about soliton theory.

When I went to do my PhD I chose Martin Kruskal who was the discoverer of solitons and started the whole field rolling in the sixties. I went to Princeton to work with him. I started a very long collaboration with him which was wonderful, and I've been in that field ever since.

Gazette: What year did you complete your undergraduate degree?

Joshi: I graduated in 1982. I went to Princeton in August/September 1982.

Gazette: And your PhD?

Joshi: It was awarded in 1987.

My PhD supervisor died a couple of years ago. He was an amazing man. He was a strong, very real, idiosyncratic person. I remember the first conversation I had with him when I first met him. It had to do something with chaos. We were discussing a topic I could work on. I said, 'chaos as in the television series?' So we started talking about television series and our favourite TV programs.

He used to wear two holsters, and they were full of pens, pencils, letter openers, chalk holders, spare leads, spare erasers, magnifying glass. Anything you might want in terms of stationery, he had it in one of those holsters.

Gazette: How did you end up with him as advisor?

Joshi: As I said before, I got interested in soliton theory, so I looked up every paper I could in that area. And I saw that in a lot of them, although he wasn't an author, he was mentioned in acknowledgements, for having stimulating discussions with. I knew he was the inventor of the word soliton, he was still active in the area and giving other people interesting ideas, so I deliberately went to Princeton to work with him.

My advisor was never scared to take apart a theorem. We would try and understand something together. And he would say, 'but this is the wrong way to do it'. And we'd go back right to the beginning and we would build it up from scratch again. I found this amazing, because nobody else I knew made that kind of effort to understand something so thoroughly.

Gazette: Did you have a scholarship?

Joshi: I had some kind of support, but they don't call it a scholarship [in the US]. You get teaching assistanceships or research assistanceships.

Working life

Gazette: What is the best career advice you have ever received?

Joshi: I don't think I ever received any advice. Nobody ever said to me 'go into this direction' or 'move to that university'. I just followed my instincts. I'm a very stubborn person, and I don't let things get in my way.

Gazette: Could you see yourself working in the industry?

Joshi: I've never worked in the industry. I did look at it very seriously when I was at university. I looked at doing engineering of some kind. I really wasn't sure what I wanted to do at the end of uni: I looked up some aeronautical companies, I looked into modelling for a gasline company. I saw myself as happy if I could do some mathematics in some group, it didn't matter where that group was. But once I did honours, I just wanted to do a PhD.

Gazette: When did you get married?

Joshi: Half-way through my PhD. My husband was also doing a PhD at the time, in geophysical fluid dynamics. But he'd done applied maths in honours as well, so we went through this together. The only reason he was doing a PhD was because I was doing a PhD. We were very lucky that we could go to the same place.

Gazette: You had your children early on in your career — your son while you were a postdoctoral researcher and your daughter a few years later when you were a lecturer at UNSW. What was your experience like of combining family and working life?

Joshi: There were lots of problems. I deliberately wanted to breastfeed my children for a year after they were born, as they were both premature, and that was very very difficult because they went to childcare. I had to work and people just didn't think about those kinds of issues for staff members in those days. It was a difficult time. And I would say that probably workplaces weren't that friendly for child rearing, when they got sick you couldn't really take the time off. There was no parental leave in those days.

Gazette: What impact did this have on your work?

Joshi: Around the times when they were very young, it certainly had an impact. I also recall that at one stage I had to go to a conference. My husband and my son had moved to Adelaide and my daughter and I hadn't moved from Sydney yet because I was waiting for an opportunity in Adelaide. So I happened to mention in the tea room that I was looking for somebody to stay at home with my daughter so I could go to the conference, just for a week. One person told me that I was a bad mother because I would even contemplate going away and leaving my child with a family member. Those kind of things are hard. But I don't want to talk about all those hard things.

Gazette: What positions have you held?

Joshi: I came back to Australia [after my PhD] and moved around a bit. I stayed at ANU for about $3\frac{1}{2}$ years. Then I was at UNSW for $5\frac{1}{2}$ years. But my husband didn't have a permanent position. So in an effort to find a permanent position for him he moved to Adelaide to join the DSTO and I had to resign my job at UNSW to follow him because I couldn't get a situation where I could get leave without pay and possibly come back. So I went to Adelaide on an ARC Senior Research Fellowship for five years and then I got a Chair at the University of Sydney. Thankfully DSTO is very forward-looking as an organisation so they allowed my husband to work in Sydney even though he belongs to a group in Adelaide.

Gazette: You are a member of . . .

Joshi: The American and the Australian Mathematical Societies. I joined the American Mathematical Society when I was a graduate student and I'm still a member because it keeps me in touch.

Gazette: What about the Australian Academy of Sciences?

Joshi: I was elected as a Fellow in March 2008, for my overall career contribution. I'm only the third female mathematician to be elected: the first was Hannah Neumann and the second was Cheryl Praeger. In general, the ratio of females to males in the Academy is very small. I recently tried to find out how many female Fellows were from Sydney University. There seems to be only two currently including myself — that's a bit of a shame.

Gazette: Postgraduate students?

Joshi: I seem to attract postgraduate students. I somehow fall between two stools: a lot of students get very interested in doing problems that are motivated by physics, but in many physics departments there isn't a mathematical or theoretical specialisation. Students then look around and find people like me. These are usually students who are very good at mathematical calculations, who want to do deeper things, but are motivated by something in the natural world. I've had four PhD students complete their PhDs with me. The fact that I moved around has made that a little more difficult. I didn't have any PhD students in Adelaide. I currently have two PhD students, and may have one more next year. I love working with students — I find it incredibly satisfying. I love working with undergraduate students too.

When I was an ARC Senior Research Fellow, I didn't have to teach. I found that I really missed the teaching and the connection with students; the fact that you can explain things to them, and see the spark in their eyes.

Looking back

Gazette: What achievement are you most proud of?

Joshi: One example I always use when I talk to school students is how I solved the Towers of Hanoi problem. This was a long time ago. I remember doing that calculation and realising that I could do it without looking it up in any books in the library or going and asking somebody for help. That was really revealing and very satisfying. That was my first problem . . . . I wasn't that young, but I remember feeling as though this was something that built my confidence.

I'm very proud of the work I did as part of my PhD. Because it involved solving a set of equations I don't think there's ever been a methodology invented for. It was non-analytic, involved highly transcendental functions. I remember, one summer, I woke up at 1 am every night because I had to solve this problem. And I'd go away and fiddle and do calculations on a piece of paper. And then I'd do it again, and again, and again until finally I got an answer. My advisor hadn't expected the answer that I got.

Gazette: Can you give advice to early career mathematicians?

Joshi: Have confidence. Once you get there, you've come through as a mathematician. You've actually worked out something for yourself. You've solved a problem. And that means that you've go the ability to do more. So start with confidence! Don't be scared of developing your own point of view. In any field of Science there are hierarchies of accepted view points. There's a great deal of pressure to work within an acceptable hierarchy. That can be very stultifying. You've got to develop your own point of view and communicate this to others. And learn how to communicate it so you don't offside people.

The other thing I'd like to say is that it is important to tackle the harder problems. You have to publish, publish, publish if you want to go into an academic career. But you should also tackle the harder problems and not just fall into the trap of publishing just the easy results.

Gazette: Women in maths — have all the problems been solved?

Joshi: I think attitudes have changed. People are more open and generous nowadays. But as I said before, it's a case of benign negligence, people don't see that there are a lot of blind spots.

Looking to the future

Gazette: What direction would you like to see the AustMS go into?

Joshi: I think it has to become a more strategically focused society. It has to allow itself to be more vocal and more visible in a national setting, rather than being introverted and only maintaining an annual meeting. It has to take the bull by the horns and become a lobbyist.

I guess we'll have to talk about that amongst ourselves. We've said for a long time that mathematics has been undervalued and treated badly on the funding scheme. We have to think about not just complaining, but finding solutions and proposing them to the government.

Gazette: Who are your main collaborators?

Joshi: For a long time, it was Martin Kruskal. One more recent one is Alexander Kitaev. He's a very good mathematician. We looked at the solutions of the first Panlevé equation — ordinary differential equations. This comes up quite often in places like random matrix theory. People had a qualitative idea about what the solutions did. There's this particular solution, called a tritronquée solution — nobody really knew where the first real pole of this solution was located. If you come in from infinity then it seems to be bounded on one half of the real line and then you encounter a pole. The solutions are highly transcendental. We were the first to provide a way of approximating the position of the pole to any order of accuracy, and to prove that the solution was pole free to the right.

I've worked with lots of other people. For example, Frank Nijhoff, at the University of Leeds. Every time we meet we talk about something that has elliptic functions and hyperelliptic functions — we meander all over the place. We've written several papers together, and I'm sure there are many more to be written.

I work with people in Japan, and others in the UK, France, Finland.

Gazette: Why do you do mathematics?

Joshi: Because I discover new patterns.

Gazette: Hobbies?

Joshi: Oh dear. I don't have time to pursue my hobbies! I love doing a lot of different things. I read. I'd love to read more. I love going on walks.

Gazette: So are you joining the walk this afternoon?

Joshi: No, I'm not, actually. I'm doing the winery tour. I love wine tastings, and figuring out what is a good wine. I love doing things like knitting. I've decided that I love architecture. I love going to art galleries. Being exposed to different types of art. And I love travelling.

Gazette: Thank you very much, Nalini!

Joshi: Thank you.

The Access Grid

AG on a personal computer; and using VPCScreen

Bill Blyth* and Jason Bell**

Introduction

The Access Grid (AG) is a fully featured and flexible video collaboration tool. Although there isn't a typical Access Grid Room (AGR), an AGR usually has three (or four) screens which are linked to operate as one large projection screen, referred to as the display wall. In addition, multiple cameras (usually three) provide video streams. There is, as well, a single audio stream. For an overview, see [1].

However, the AG software does not require a fully featured AGR. For example, the AG software can be run on a desktop or laptop: it's free (and open source) and all that is required is a video source (such as a web-cam), an audio input and output device (such as a set of headphones with a microphone), and a good internet connection. In this case, this AG configuration is referred to as a Personal Interface to the Grid (a PIG). Another low-cost setup is to use a desktop with two large LCD monitors (to give more screen 'real estate' for displaying video streams and content) with a single or multiple video sources and an echo-cancelling microphone.

Within AMSI, most of the teaching of advanced mathematics AG sessions and AG seminars are conducted in a presentation mode where interactive remote control of the software is not required. Although current practice is usually to transmit the presentation via VNC[1], in these cases the use of VPCScreen (rather than VNC) is recommended as it has several advantages (see below).

Personal interfaces to the grid (PIGs)

There are many versions of video conferencing software available. The Australian Research Collaboration Service, ARCS, provides support for Access Grids which is 'great for room-based video-conferencing' and EVO[2] which is 'particularly suited to desktop installations' (see http://www.arcs.org.au). If video conferencing is desired for communicating between a few isolated researchers, then EVO is likely to be a good choice since it requires less bandwidth and is more forgiving with respect to the quality of the internet connection. The AG is designed to support fully-featured collaboration (not just video conferencing) and generally has higher bandwidth requirements. It is suggested that a good connection to support a PIG, should have a minimum of an upload rate of about 1 MB/s and a download rate

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[1] Virtual network computing (VNC) is a graphical desktop sharing system.

[2] EVO is a PC-based multipoint collaboration tool which features advanced videoconferencing.

of at least 3 MB/s. Cable or ADSL2 internet connections have been successfully used, but some adjustments may be required for acceptable collaboration (for example, reducing frame rates for each stream). Obviously, the more sites connected (and video streams transmitted), the more bandwidth required!

Usually, different video conferencing systems cannot be used together, but EVO to AG bridges are under development and hoped to be available in Australia very soon. This would mean that an EVO user could use a lower bandwidth internet connection and participate within an AG session.

For most mathematicians wishing to use the AG, it is recommended that for an established AGR, support should be provided by specialist IT staff. However, a limited number of PIGs can be used

- to monitor AG sessions,
- to record AG sessions (using AGVCR),
- to participate in small interactive sessions (rather than use a full AGR),
- as a personal AG (that is, join in from your office as an example),
- to assist with resolving clashes (when the main AGR is already booked).

With respect to clashes, note that the AMSI program of collaborative teaching of advanced mathematics (usually as Honours courses) tries to avoid clashes, but this cannot be guaranteed since the Honours mathematics classes are not timetabled nationally. The AGR Honours classes are timetabled locally as part of the host university's Honours timetable — when remote sites express an interest in the subject the Honours lecturer and Honours coordinators negotiate to try to avoid possible clashes. In the current semester, an RMIT student wished to take an Honours course remotely, but there was a partial clash since the RMIT AGR was already booked for another AMSI AGR Honours course. This partial clash could have been dealt with by using a PIG (although RMIT was fortunate to be able to use the AGR at the Victorian Partnership for Advanced Computing, VPAC, which is a five-minute walk away).

The video and audio hardware for a PIG is different to that used in a full AGR. Any decent webcam is fine for the video. However, as in all AG facilities, it is essential to have high-quality audio equipment and special attention is required to eliminate echoing. A quality headset combination with speakers that enclose the ears and a microphone is good since audio feedback between the mike and the speakers is avoided. Another option is to use an echo cancelling mike and speaker set that is plugged in via a USB port[1].

The AG homepage, http://www.accessgrid.org/, has a wealth of documentation, tutorials and downloads available. The AG software is platform independent: it runs under Windows, Mac or Linux (although currently there are some audio issues with the Mac platform). To install the AG software, download the installer from the right-side panel at http://www.accessgrid.org/software. It is suggested

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[1] Examples of suggested hardware can be found at http://www.arcs.org.au/products-services/collaboration-services/video/audio-devices and http://www.accessgrid.org/hardware.

to choose a stable release version. On any Windows computer (laptop or desktop, running under XP or Vista), it is highly recommended to choose the Access Grid bundle[2].

This 'bundle' includes the all-in-one installer for Windows and installs all of the required versions of the various pieces of software that are used for the AG. For installation, check the boxes for all the add-ons (such as VPCScreen and AGVCR), as this will provide superior functionality. After installation, the Venue Server should be changed (in the box below Navigation) to
https://vv3.ap-accessgrid.org:8000/Venues/default
which is the address for the Asia Pacific Access Grid Venue Server, the most commonly used Venue Server in Australia.

A small- to medium-sized room AG node

Whilst it is possible for two people to crowd around a laptop PIG, it is not ideal. For small groups in a small- to medium-sized room, a low cost AG node can be run by augmenting a PIG with larger monitors: usually two 40+ inch LCD screens are favoured. In this case, one echo cancelling mike and speaker will be adequate. Some rooms are set up in this manner: for example, the AGR at the Mackay campus of CQ University Australia (see Figure 1). Another variation, is the University of Melbourne AGR2GO which is a mobile AG node (with a laptop and two 40-inch LCD screens) which can be wheeled on a trolley to any room which has wheelchair access and an appropriate internet connection.

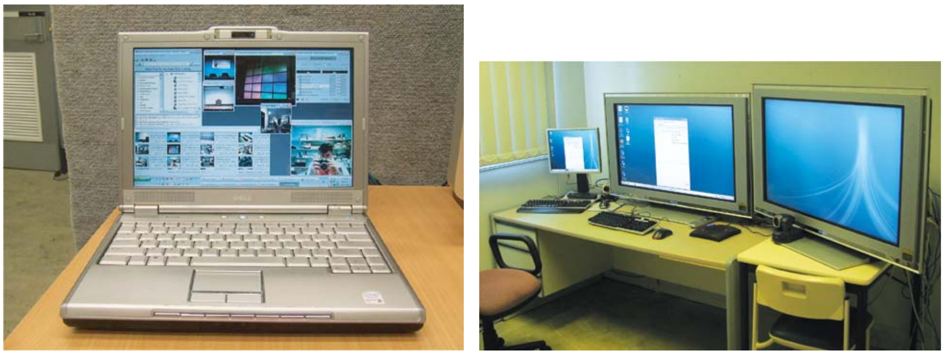

Figure 1. Left: A PIG: an AG node on a laptop. Right: The CQU Mackay small room AGR.

VNC or VPCScreen

Computer scientists and IT support professionals are familiar with using the software VNC to remotely control computers. Within the AG context, VNC is used to provide the capability to remotely control presentations, webpages and other electronic content within an AG session. To facilitate this, the AG shared application VenueVNC is generally used to support the sharing of electronic material.

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[2] http://www.accessgrid.org/files/AGTK3.1_easyinstaller.exe.

VenueVNC simply acts like a wrapper for VNC, in which it passes additional information such as hostname and password, thus making it simpler for AG operators to connect to VNC. One of the benefits of using VNC (or more specifically VenueVNC) is the ability for each site to be able to remote control and operate the software on the presenter's computer. This allows remote sites the ability to be able to write directly on the same Word or PDF documents, control presentations at different times, or take turns at controlling the same Maple or Mathematica file.

However, the use of VNC comes at a cost. This cost can be associated with the fact that VNC clients utilise a separate unicast (not multicast) connection to a VNC server, in which it has been found that large (more than eight) simultaneous connections cause delays in interactivity. These problems can be addressed: the Canadian coast to coast AGR seminar series uses VNC Reflector and multicast versions of VNC are also available.

There are further problems with VNC. The add-on AGVCR is the cassette recorder for the AG session: everything is recorded (all of the video and audio streams) for the AG session. Unfortunately, if VNC is being used to transmit the presentation material, then this presentation material is not recorded. Another VNC problem becomes evident if an attempt is made to show a movie (such as an mpeg file) and an animation (such as a Maple or Mathematica animation). VNC usually shows a black region where the movie or animation is running at the host AGR.

For standard lecture presentations and seminar presentations, remote control is never desired, so VNC is not always a good choice: the use of VPCScreen is recommended. VPCScreen captures and streams: the whole screen, a screen region or a selected window. This can be recorded with AGVCR (see documentation at the Access Grid home page, or [2]). Unlike VNC, VPCScreen can stream a movie or animation (the VPCSCreen software has a fixed frame rate of 10 fps, but this is generally adequate). Maple animations at a Maple frame rate of 18 fps have been tested and they work well. (Note that usually a lower frame rate, say about 5, is used for animation demonstrations.)

Conclusion

Collaborative teaching of advanced mathematics across Australia via the Access Grid is expanding with the participation of New Zealand. National seminars are also offered over the AG. For many participants, the use of a fully featured AGR is recommended. However, there are occasions (such as dealing with clashes or monitoring) where the use of a PIG is valuable. For many mathematicians, AG sessions are mostly conducted in a presentation mode where interactive remote control of the software is not required. In these cases the use of VPCScreen (rather than VNC) is recommended. Use of VPCScreen results in transmission of the computer screen (or part thereof) as a video stream within the AG multicast broadcast. Besides scaling up well to many AG nodes, it can be recorded by the AGVCR so that it is simple to record the full AG session.

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Bill Blyth is Adjunct Professor of computational mathematics at RMIT University and was Head of the Department of Mathematics for $6\frac{1}{2}$ years. He is Chair of the Engineering Mathematics Group of Australia, a Center Affiliate at the International Centre for Classroom Research (at the University of Melbourne) and led the design, construction and initial delivery phases of the RMIT University AGR. He is currently at The Australian Mathematical Sciences Institute, AMSI, as the national coordinator of AMSI's AGR project. His PhD was in theoretical physics at Imperial College, London. He has an unusually broad range of research interests in mathematics education (in technology-rich classrooms) and the numerical solution of differential and integral equations. He has published more than 60 refereed papers.

Jason Bell is a Research Systems Support Officer at CQ University Australia and an Access Grid specialist within the Collaboration Services Team at ARCS. Though his duties can vary substantially, one of his main roles is the focus on Access Grids. He is the main developer and tester of the AG Global Quality Assurance program and runs a weekly AG test session for the Asia Pacific region. He has also developed numerous guides, and has provided several presentations and hands on workshops on the topic of AGs. He has contributed code to AG Toolkit itself. His B.I.T. (Hons) was from CQU and has research interests include nonlinear optimisation, evolutionary algorithms, high performance computing, distributed and parallel computing, robotics and Access Grids. He has contributed to a few published refereed papers (non-AG related) as well as an extensive list of AG PowerPoint presentations which can be found on the ARCS website.

Obituary

Emeritus Professor Ernest Oliver (Ernie) Tuck
1 June 1939 – 11 March 2009

Sadly, at 11 pm on 11 March 2009, a little short of his 70th birthday, Professor Ernest Oliver (Ernie) Tuck died from prostate cancer. His battle lasted almost two years from the time of diagnosis, but none of the treatments he underwent were able to successfully arrest its progress. Without doubt, Australia has lost one of its leading applied mathematicians and the world has lost a notable fluid mechanist.

Many will remember the last ANZIAM (Australian and New Zealand Industrial and Applied Mathematics) conference Ernie attended, in Katoomba, February 2008, and the talk he gave there. It epitomised his love of mathematics and the joy he experienced in telling others about it. Early on in the conference, he was struck by great pain due to the spread of cancer to his bones. It was clear to those who knew him that he was very ill. But he still delivered his talk — on the Riemann zeta function (!) — at the scheduled time, with the usual enthusiasm, walking of the floor, gesticulations, and writing on the whiteboard, and he thoroughly enjoyed doing so. We suspect he did not feel any pain during that seminar, as he was totally engaged with the thrill of giving a mathematical talk! His enthusiasm was so infectious it has since led to a publication by a member of the audience, Sir Michael Berry of the University of Bristol (with P. Shukla): 'Tuck's incompressibility function: statistics of zeta zeros and eigenvalues', *J. Phys. A* 41 (2008) 385202. Ernie was chuffed by the appearance of his name in that title. The title also amused him; to quote Ernie from December last year: 'I am not entirely sure what Berry means by 'incompressibility'; it certainly has nothing to do with fluids.'

Ernie loved both teaching and research in mathematics. He was an enthusiastic and passionate teacher, from first-year university calculus (he certainly did avoid linear algebra) through to specialised fourth-year courses on such subjects as aerodynamics. He inspired his undergraduate students, as well as his higher-degree research students. He treated his research students as colleagues and friends, something they much appreciated. In addition, they soon learned to admire and value his insight. It wasn't always an easy road for his research students. There was a time when Ernie was Dean of the Faculty of Mathematical and Computer Sciences, with many administrative duties, in addition to his teaching and research, four PhD students and a research assistant. But through such circumstances he taught his students to stand on their own two feet and believe in their research, something which they have greatly appreciated since completing their PhDs.

A short history

Ernie was born in Adelaide, South Australia. He and his younger brother were raised by their mother after his father, a World War II veteran, was killed in an automobile accident when Ernie was just six years old. This family tragedy had a happy consequence later, when Ernie met his future wife and often-acknowledged best friend, Helen, at a Legacy gathering for children of military veterans.

Ernie was an undergraduate student at the University of Adelaide from 1956–1959, and received his First Class Honours degree in mathematics in 1960, supervised and mentored by Professor Ren Potts. Ren's interests in operations research and statistics rubbed off on his student, and provided the topic of Ernie's first research paper, 'Stability of following in two dimensions', *Operations Research*, 9 (1961) 479–495. After completing his Honours degree, Ernie won a Legacy Scholarship that funded his postgraduate studies at the University of Cambridge. Ernie was both proud and grateful for this opportunity provided by Legacy, and remained strongly attached to Cambridge throughout his life. Thus at the close of 1960, having proposed to Helen the day before, he embarked by ship to the UK to study at the University of Cambridge. Helen followed Ernie to Cambridge after his first year, and they were married in the chapel of Trinity College in 1961.

Ernie's PhD supervisor was Professor Fritz Ursell, a major figure in British applied mathematics and, currently, Emeritus Professor at the University of Manchester. In addition to Ursell's influence, Ernie's subsequent focus on ship hydrodynamics and related fields may be attributed to his observations on the long voyage to England, and to a deep inquisitiveness in science, technology, and engineering applications. Another possible connection with the field of hydrodynamics is suggested by the fact that Sir Horace Lamb had been the first Professor of Mathematics at the University of Adelaide.

With guidance and encouragement from Ursell, Ernie focused his PhD research on one of the first applications of slender-body theory to ships. Ernie's approach was revolutionary, based on the method of matched asymptotic expansions, and applied to the prediction of the wave resistance of a ship moving in steady motion on the free surface. When Ursell moved to Manchester after Ernie's first year, an arrangement was made where Ernie spent his second year at Cambridge and his third at Manchester. After his second year he participated in a small meeting held at Wageningen, Netherlands, which was organised primarily to discuss his research and the complementary work of Gerrit Vossers on the same topic. That meeting was supported by the US Office of Naval Research. In 1963 Ernie was awarded a PhD for his thesis entitled 'The steady motion of a slender ship'. Clearly Ernie was an early adopter of the three-year rule for PhD completions!

After Cambridge, Ernie moved to a position at the David Taylor Model Basin, a US Naval Laboratory, to work with Francis Ogilvie and Nick Newman. This facility is a major laboratory, with a number of towing and modelling tanks inside a most impressive building almost exactly one kilometre long. Ernie and Helen's first son, Warren, was born during this time. From there Ernie moved to the California Institute of Technology, to work with Ted Wu.

Then, in December 1967, the family returned to Australia where Ernie took up a Readership at the University of Adelaide. The story of his return to Adelaide is a little intriguing. Apparently a former colleague of Ernie at the David Taylor Model Basin received a request for a reference for Ernie from what he described as 'that other University in Adelaide' (in 1967 that of course was a unique specification). The requested reference was provided but another letter was sent to Ren Potts at the University of Adelaide asking the question, 'how could you let such a talented person' go elsewhere in Adelaide? Clearly Ren could not, and the rest is history.

After his return to Adelaide Ernie and Helen's second son, Geoff, was born.

In 1974, at the age of just 34, Ernie was promoted to a Personal Chair. On the retirement of Professor Ren Potts in 1990, Ernie then became the Chair of Applied Mathematics and also the Elder Professor of Applied Mathematics (named after Sir Thomas Elder, a renowned Australian pastoralist, businessman and benefactor of the University). Ernie held those positions until his retirement in June 2002. During his time in applied mathematics he frequently held the position of Head of Department and he was also Dean of the Faculty of Mathematical and Computer Sciences from 1993 to 1996.

On his retirement he was awarded the title of Professor Emeritus. He really did not retire from research at all but continued the work he loved — supervising research students, pursuing his research interests, collaborating with colleagues, writing papers and grant applications, attending conferences, organising the ICTAM 2008 conference, and more. Even near the very end he was fervently hoping to attend the February 2009 ANZIAM conference, and was bitterly disappointed that his health did not allow this. Ernie did however live to see his final research student, Leo Lazauskas, qualify for his PhD early this year.

The man and his work

Ernie's primary field of expertise was fluid mechanics. He worked on a wide variety of topics related to ship hydrodynamics, aerodynamics, acoustics, biofluid mechanics, and numerical analysis. His contributions to these fields were based primarily on analytic methods, but early on in his career 'high-speed' mainframe computers were becoming useful to scientists and engineers on a broad scale and Ernie was quick to embrace the developing field of numerical computation. This enabled him to produce practical and illustrative results based on his theoretical analyses. One of his most exciting computations was for the nonlinear waves generated by a submerged two-dimensional dipole in steady motion (*J. Fluid Mech.*, 22 (1965) 401–414). The streamlines include jets emerging from the free surface, which generated much discussion; Helen embroidered a splendid wall hanging with images of the streamlines.

Ernie had great expertise as a modeller, and published over 170 articles, with the vast majority being in the top journals in fluid mechanics; his papers are clear, concise, and stimulating. He also had a personal interest in games theory, and published articles on both blackjack and backgammon. After retirement Ernie became interested in Riemann's Hypothesis and wrote three papers in this area. This is usually regarded as a rather 'Pure' mathematical area but Ernie realised that the analytical skills honed in his hydrodynamic research could find application here. We have already referred to a recent publication citing his work in this area. For a complete list of Ernie's publications, see www.maths.adelaide.edu.au/ernie.tuck/.

Ernie's research is characterised by the recognition of new or unsolved problems, application of novel mathematical methods, and careful numerical analysis. He was particularly adept at solving complex problems with simple approximations, as in his applications of matched asymptotic expansions. When he first employed this method to analyse the wave resistance of slender ships it was relatively new, and unknown to most. Subsequently he found other problems to which the same method was applicable, including the 'squat' of ships in shallow water, various types of flow or wave transmission through small gaps, end effects on blunt slender bodies, and bodies moving near a plane wall or in close proximity to other bodies. Several other topics which recur throughout the list of his publications include the strip theory of ship motions, Michell's thin-ship theory of wave resistance, planing, bodies with zero wave resistance, nonlinear free-boundary problems, numerical solution of integral equations, low-Reynolds number flows, wave resistance of multihull vessels, and lifting-surface theory.

Ernie's work on 'squat' (or sinkage) of ships in shallow water is an interesting example of his international stature in the field of ship hydrodynamics. He was asked to work on this problem soon after he arrived at the David Taylor Model Basin, motivated by the grounding of an aircraft carrier in the Gulf of Mexico. His brilliant analysis, which combined slender-body theory with the governing equations for shallow-water waves, revealed the nonlinear effect of a ship's speed on squat, particularly in the vicinity of the critical Froude number. He first reported his results in discussion of a paper by German researchers, where he showed how his elegantly simple theory could explain their experimental results. Subsequently he published this seminal work in the *Journal of Fluid*

Mechanics, 26 (1966) 81–95. He also simplified the essential results in Navigation (*The Journal of the Australian Institute of Navigation*), 3 (1970) 321–324,for the benefit of ship operators. The importance of his work was recognised after a widely publicised accident: on 7 August 1992 the famous Cunard flagship Queen Elizabeth 2 struck a shoal after leaving Martha's Vineyard, enroute to New York. The ship was behind schedule, operating at high speed, and passed over a shoal area because neither the pilot nor the ship's officers understood how the speed affected the squat. Ernie took a great interest in this accident, and subsequently he contributed a lucid overview of squat to the Workshop on Ship Squat in Restricted Waters held in Washington, October 1995.

In 1988 Ernie was elected a Fellow of the Australian Academy of Science and, in 1996, a Fellow of the Australian Academy of Technological Sciences and Engineering. Being a Fellow of both academies is a rare distinction and a recognition of Ernie's success in both scientific work and in application. In 1999, Ernie was awarded the Thomas Ranken Lyle Medal from the Australian Academy of Science, which is awarded once every two years and recognises outstanding achievement by a scientist in Australia for research in mathematics or physics. In 1999 he was also awarded the ANZIAM Medal by the Australian and New Zealand Industrial and Applied Mathematics (ANZIAM) society, a division of the Australian Mathematical Society. This medal has been awarded every two years since 1995 for research achievements and contributions to Applied Mathematics and ANZIAM, and Ernie was the third recipient. It is worth noting that the first ANZIAM Medal was awarded to Ren Potts. This reflects the extraordinary strength of Adelaide Applied Mathematics in the era of Ren Potts and Ernie Tuck. Ernie was also the proud recipient of a Centenary Medal from the Federal Government at the time of our national centenary (2001).

An international award of which Ernie was proud was his selection as the Georg Weinblum Memorial Lecturer for 1990–1991. Named after an internationally famous ship hydrodynamicist the award required Ernie to deliver the Memorial Lecture firstly in Berlin (1990) and later in Washington in 1991.

In addition to frequent participation in ANZIAM conferences, Ernie made many longer trips to attend the (ONR) Symposia on Naval Hydrodynamics, IUTAM Congresses, the International Workshops on Water Waves and Floating Bodies (IWWWFB), and more. He was particularly active in small informal meetings, stimulated no doubt by his early exposure to seminars at Cambridge, the ad hoc meeting in Wageningen, and his early membership of the Analytical Ship-Wave Panel (H-5) of the US Society of Naval Architects and Marine Engineers.

Ernie successfully supervised 25 PhD and 4 research Masters students during his time at the University of Adelaide. At his retirement symposium in January 2003, 16 of these students were able to attend, a number of whom travelled great distances for the occasion. This aptly demonstrates the very high esteem in which Ernie was held by his students. During the period in the mid-1990s when Ernie was Dean and then on study leave, he gave his departmental office over to his research assistant and four postgraduates — including open access to his whole library. This provided a great (if crowded) atmosphere for the inhabitants, each of whom went on to write at least three papers with Ernie. At about the same time in the period between 1993 and 2001 Ernie's students were awarded best student paper at the annual ANZIAM conference on four separate occasions.

Ernie was very interested in promoting mathematics in general and particularly in motivating young people towards an interest in maths, both through his teaching and by other means. As part of this interest he and a long-time friend, Professor Neville de Mestre of Bond University, produced a book aimed at secondary school students, titled 'Computer Ecology and Chaos' and published in 1991.

Ernie was Chair of the ANZIAM Society for two years and Deputy Chair for three years. He was Editor of the main Australian Applied Maths Journal — now the ANZIAM Journal — for eight years from 1985–1992. He was an early advocate of LaTeX, and in 1992 he established TeXAdel, an organisation responsible for automating the production of the Australian Mathematical Society journals.

Very recently and, remarkably, while his health was already very much affected, Ernie was the President of the ICTAM (International Congress of Theoretical and Applied Mechanics) 2008, held in Adelaide in August last year. ICTAM is held every four years and attracts approximately 1000 delegates, so that this was a major contribution by Ernie to his discipline. Some may recall the pleasure he expressed during the conference opening that he was able to be there and participate as its President.

Other interests

Everyone who knew Ernie knew he had a warm and loving personality. He was shy as a young man, but never shy about maths, or his family. He loved Helen dearly, and their two sons Warren and Geoff. Later, after Warren and Geoff had grown up and married, their families were at the centre of his life. He doted on his young grandchildren, spending as much time as possible with them over the last few years. He loved playing rough and tumble games with his youngest granddaughter Samantha, and then he became the clown. He enjoyed seeing granddaughter Georgia bring her dog into the hospice in his last week, and was touched that another, eight-year-old Isabel, was saving her money for a plane ticket from Hobart to Adelaide so that she could come to see him. Just several days before his death, he was watching his oldest grandchild, Matthew, play cricket, as he had done on many other Saturdays. Ernie loved other sports also, including Australian Rules football, and basketball. He also loved playing backgammon, bridge, blackjack, . . . though he could not resist analysing the games mathematically! Another simple activity that gave him much pleasure was going for long drives with Helen, on weekends, or on trips to interstate conferences. His father's love of motorcycles possibly led to Ernie's love of cars; he frequently attended the Rowley Park Speedway before it closed. Again, mathematics came into this leisure activity: in his honours lectures on aerodynamics, he also talked about using spoilers to produce negative lift to keep sports cars glued to the road.

The Tuck Fellowship

To honour Ernie and continue his legacy a new fellowship has been proposed — The Tuck Fellowship — to support the participation at conferences by students and younger research workers, something Ernie fostered throughout his lifetime. After consultation with Ernie to ascertain his preferences, it was decided to associate this fellowship with the International Workshop on Water Waves and Floating Bodies (or IWWWFB), an annual conference at the heart of his research interests and with an emphasis on participation of young researchers. One student or young researcher will be fully funded to attend this workshop each year, with applications accepted from eligible people across the globe. More information is available at www.iwwwfb.org/tuck.htm. As suggested by Ernie, if sufficient funds are raised, a student or young researcher will also be sponsored to attend the ANZIAM conference. Ernie was delighted and very much moved when told that his suggestion had been adopted — he cared very much for the 'local' Applied Mathematics society. Donations to the University of Adelaide to establish this fellowship are currently being sought. Those who wish to honour Ernie in this way may obtain a donation form at www.maths.adelaide.edu.au or at www.iwwwfb.org/tuck.htm.

Closing remarks

Ernie will be remembered as a brilliant Australian mathematician and a caring and fun-loving man. He will be missed by many. In closing we would like to express, on behalf of all, sympathy to Helen and her family at the untimely loss of Ernie, but also gratitude for the wonderful academic career we have had the privilege to observe and share.

Elizabeth Cousins[1], Peter Gill[1], Nick Newman[2] and Yvonne Stokes[1]

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Obituary

Ernest Oliver (Ernie) Tuck
1 June 1939 – 11 March 2009

I have been fortunate to count Ernie Tuck as a friend for more than forty years. We met at the 1968 Applied Mathematics Conference at Victor Harbor on a Sunday. By Tuesday we were bodysurfing together at Port Elliot beach after lectures finished for the day. During that swim in the surf, Ernie's wife, Helen, frantically blew the car's horn to warn us of the school of sharks in the water. But it was not their dinner time, and luckily for us they turned out to be porpoises.

I think that Helen eventually forgave me for encouraging such bravado in a young, aspiring and brilliant academic, for we began to visit each others' homes when in Adelaide, Canberra, the Gold Coast or the United Kingdom, and became lifelong friends.

We continued to meet regularly for many years at the February Applied Mathematics Conferences. Ernie was a great supporter of these events, logging more than thirty attendances since they began way back in 1965.

Not many of you will know that Ernie was the first academic to take study leave at the fledgling Bond University in 1990, just one year after it opened. It was there that he suggested we write a book for high-school children on chaos theory. There were many zany and memorable moments as we conjured up the text and the numerous cartoons that enliven Computer Ecology and Chaos.

Prior to the 1999 ICIAM meeting in Edinburgh, Ernie, Helen, Margaret and I travelled through the Lakes District and western Scotland for a week. Besides stopping for morning and afternoon tea, Ernie always insisted that we call each day at a typical pub for lunch. He was such a delightful travelling companion that we couldn't refuse his enthusiasm to include this in the daily itinerary, but I couldn't convince him to go swimming in the sea at Skye.

We had a common research interest in low-Reynolds-number flows, and his ability to open up new aspects of this subject amazed many of his colleagues. His main research interest in fluid mechanics was in ship waves and associated problems for which he had an international reputation matched by only a few.

He loved cricket and Aussie Rules. One afternoon Ernie and I visited the Adelaide Oval to watch an interstate cricket match. It started to rain for a short period, the players went off, and then it fined up. Out came the groundsmen with a small tractor to which was attached a long heavy rope to mop up the raindrops on the field. The tractor was driven around the perimeter and the unattached end of the rope was held on the ground by one person at the wicket. As the water droplets were being spread, Ernie said ‘I wonder what curve the rope is forming?’ Out came a piece of paper and a ballpoint and applied maths was in action again. He was like this with many observations of natural phenomena such as skimmer boards on the edge of the beach, honey dripping from a spoon, oil tankers in shallow water, waves breaking, hydrofoils, catamarans, tsunamis, backgammon, iceberg towing, flow out of hoses, waterfalls and paint flow. His interests were exceedingly widespread.

Ernie was extremely proud of the mathematical achievements of his two sons. Geoffrey completed his PhD in applied maths at Adelaide in 1994 with Hugh Possingham as supervisor, while Warren recently graduated in Engineering at the age of 43.

For readers who were fortunate enough to attend any of Ernie’s lectures at Applied Mathematics conferences, you will remember a dynamic delivery full of humour and surprises, laced with original mathematical expertise and ideas. His invited talk at ICIAM 2003 in Sydney on multi-hulled vessels was beautifully presented, and did Australia proud as Ernie was one of only two Australians among the twenty-seven invited speakers.

It is a shame that such a brilliant mind can no longer produce insightful research in many areas. In the world of applied mathematics we will miss his friendship, his love of students, his poignant questions at the end of anybody’s lecture, and his helpful ideas. I am glad to have known him both as a mathematician, a good family man, and a great human being.

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Technical papers

Crime investigation: an introduction to error-correcting codes

Gerry Myerson*

Abstract

An expository introduction to coding theory, inspired by a problem from Norman Do's Puzzle Corner.

The puzzle

> While questioning a witness, a judge is only allowed to ask questions which are to be answered 'yes' or 'no'. The judge has carefully calculated that, as long as the witness answers every question truthfully, then she can solve the case in no more than 91 questions. Show that the judge can solve the case in no more than 105 questions if it is known that the witness may answer at most one question falsely.

In fact, under the stated conditions, the judge can solve the case in 98 questions. It isn't necessary for her, or the witness, to understand anything about coding theory to implement the solution, but the solution is best understood in the context of error-correcting codes.

Error-correcting codes

Coding theory is concerned with the efficient transmission of messages across space and/or time in such a way that errors in the transmission can be detected and/or corrected. For our purposes, a 'message' will be a finite string of 0s and 1s. A code will be a collection of such messages, or codewords, all of the same length. If two codewords differ in a single bit — if, say, $a = 011010111100$ and $b = 011010110100$ are both codewords — then it will be impossible for anyone receiving the message a to know whether the intended message was a, or whether the intended message was b, and a single error occurred in transmission.

Perhaps the simplest example of a one-error-detecting code is $\{\,00, 11\,\}$. If one error is made in transmitting either codeword, the result will be 01 or 10, neither of which is a codeword. Thus, the recipient will be able to tell that an error has

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Dedicated to the memory of the late Peter Pleasants, former colleague at Macquarie and enthusiastic contributor to the Puzzle Corner.

occurred. Of course, the recipient will be unable to tell *what* error occurred; on receiving 01, the recipient will be unable to tell whether the intended message was 00 or 11.

Perhaps the simplest example of a single-error-correcting code is $\{ 000, 111 \}$. Now if a single error occurs during transmission, the recipient will not only be able to detect it but to unambiguously correct it.

Crime investigation

To see what this has to do with crime investigation (or, at any rate, with puzzles about crime investigation), imagine that the judge can solve the case by asking a truthful witness a single yes–no question. Then to solve the case with a witness who may lie at most once, all she needs to do is ask the same question three times. Coding 'yes' as 1 and 'no' as 0, if there are no lies then the answers must be 000 or 111. A single lie corresponds to a single error in transmission, which can be detected and corrected.

The Hamming code

A somewhat more sophisticated one-error-correcting code is the Hamming code of length 7. This consists of the following 16 codewords:

1000011	0100101	0010110	0001111
0111100	1011010	1101001	1110000
0110011	1010101	1100110	1111111
1001100	0101010	0011001	0000000

To prove that this code can correct any single error, we need to show that every pair of codewords differs in at least three places. However, this code has some additional structure which makes it unnecessary to compare each pair of words; it is a vector space.

The set of all n-bit strings of 0s and 1s can be viewed as an n-dimensional vector space — not a real or complex vector space, but a vector space over $\boldsymbol{Z}_2 = \{ 0, 1 \}$, the field of two elements. In this field, $1 + 1 = 0$. An n-bit string can be identified with an n-component vector with entries from $\boldsymbol{Z}_2$. Vectors are added, and multiplied by scalars, in the usual component-wise way (but the only scalars are 0 and 1).

The Hamming code given above is a four-dimensional subspace of the seven-dimensional vector space of all seven-bit strings from $\boldsymbol{Z}_2$. It is easily checked that a basis is given by

$$\{ 1000011, 0100101, 0010110, 0001111 \}$$

The difference of any two codewords is again a codeword, and the number of bits in which any two codewords differ is just the number of ones in their difference. Since there is no non-zero codeword with fewer than three 1s, there are no two codewords that differ in fewer than three locations. Thus, the code can correct any single error in transmission.

The Hamming code has another useful property; every seven-bit string is either in the code, or differs in exactly one location from a string that is in the code. A simple counting argument establishes this, for each word in the code, together with the strings that differ from it in exactly one location, form a set of size 8, and the 'three locations' condition ensures that these sets of 8 are disjoint. There are 16 disjoint sets of 8, which accounts neatly for all 128 binary strings of length 7. For this reason the Hamming code is said to be a *perfect* code.

Crime investigation, revisited

Now a judge who can solve a case by asking a truthful witness four questions can solve the case by asking a witness seven questions, if the witness may lie at most once. The judge asks the four questions, then asks the three supplementary questions: 'If you had answered questions 2, 3 and 4 truthfully, would you have said 'yes' to an odd number of them?'; 'If you had answered questions 1, 3, and 4 truthfully, would you have said 'yes' to an odd number of them?'; 'If you had answered questions 1, 2, and 4 truthfully, would you have said 'yes' to an odd number of them?'

The judge views the seven answers received as a 7-bit string $\boldsymbol{y} = a_1a_2a_3a_4a_5a_6a_7$; remember, each 'yes' is a 1, each 'no' is a 0. If all seven answers are truthful, then $a_5 = a_2 + a_3 + a_4$, $a_6 = a_1 + a_3 + a_4$, and $a_7 = a_1 + a_2 + a_4$. These conditions are satisfied by the basis vectors given earlier for the Hamming code, hence, they are satisfied by all the codewords. The conditions are easily seen to determine a four-dimensional space, so in fact the seven answers are all truthful if and only if the string is a codeword.

Thus, if the string is a codeword, then all the answers are truthful, and the judge can solve the case. If the string is not a codeword, then there is a unique codeword from which it differs in a single location; the judge 'corrects' the string to that codeword, and solves the case.

Identifiers, matrices, and simplifications

We can make things a little easier for the judge and the witness. First, we can assign the first four questions the identifiers 011, 101, 110 and 111, in that order. Then the supplementary questions become: 'if you had answered the questions whose identifier had first bit 1 truthfully, would you have said 'yes' to an odd number of them?'; 'if you had answered the questions whose identifier had second bit 1 truthfully, would you have said 'yes' to an odd number of them?'; 'if you had answered the questions whose identifier had third bit 1 truthfully, would you have said 'yes' to an odd number of them?'

Second, we can spare the judge the work of hunting through the 16 codewords to find the closest match. The judge simply has to compute $\boldsymbol{s} = H\boldsymbol{y}$, where

$$H = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

and all arithmetic is done in $\boldsymbol{Z}_2$. Note that $H = [B \mid I]$ where B is the matrix whose columns are the identifiers given above for the first four questions and I is the 3×3 identity matrix. If we let $G = [I \mid B^t]$, where now I is the 4×4 identity matrix, so

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

then the row space of G is the nullspace of H. But the row space of G is the Hamming code, for which reason G is called the *generator matrix* for the code. Thus, $\boldsymbol{s} = \mathbf{0}$ if and only if $\boldsymbol{y}$ is a codeword, for which reason H is called the *parity-check matrix.* Moreover, if $\boldsymbol{s} \neq \mathbf{0}$, then $\boldsymbol{s}$ is one of the columns of H (since every non-zero three-bit string is a column of H), say, the jth column; then a_j is the bit of $\boldsymbol{y}$ that needs to be corrected.

For example, if $\boldsymbol{y} = 1011011$, then $H\boldsymbol{y} = 001$, which is the seventh column of H, so the seventh bit of $\boldsymbol{y}$ must be corrected, and the truthful answers are 1011010.

We can make life even easier for judge and witness. The supplementary questions can be: 'Did you lie about a question whose identifier has first bit 1?'; 'Did you lie about a question whose identifier has second bit 1?'; 'Did you lie about a question whose identifier has third bit 1?'

If the witness answered the first four questions truthfully, then he can answer 'yes' to at most one of the supplementary questions. If one of his first four answers was a lie, then he must answer all the supplementary questions truthfully, which means he must answer 'yes' to at least two of them. Thus, if he answers 'yes' to at most one of the supplementaries, he must have answered the first four truthfully, and the judge can solve the case; if he answers 'yes' to two or more of the supplementaries, then these answers must all be truthful, and the judge can easily work out which one of the first four questions elicited a deceitful answer, and solve the case. For example, if the witness responds in the affirmative to the first and third supplementaries, then he must have lied about the question with identifier 101, which was the second question.

There is a very simple relation between this scheme and the previous one. If we view the answers to this set of seven questions as the seven-bit string $\boldsymbol{z} = a_1a_2a_3a_4c_5c_6c_7$, then $c_5 = a_2 + a_3 + a_4 - a_5$, $c_6 = a_1 + a_3 + a_4 - a_6$, and $c_7 = a_1 + a_2 + a_4 - a_7$, as one can easily check by running through the logic. Thus, the new supplementary questions are giving us the same information as the old ones did, but this is a more readily usable form.

Mamma's little baby loves shortening, shortening

So much for the judge who only needs correct answers to four questions. Before we scale things up to 91 questions, we introduce the concept of *shortening* a code. To shorten a code, we choose any component, delete every codeword having a one in that component, then delete that component from the remaining codewords. For example, to shorten the Hamming code at the fifth component, we delete the eight codewords that have a 1 in the fifth location, and delete the fifth location from the remaining eight codewords, leaving 000000, 001101, 010110, 011011, 100011, 101110, 110101 and 111000. A shortened code has length one less than the original code, and it's easy to see that if the original code had no two words differing in fewer than d locations then the same is true of the shortened code; in particular, if the original was a single-error-correcting code, then so is the shortened code. Also, if the original code was a vector space, then the shortened code is, in general, a vector space of dimension one less. Finally, what you do once, you can do again; a code can be shortened repeatedly, and the result will always be an error-correcting code if the original was.

More Hamming codes, and solving the puzzle

Now what I have been calling the Hamming code is more accurately the simplest in an infinite family of (binary) Hamming codes. Given any k, one can construct the matrix $H = [B \mid I]$ whose columns are the non-zero k-bit strings, and the matrix $G = [I \mid B^t]$ with $2^k - k - 1$ rows and $2^k - 1$ columns. Then the row-space of G is a Hamming code. It is a $2^k - k - 1$ dimensional vector space whose members are strings of length $2^k - 1$, and it has no non-zero element with fewer than three 1s, so it can correct any single error. Also, H is a parity-check matrix for this code, as the code is the nullspace of H. Decoding, error-detecting, and error-correcting can be performed for these Hamming codes in much the same way as for the length 7 code we have been working with.

Taking $k = 7$, there is a 120-dimensional error-correcting code of length 127. Shortening this code 29 times, we arrive at a 91-dimensional code of length 98, which is exactly what our judge needs in order to solve the case presented in the opening paragraphs of this essay. There is no need for judge or witness to construct the matrices G and H associated with this code. The judge simply does the following:

1. Attach to each of the 91 questions a different seven-bit identifier, each identifier having at least two 1s. Note that there are 128 seven-bit strings, of which only eight have fewer than two 1s, so there are more than enough identifiers available.
2. Ask the 91 questions.
3. Ask the seven supplementary questions: 'Did you lie about a question whose identifier has first bit 1?'; 'Did you lie about a question whose identifier has second bit 1?'; ... 'Did you lie about a question whose identifier has seventh bit 1?'

If the witness answers the 91 questions truthfully, then he can affirm at most one of the supplementary questions. If the witness fibs about one of the 91 questions, then he must answer the supplementary questions truthfully, so must affirm at least two of them. Thus, if the witness denies all, or all but one, of the supplementary questions, he must have told the truth in answer to all of the 91 questions. If the witness affirms two or more of the supplementaries, he must be answering all the supplementaries truthfully, and the judge can then easily work out which one of the 91 questions brought forth a lie. Either way, she can then solve the case.

Judge not, lest ye be judged

Is our solution best possible? Can a cleverer judge come up with a scheme guaranteed to solve the case with only 97 questions?

Let's look at something smaller. In some other courtroom, the only thing the judge needs to know to solve a case is the remainder when the witness' age is divided by 20. There are, of course, 20 possibilities, and the judge, we assume, is unable to rule any of them out a priori. If the witness is truthful, then it is easy to design a scheme whereby the judge can solve the case with no more than five questions and might, depending on the witness' age, be able to solve it in fewer than five questions; no scheme can guarantee solving the case with fewer than five questions.

Now suppose the witness can lie at most once. Taking $k = 4$ in the previous section, we see there is an 11-dimensional Hamming code of length 15. Shortening this code six times, we arrive at a five-dimensional error-correcting code of length 9. The judge can use this to solve the case with nine yes-no questions.

But in fact the judge can solve the case with eight questions. Consider the code of length 8 which contains 11010000, 11100100, 10101010, 11111111, 00000000, and all cyclic shifts of these strings (e.g. the cyclic shifts of 11010000 are 10100001, 01000011, and so on). The reader can verify that there are 20 codewords, and every pair of codewords differs in at least three places (we owe this example to [1, Exercise 2.16]). Now all the judge has to do is assign the 20 possibilities for the witness' age to the 20 codewords (any one-one correspondence will do) and ask the witness the eight questions, 'Does the remainder when your age is divided by 20 correspond to a codeword whose first bit is a one?'; 'Does the remainder correspond to a codeword whose second bit is a one?'; …'Does the remainder correspond to a codeword whose last bit is a one?' The answers will either yield a codeword, or a string that differs in exactly one place from exactly one of the codewords, and in either situation the judge can solve the case.

To sum up, we can distinguish among 20 possibilities with eight questions if one lie is permitted; equivalently, there is an eight-dimensional error-correcting code with 20 codewords.

Now let's go back to the first courtroom. How many possibilities is the judge trying to distinguish? Certainly no more than 2^{91}, as she can guarantee to solve the case with 91 questions to a truthful witness. Presumably more than 2^{90}, as otherwise the 91 questions to a truthful witness could have been reduced to a smaller number. More than this we cannot say; we know only that there are m possibilities, for some m, $2^{90} < m \leq 2^{91}$.

The ability to solve the case with 97 questions is equivalent to the existence of an error-correcting code of length 97 with m codewords. If $m = 2^{91}$, then there certainly is no such code; a simple counting argument (known in the technical literature as 'the sphere-packing bound') shows this. It is not known for which values of m there is an error-correcting code of length 97 with m codewords; indeed, it is not even known for which values of m there is an error-correcting code of length 16 with m codewords. For these reasons, we are unable to say whether the judge can solve the case with 97 questions.

Extras

That concludes our crime investigation, but only scratches the surface of error-correcting codes. Here are a few topics I feel I ought to mention, even though I can hardly say anything about them here.

The Hamming codes can correct any single error but are useless if more than one error occurs in transmission. A great deal of effort has gone into finding codes capable of correcting multiple errors by dint of having every pair of codewords differ in many locations.

In some applications it is common for errors to occur in bursts, that is, for several consecutive bits to be affected by noise in transmission. In other applications, transposition errors are common. Codes can be specially designed to deal with burst and/or transposition errors.

The triple repetition code introduced above as the simplest error-correcting code has a rate of 1/3; that is, it conveys 1 bit of information for every 3 bits transmitted. The Hamming code of length 7 has a rate of 4/7. All other things being equal, the higher the rate, the better the code. However, all other things are not equal — there are trade-offs (inequalities) connecting the rate, the length, and the error-correcting capabilities of codes. A great deal of research goes into finding the theoretical restrictions on these quantities, and into constructing codes which push up against the theoretical restrictions.

Now suppose you have a code with a high rate and a high error-correcting capability — but suppose the only way to decode an incoming message is to compare it with each word in the code to see which one is closest. That won't be very practical, if the code is large. Ease of decoding is another code characteristic that must be taken into account in applications.

All the codes discussed in this essay are binary. This makes our computers happy, but codes in which the symbols used come from a set of size exceeding two are important in both theory and practice. An example is the *International Standard Book Number* (ISBN) with which books are issued nowadays. Since 2007, the ISBNs have formed a code of length 13 on the ten digits; previously, they formed a code of length 10 on the 11 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X.

There are many texts one can consult to follow up on some or all of these issues. Some published in the last 20 years are:

Adámek, J. (1991). *Foundations of Coding.* John Wiley.

Bierbrauer, J. (2004). *Introduction to Coding Theory.* Chapman and Hall.

Hoffman, D.G., Leonard, D.A., Lindner, C.C., Phelps, K.T., Rodger, C.A. and Wall, J.R. (1991). *Coding Theory.* Marcel Dekker.

Ling, S. and Xing, C. (2004). *Coding Theory.* Cambridge University Press.

McEliece, R.J. (2004). *The Theory of Information and Coding*, Student edn. Cambridge University Press.
Pless, V. (1998). *Introduction to the Theory of Error-Correcting Codes* John Wiley.
Pretzel, O. (1992). *Error-Correcting Codes and Finite Fields.* Oxford University Press.
Roman, S. (1997). *Introduction to Coding and Information Theory.* Springer.
Roth, R. (2006). *Introduction to Coding Theory.* Cambridge University Press.
Vermani, L.R. (1996). *Elements of Algebraic Coding Theory.* Chapman and Hall.
Welsh, D. (1998). *Codes and Cryptography.* Oxford University Press.

History

The puzzle appeared in Norman Do's Puzzle Corner 4 in this *Gazette*, 35 (2007) 202–207. It also appears at http://www.kalva.demon.co.uk/soviet/sov91.html according to which it was part of the 25th All Soviet Union Math Competition, held in 1991. This competition, with solutions, can be found in Arkadii Slinko's book, USSR Mathematical Olympiads 1989–1992, published by the Australian Mathematics Trust.

Very closely related is a problem Ulam posed in the last chapter of his autobiography, Adventures of a Mathematician (Scribner, 1976). See the solution given in [2].

References

[1] Hill, R. (1990). *A First Course in Coding Theory.* Oxford University Press.
[2] Pelc, A. (1987). Solution of Ulam's problem on searching with a lie. *J. Combin. Theory Ser. A* **44,** 129–140.

A geometric approach to saddle points of surfaces

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Abstract

We outline an alternative approach to the geometric notion of a saddle point for real-valued functions of two real variables. It is argued that our treatment is more natural than the usual treatment of this topic in standard texts on calculus.

1. Introduction

What is a saddle point of a surface in 3-space? A reasonable answer is: a saddle point is like the centre point of a horse saddle or the low point of a ridge joining two peaks. In other words, a saddle point is that peculiar point on the surface which is at once a peak along a path on the surface and a dip along another path on the surface. Another answer that is mundane but more likely to fetch points in a Calculus test is as follows. A *saddle point* of a real-valued function of two real variables is a critical point (that is, a point where the gradient vanishes) which is not a local extremum. The first answer gives an intuitive description of a saddle point, while the second is the mathematical definition commonly given in most texts on Calculus. (See, e.g. [1, §9.9] or [6, §3.3].) A typical example is the hyperbolic paraboloid given by $z = xy$ or by (the graph of) the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) := xy$. Here the origin is a saddle point. Indeed if we look at the paths along the diagonal lines $y = -x$ and $y = x$ in the plane, then we readily notice that the origin is at once a peak and a dip. Also, the origin is the only critical point of f and clearly f does not have a local extremum at the origin.

The aim of this paper is first, to point out that there is a significant disparity between the two answers, and second, to suggest an alternative approach to saddle points which may take care of this. The first point is easy to illustrate. There are surfaces or rather, functions of two variables where the conditions in the second answer are met but the geometric picture is nowhere close to the description in the first answer. For example, if $f\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x, y) := x^3$ or by $f(x, y) := x^2 + y^3$, then the origin is a saddle point according to the usual mathematical definition, but the corresponding surface (Figure 1) hardly looks like a saddle that you might want to put on a horse for any rider! Another unsatisfactory aspect is

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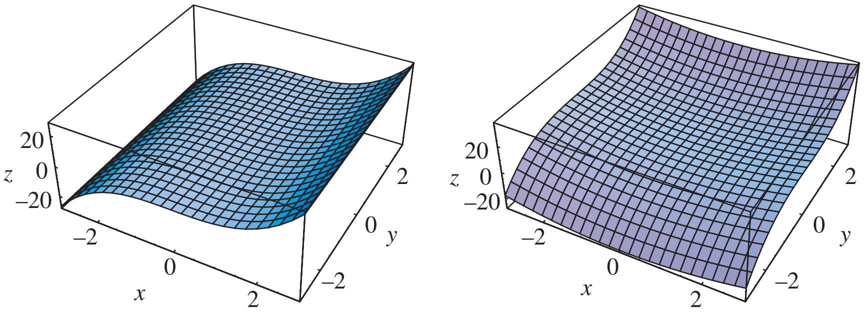

Figure 1. Graphs of $z = x^3$ and $z = x^2 + y^3$ near the origin.

the *a priori* assumption that the saddle point is a critical point, that is, a point at which the gradient exists and is zero. This is quite unlike the usual definitions of analogous concepts in one variable calculus, such as local extrema or points of inflection, where one makes a clear distinction between a geometric concept and its analytic characterisation (See, for instance, [3] and its review [8].) The definition we propose here seems to fare better on these counts in the case of functions of two variables. The basic idea is quite simple and, we expect, scarcely novel. However, we have not seen in the literature an exposition along the lines given here. For this reason, and with the hope that the treatment suggested here could become standard, we provide a fairly detailed discussion of the definition, basic results and a number of examples in the next three sections. Alternative approaches and extensions are briefly indicated in a remark at the end of the paper and we thank the referee for some of the suggestions therein.

2. Definition of a saddle point

Let D be a subset of $\mathbb{R}^2$. A *path* in D is a continuous map from $[a, b]$ to D. Here, and hereafter, while writing open or closed intervals such as (a, b) or $[a, b]$, it is tacitly assumed that $a, b \in \mathbb{R}$ with $a < b$. Given any $\boldsymbol{p} \in D$, a path $\gamma \colon [a, b] \to D$ is said to *pass through* $\boldsymbol{p}$ if $\gamma(t_0) = \boldsymbol{p}$ for some $t_0 \in (a, b)$. A path $\gamma \colon [a, b] \to D$ is said to be *regular* if γ is differentiable on (a, b) and $\gamma'(t) \neq \mathbf{0}$ for all $t \in (a, b)$. Two regular paths $\gamma_1 \colon [a_1, b_1] \to D$ and $\gamma_2 \colon [a_2, b_2] \to D$ are said to *intersect transversally* at some $\boldsymbol{p} \in D$ if there are $t_i \in (a_i, b_i)$ such that $\gamma_i(t_i) = \boldsymbol{p}$ for $i = 1, 2$ and moreover, $\gamma_1'(t_1)$ and $\gamma_2'(t_2)$ are not multiples of each other. In other words, the two paths pass through $\boldsymbol{p}$ and their tangent vectors at $\boldsymbol{p}$ are not parallel.

Example 1. (i) $\gamma \colon [-1, 1] \to \mathbb{R}^2$ defined by $\gamma(t) := (t, t^2)$ is a regular path, while $\widetilde{\gamma} \colon [-1, 1] \to \mathbb{R}^2$ defined by $\widetilde{\gamma}(t) := (t^2, t^3)$ is not a regular path.

(ii) If $\gamma_1, \gamma_2 \colon [-1, 1] \to \mathbb{R}^2$ are defined by $\gamma_1(t) := (t, -t)$ and $\gamma_2(t) := (t, t)$, then γ_1 and γ_2 are regular paths in $\mathbb{R}^2$ which intersect transversally at the origin. Further, the path $\gamma_3 \colon [-1, 1] \to \mathbb{R}^2$ defined by $\gamma_3(t) := (2t + t^2, 2t - t^2)$, is also regular and passes through the origin. The paths γ_1 and γ_3 intersect transversally at the origin, whereas the paths γ_2 and γ_3 do not.

Let $D \subseteq \mathbb{R}^2$, $\boldsymbol{p} \in D$ and $\gamma\colon [a,b] \to D$ be a regular path in D passing through $\boldsymbol{p}$ so that $\gamma(t_0) = \boldsymbol{p}$ for some $t_0 \in (a,b)$. Now, any $f\colon D \to \mathbb{R}$ can be restricted to (the image of) γ so as to obtain a real-valued function of one variable $\phi\colon [a,b] \to \mathbb{R}$ defined by $\phi(t) := f(\gamma(t))$. We shall say that f has a local maximum (or, similarly, a local minimum) at $\boldsymbol{p}$ along γ if ϕ has a local maximum (or, similarly, a local minimum) at t_0.

Definition 1. Let $D \subseteq \mathbb{R}^2$ and $\boldsymbol{p}$ be an interior point of D. A real-valued function $f\colon D \to \mathbb{R}$ has a *saddle point* at $\boldsymbol{p}$ if there are regular paths γ_1 and γ_2 in D intersecting transversally at $\boldsymbol{p}$ such that f has a local maximum at $\boldsymbol{p}$ along γ_1, while f has a local minimum at $\boldsymbol{p}$ along γ_2.

The above definition is a faithful abstraction of the idea that a saddle point is the point at which the graph of the function is at once a peak along a path and a dip along another path. The condition that the two paths intersect transversally might seem technical. But its significance will be clear from Example 2(iii) below.

It may be remarked that in our definition of a saddle point, we have permitted ourselves as much laxity as is usual while defining local extrema. To wit, if a function is locally constant at $\boldsymbol{p}$, then it has a local maximum as well as a local minimum at $\boldsymbol{p}$. In the same vein, a locally constant function at $\boldsymbol{p}$ has a saddle point at $\boldsymbol{p}$. More generally, if a function is locally constant along two regular paths intersecting transversally at $\boldsymbol{p}$, then it has a saddle point at $\boldsymbol{p}$. If we don't want to be so indulgent, then we can use the stronger notion of a *strict* saddle point. A *strict saddle point* is defined simply by replacing in Definition 1, local maximum by strict local maximum and local minimum by strict local minimum. Indeed, it is the notion of a strict saddle point that comes closest to our geometric intuition about saddle points. In almost all the examples as well as the criteria for saddle points discussed here, it is seen that the function has, in fact, a strict saddle point.

Example 2. (i) [Hyperbolic paraboloid] The function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := xy$ has a saddle point at $(0,0)$. To see this, it suffices to consider the paths γ_1 and γ_2 in Example 1(ii). Similarly, one can see that if $a,b,c,d \in \mathbb{R}$ with $ad-bc \neq 0$, then $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := (ax+by)(cx+dy)$ has a saddle point — in fact, a strict saddle point, at $(0,0)$.

(ii) [Monkey saddle] The function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := x^3 - 3xy^2$ has a strict saddle point at the origin. To prove this, it helps to look at the level curves of f. We then find that it suffices to consider the parabolic paths given by $t \mapsto (-t\sqrt{3}+t^2,\ t+t^2\sqrt{3})$ and $t \mapsto (t\sqrt{3}-t^2,\ t+t^2\sqrt{3})$ for $t \in [-\sqrt{3},\sqrt{3}]$. The surface $z = f(x,y)$ or the graph of f near the origin is shown in Figure 2 on the left. It may be interesting to try and visualise these paths on this surface.

(iii) [Fake saddle] Consider $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := x^3$. In this case $\gamma_1,\gamma_2\colon [-1,1] \to \mathbb{R}$ defined by $\gamma_1(t) := (-t^2,t)$ and $\gamma_2(t) = (t^2,t)$ are regular paths passing through $\mathbf{0}$. Also, f has a strict local maximum at $\mathbf{0}$ along γ_1 and a strict local minimum at $\mathbf{0}$ along γ_2. However, γ_1 and γ_2 do not intersect transversally at $\mathbf{0}$. In fact, as the surface on the left in Figure 1 indicates, f does not have a saddle point at $\mathbf{0}$. A formal proof of this is given later in Example 4(iii).

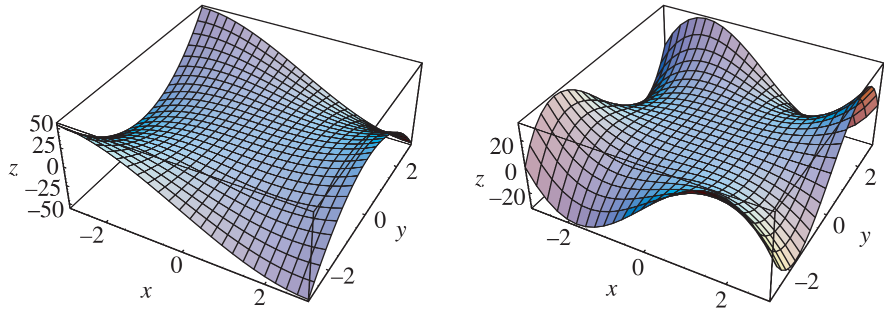

Figure 2. Monkey saddle $z = x^3 - 3xy^2$ and dog saddle $z = x^3y - xy^3$

We now show that a saddle point is necessarily a critical point. In what follows, by $\nabla f(\boldsymbol{p})$ we denote the gradient of a function f at an interior point $\boldsymbol{p}$ of its domain.

Proposition 1. *Let $D \subseteq \mathbb{R}^2$ and $\boldsymbol{p}$ be an interior point of D. If $f\colon D \to \mathbb{R}$ is differentiable at $\boldsymbol{p}$ and has a saddle point at $\boldsymbol{p}$, then $\nabla f(\boldsymbol{p}) = \mathbf{0}$.*

Proof. For $i = 1, 2$, let $\gamma_i\colon [a_i, b_i] \to D$ satisfy the conditions in Definition 1 with $n = 2$, and let us write $\boldsymbol{p} = \gamma_i(t_i)$ with $t_i \in (a_i, b_i)$ and $\phi_i := f \circ \gamma_i$. Since f is differentiable at $\boldsymbol{p}$ and γ_i is regular, by the chain rule, $\phi_i'(t_i)$ exists and equals $\nabla f(\boldsymbol{p}) \cdot \gamma_i'(t_i)$ for $i = 1, 2$. On the other hand, since ϕ_i have local extrema at t_i, we have $\phi_i'(t_i) = 0$ for $i = 1, 2$. Now, since $\gamma_1'(t_1)$ and $\gamma_2'(t_2)$ are linearly independent vectors in $\mathbb{R}^2$, we can conclude that $\nabla f(\boldsymbol{p}) = \mathbf{0}$.

3. Discriminant test

The discriminant test or the second derivative test is a high point of any exposition of local extrema and saddle points of functions of two real variables. It facilitates easy checking of saddle points in many, but not all, cases. The classical definition of a saddle point given in our introduction is, in fact, tailor-made so that the discriminant test can be proved easily. Some texts (e.g. [2, p. 347]) even take an easier option to *define* a saddle point as a critical point where the discriminant is negative. This may appear a bit like putting the cart before the horse. But the importance of the discriminant test can hardly be overemphasised and it seems imperative that it remains available with our geometric notion of a saddle point.

Let us recall that a *binary quadratic form* (over $\mathbb{R}$) is a polynomial of the form

$$Q(\boldsymbol{h}) = Q(h_1, h_2) := ah_1^2 + 2bh_1h_2 + ch_2^2,$$

where $\boldsymbol{h} = (h_1, h_2)$ is a pair of variables and a, b, c are (real) constants. We say that Q is *positive definite* if $Q(\boldsymbol{u}) > 0$ (similarly, *negative definite* if $Q(\boldsymbol{u}) < 0$) for all $\boldsymbol{u} \in \mathbb{R}^2$, $\boldsymbol{u} \neq \mathbf{0}$. In case Q takes positive as well as negative values, that is, if there are $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^2$ such that $Q(\boldsymbol{u})Q(\boldsymbol{v}) < 0$, then Q is said to be *indefinite*. In this situation, the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are necessarily nonzero and they can not be multiples of each other since $Q(t\boldsymbol{h}) = t^2Q(\boldsymbol{h})$ for any $t \in \mathbb{R}$ and $\boldsymbol{h} \in \mathbb{R}^2$.

Let $D \subseteq \mathbb{R}^2$ and $\boldsymbol{p}$ be an interior point of D. Suppose $f\colon D \to \mathbb{R}$ has continuous partial derivatives of first and second order in an open neighbourhood of $\boldsymbol{p}$. Then the *Hessian form* of f at $\boldsymbol{p}$ is the binary quadratic form defined by

$$Q_{\boldsymbol{p}}(\boldsymbol{h}) = Q_{\boldsymbol{p}}(h_1, h_2) := f_{xx}(\boldsymbol{p})h_1^2 + 2f_{xy}(\boldsymbol{p})h_1h_2 + f_{yy}(\boldsymbol{p})h_2^2.$$

With the hypothesis and notation as above, we have the following.

Proposition 2. *If $\nabla f(\boldsymbol{p}) = \boldsymbol{0}$ and the Hessian form $Q_{\boldsymbol{p}}$ of f at $\boldsymbol{p}$ is indefinite, then f has a strict saddle point at $\boldsymbol{p}$.*

Proof. The basic argument is similar to that used in many texts on calculus, but we provide a sketch for the sake of completeness. Assume that $\nabla f(\boldsymbol{p}) = \boldsymbol{0}$ and $Q_{\boldsymbol{p}}$ is indefinite. Then there are nonzero $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^2$ such that $Q_{\boldsymbol{p}}(\boldsymbol{u}) < 0$, while $Q_{\boldsymbol{p}}(\boldsymbol{v}) > 0$. By the continuity of the second order partials, there is $\delta > 0$ such that for any $\boldsymbol{q} \in \mathbb{R}^2$ with $\|\boldsymbol{q} - \boldsymbol{p}\| \leq \delta$, we have $\boldsymbol{q} \in D$ and $Q_{\boldsymbol{q}}(\boldsymbol{u}) < 0$, while $Q_{\boldsymbol{q}}(\boldsymbol{v}) > 0$. Scaling $\boldsymbol{u}$ and $\boldsymbol{v}$ suitably, we may assume that $\|\boldsymbol{u}\| \leq 1$ and $\|\boldsymbol{v}\| \leq 1$. Given any $t \in [-\delta, \delta]$ and $\boldsymbol{h} \in \mathbb{R}^2$ with $\|\boldsymbol{h}\| \leq 1$, by Taylor's Theorem, there is $\boldsymbol{q} \in \mathbb{R}^2$ on the line joining $\boldsymbol{p}$ and $\boldsymbol{p} + t\boldsymbol{h}$ such that

$$f(\boldsymbol{p} + t\boldsymbol{h}) - f(\boldsymbol{p}) = \nabla f(\boldsymbol{p}) \cdot (t\boldsymbol{h}) + \tfrac{1}{2} Q_{\boldsymbol{q}}(t\boldsymbol{h}) = \frac{t^2}{2} Q_{\boldsymbol{q}}(\boldsymbol{h}).$$

Thus, if $\gamma_1, \gamma_2\colon [-\delta, \delta] \to \mathbb{R}^2$ are defined by $\gamma_1(t) := \boldsymbol{p} + t\boldsymbol{u}$ and $\gamma_2(t) := \boldsymbol{p} + t\boldsymbol{v}$, then γ_1 and γ_2 are regular paths intersecting transversally at $\boldsymbol{p}$ such that f has a strict local maximum at $\boldsymbol{p}$ along γ_1 and a strict local minimum at $\boldsymbol{p}$ along γ_2.

Remark 1. The function f as in Example 2(ii) has a strict saddle point at $\boldsymbol{0}$, but its Hessian form at $\boldsymbol{0}$, being identically zero, is not indefinite. This shows that the converse of Proposition 2 is not true, in general. We can probe further. Observe that our proof of Proposition 2 actually shows that when the Hessian form is indefinite, the two paths satisfying the requirements for a strict saddle point, can be chosen as straight line segments. We can, therefore, ask if the 'weak converse' is true, that is, if straight line segments suffice to show that a differentiable function has a strict saddle point at $\boldsymbol{p}$, then whether the Hessian form $Q_{\boldsymbol{p}}$ is necessarily indefinite? The following example shows that the answer is negative.

Example 3. [Dog saddle] The function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) := x^3y - xy^3$ has a saddle point at the origin. To see this, it suffices to consider the paths given by $t \mapsto (t, -t/2)$ and $t \mapsto (t, t/2)$. These are straight line segments intersecting transversally at the origin for which the conditions in Definition 1 are satisfied. But the Hessian form of f at the origin is identically zero, and hence not indefinite. The graph of f near the origin is shown in Figure 2 on the right.

In order to apply Proposition 2 to specific examples, it is essential to have a useful characterisation of the Hessian form being indefinite. This is basically a well-known question of linear algebra. (See, e.g. [4, 5].) Again, we include the requisite result and a quick proof for the sake of completeness.

Lemma 1. *Let $Q(\boldsymbol{h}) := ah_1^2 + 2bh_1h_2 + ch_2^2$ be a binary quadratic form. If $ac - b^2 < 0$, then Q is indefinite.*

Proof. Observe that (i) if $a \neq 0$, then $Q(1,0)Q(b,-a) = a^2(ac-b^2) < 0$, (ii) if $a = 0$ and $c \neq 0$, then $Q(0,1)Q(c,-b) = c^2(ac-b^2) < 0$, and (iii) if $a = 0$ and $c = 0$, then $Q(1,1)Q(1,-1) = -4b^2 < 0$. Thus, in any case, Q is indefinite.

In the remainder of this section, let $D \subseteq \mathbb{R}^2$ and $\boldsymbol{p}$ be an interior point of D. Further, let $f\colon D \to \mathbb{R}$ be such that f has continuous partial derivatives of first and second order in an open neighbourhood of $\boldsymbol{p}$. We define the *discriminant* of f at $\boldsymbol{p}$ to be the real number

$$\Delta f(\boldsymbol{p}) := f_{xx}(\boldsymbol{p}) f_{yy}(\boldsymbol{p}) - f_{xy}(\boldsymbol{p})^2.$$

With the hypothesis and notation as above, we have the following.

Theorem 1 (Discriminant Test)**.** *If* $\nabla f(\boldsymbol{p}) = \mathbf{0}$ *and* $\Delta f(\boldsymbol{p}) < 0$*, then* f *has a strict saddle point at* $\boldsymbol{p}$*.*

Proof. Apply Lemma 1 to the Hessian form of f at $\boldsymbol{p}$ and use Proposition 2.

In fact, as an application of Proposition 1, we can obtain the following stronger version of Theorem 1.

Corollary 1. *Assume that* $\Delta f(\boldsymbol{p}) \neq 0$*. Then*

$$f \text{ has a saddle point at } \boldsymbol{p} \iff \nabla f(\boldsymbol{p}) = \mathbf{0} \text{ and } \Delta f(\boldsymbol{p}) < 0.$$

In particular, when $\Delta f(\boldsymbol{p})$ *is nonzero,* f *has a saddle point at* $\boldsymbol{p}$ *if and only if it has a strict saddle point at* $\boldsymbol{p}$*.*

Proof. If f has a saddle point at $\boldsymbol{p}$, then it can not have a strict local extremum at $\boldsymbol{p}$. Hence, by the discriminant test for local extrema of functions of two variables [6, §3.3], $\Delta f(\boldsymbol{p})$ can not be positive. Thus, in view of Proposition 1, we have $\nabla f(\boldsymbol{p}) = \mathbf{0}$ and $\Delta f(\boldsymbol{p}) < 0$. The converse follows from Theorem 1. The last assertion follows from the equivalence just proved and Theorem 1.

Remark 2. Example 2(i) can be treated with the help of the discriminant test. On the other hand, if f is as in Example 2(ii) or Example 3, then $\Delta f(\mathbf{0}) = 0$, and hence the discriminant test is not applicable. This shows that the converse of Theorem 1 is not true, in general.

4. Examples

The aim of this section is to discuss a variety of examples, which not only illustrate our definition of a saddle point but also enable the reader to compare it with the definition usually found in Calculus texts. In the latter case, we call it a saddle points in the classical sense. Note that if $f\colon \mathbb{R}^2 \to \mathbb{R}$ is a constant function and $\boldsymbol{p}$ is any point of $\mathbb{R}^2$, then f has a saddle point at $\boldsymbol{p}$ in our sense but not in the classical sense. On the other hand, if $f\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x,y) := x^3$, then as we show in Example 4(iii) below, f has a saddle point at the origin in the classical sense, but not in our sense. However, a strict saddle point in our sense is a saddle point in the classical sense. This is the case when the discriminant test (Theorem 1) is applicable.

Example 4. (i) Consider $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := \min\{|x|,|y|\}$ if $xy \geq 0$ and $f(x,y) := -\min\{|x|,|y|\}$ if $xy < 0$. Using the paths γ_1 and γ_2 in Example 1(ii), namely, those given by $t \mapsto (t,-t)$ and $t \mapsto (t,t)$, we see that f has a strict saddle point at $(0,0)$. Thus, a nondifferentiable function can have a saddle point (in our sense).

(ii) Let $c_1, c_2 \in \mathbb{R}$ with $0 < c_1 < c_2$. Consider $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := (y - c_1x^2)(y - c_2x^2)$. Using the paths given by $t \mapsto (t, ct^2)$ and $t \mapsto (0,t)$, where $c \in \mathbb{R}$ satisfies $c_1 < c < c_2$, we see that f has a strict saddle point at $(0,0)$. Note that for $\lambda \in \mathbb{R}$, $\lambda \neq 0$, we have $f(t,\lambda t) = t^2(\lambda - c_1t)(\lambda - c_2t) > 0$ for $0 < |t| < |\lambda|/c_2$. Also, $f(t,0) = c_1c_2t^4 > 0$ and $f(0,t) = t^2 > 0$ for all $t \neq 0$. Hence f has a strict local minimum at $(0,0)$ along every straight line through the origin. Thus, in this example, straight line segments alone can not work to show that f has a saddle point at $(0,0)$, but a combination of a parabola and a straight line segment does.

(iii) Consider $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) := x^m$, where m is an odd positive integer. If $m = 1$, then clearly, f has no saddle points (in any sense). Assume now that $m > 1$. Then f is differentiable and $\nabla f(0,y_0) = (0,0)$ for any $y_0 \in \mathbb{R}$. Since f takes both positive and negative values in every open neighbourhood of $(0,y_0)$, we see that f has a saddle point at $(0,y_0)$ in the classical sense for every $y_0 \in \mathbb{R}$. On the other hand, if it had a saddle point (in our sense) at $(0,y_0)$ for some fixed $y_0 \in \mathbb{R}$, then we would find paths $\gamma_i\colon [a_i,b_i] \to \mathbb{R}^2$ satisfying the conditions of Definition 1. Write $\gamma_i(t) := (x_i(t), y_i(t))$ and let $t_i \in (a_i,b_i)$ be such that $\gamma_i(t_i) = (0,y_0)$. Then 0 is a local extremum of x_i^m at t_i. But since x and x^m have the same sign for any $x \in \mathbb{R}$, it follows that each x_i has a local extremum at t_i. Consequently, $x_i'(t_i) = 0$ for $i = 1,2$, and therefore γ_1 and γ_2 can not intersect transversally at $(0,y_0)$. Thus f does not have any saddle point.

Our next set of examples generalise some of the simplest and most natural types of functions of two variables, such as xy, $x^2 - y^2$, $x^3 - 3xy^2$ and x^m, which we have seen earlier. The arguments in the general case are a bit involved and make good material for starred exercises in calculus texts (although we have yet to see them in print), especially for those who may choose to adopt our definition of saddle point.

Example 5. (i) Let $m,n \in \mathbb{N}$ (where $\mathbb{N}$ denotes the set of positive integers) and $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) := x^my^n$. Then

$$f \text{ has a strict saddle point at } (0,0) \iff \text{both } m \text{ and } n \text{ are odd.}$$

The reverse implication is easy. Indeed, if m and n are odd, then $m+n$ is even and $f(t,-t) = -t^{m+n} < 0$, while $f(t,t) = t^{m+n} > 0$ for all $t \neq 0$. Thus it suffices to consider the paths γ_1 and γ_2 in Example 1(ii). To prove the forward implication, first suppose m and n are both even. Then $f(x,y) \geq 0$ for all $(x,y) \in \mathbb{R}^2$ and hence f can not have a strict local maximum at $(0,0)$ along any path passing through $(0,0)$. So f can not have a strict saddle point at $(0,0)$. Next, suppose m is odd and n is even. For $i = 1,2$, let $\gamma_i\colon [a_i,b_i] \to \mathbb{R}^2$ be regular paths intersecting transversally at $(0,0)$ such that f has a strict local maximum (or a strict local minimum) along γ_1 (resp: γ_2). Write $\gamma_i(t) := (x_i(t), y_i(t))$ and let $t_i \in (a_i,b_i)$ be such that $\gamma_i(t_i) = (0,0)$. Then there is $\delta_1 > 0$ such that

$$0 < |t - t_1| < \delta_1 \Longrightarrow x_1(t)^m y_1(t)^n < 0 \Longrightarrow y_1(t) \neq 0 \text{ and } x_1(t) < 0,$$

where the last implication follows since n is even and m is odd. Thus, x_1 has a strict local maximum at t_1 and so $x_1'(t_1) = 0$. Similarly, there is $\delta_2 > 0$ such that

$$0 < |t - t_2| < \delta_2 \Longrightarrow x_2(t)^m y_2(t)^n > 0 \Longrightarrow y_2(t) \neq 0 \text{ and } x_2(t) > 0.$$

Consequently, $x_2'(t_2) = 0 = x_1'(t_1)$, which contradicts the assumption that γ_1 and γ_2 intersect transversally at $(0,0)$. The case when m is even and n is odd is similar. Thus, we have shown that if both or one of m and n is even, then f does not have a strict saddle point at $(0,0)$.

(ii) Let $m, n \in \mathbb{N}$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) := x^m - y^n$. Then

$$f \text{ has a saddle point at } (0,0) \Longleftrightarrow \text{both } m \text{ and } n \text{ are even.}$$

The reverse implication is again easy. Indeed, if m and n are both even, we have $f(0,t) = -t^n < 0$, while $f(t,0) = t^m > 0$ for all $t \neq 0$. Thus it suffices to consider the paths $t \mapsto (0,t)$ and $t \mapsto (t,0)$. To prove the forward implication, observe that if $m = 1$ or $n = 1$, then $\nabla f(0,0) \neq (0,0)$, and hence by Proposition 1, f can not have a saddle point at $(0,0)$. So we now assume that $m \geq 2$ and $n \geq 2$. Suppose m or n is odd. Note that if $g\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by $g(x,y) := -f(y,x) = x^n - y^m$, then clearly, f has a saddle point at $(0,0)$ if and only if g does. Thus, we may assume without loss of generality that m is odd. Let us first consider the case when $n \geq m$. Let $\gamma_i\colon [a_i, b_i] \to \mathbb{R}^2$ be paths satisfying the conditions in Definition 1. Write $\gamma_i(t) := (x_i(t), y_i(t))$ and let $t_i \in (a_i, b_i)$ be such that $\gamma_i(t_i) = (0,0)$. Then there is $\delta_1 > 0$ such that

$$|t - t_1| < \delta_1 \Longrightarrow x_1(t)^m \leq y_1(t)^n \Longrightarrow x_1(t) \leq y_1(t)^{n/m},$$

where the last implication follows since m is odd. Since $\gamma_1(t_1) = (0,0)$, we see that $x_1 - y_1^{n/m}$ has a local maximum at t_1, and so $x_1'(t_1) - (n/m) y_1(t_1)^{(n-m)/m} y_1'(t_1) = 0$. It follows that $x_1'(t_1) = y_1'(t_1)$ if $n = m$ and $x_1'(t_1) = 0$ if $n > m$. Similarly, there is $\delta_2 > 0$ such that

$$|t - t_2| < \delta_2 \Longrightarrow x_2(t)^m \geq y_2(t)^n \Longrightarrow x_2(t) \geq y_2(t)^{n/m},$$

and this yields that $x_2'(t_2) = y_2'(t_2)$ if $n = m$ and $x_2'(t_2) = 0$ if $n > m$. Either way, the condition that γ_1 and γ_2 intersect transversally is contradicted. Next, suppose m is odd and $n < m$. In case n is odd, then considering $g\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $g(x,y) := -f(y,x) = x^n - y^m$, we obtain from the previous case that g, and hence f, does not have a saddle point at $(0,0)$. Thus, let us assume that n is even. Now, as before, there is $\delta_2 > 0$ such that

$$|t - t_2| < \delta_2 \Longrightarrow x_2(t)^m \geq y_2(t)^n \geq 0 \Longrightarrow x_2(t) \geq 0,$$

and this yields $x_2'(t_2) = 0$. Consequently, there is $\xi_2\colon [a_2, b_2] \to \mathbb{R}$ such that $x_2(t) = (t - t_2)\xi_2(t)$ for all $t \in [a_2, b_2]$ and moreover $\xi_2(t) \to 0$ as $t \to 0$. Also, since y_2 is differentiable at t_2, there is $\eta_2\colon [a_2, b_2] \to \mathbb{R}$ such that $y_2(t) = (t - t_2)\,[y_2'(t_2) + \eta_2(t)]$ for all $t \in [a_2, b_2]$ and moreover $\eta_2(t) \to 0$ as $t \to 0$. Thus,

$$|t - t_2| < \delta_2 \Longrightarrow (t - t_2)^m \xi_2(t)^m \geq (t - t_2)^n [y_2'(t_2) + \eta_2(t)]^n,$$

and hence

$$0 < |t - t_2| < \delta_2 \Longrightarrow (t - t_2)^{m-n}\xi_2(t)^m \geq [y_2'(t_2) + \eta_2(t)]^n.$$

Since n is even, upon letting $t \to t_2$, we see that $0 \geq y_2'(t_2)^n = |y_2'(t_2)|^n$, and hence $y_2'(t_2) = 0 = x_2'(t_2)$. So the condition that γ_2 is regular is contradicted.

(iii) Let $m, n \in \mathbb{N}$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) := x^m + y^n$. Then f never has a saddle point at $(0, 0)$. To see this, note that if m and n are both even, then $f(x, y) > 0$ for all $(x, y) \neq (0, 0)$, and so f cannot have a local maximum along any path passing through the origin. The remaining cases can be proved by arguments similar to those in (ii) above.

(iv) [Generalised Monkey Saddle] Let $n \in \mathbb{N}$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) := \mathrm{Re}(x + iy)^n$. Note that the surface $z = f(x, y)$ is parametrically given by $x = r\cos\theta$, $y = r\sin\theta$ and $z = r^n \cos n\theta$, where $r \geq 0$ and $-\pi < \theta \leq \pi$. If $n = 1$, then clearly, f has no saddle points. But f has a strict saddle point at $(0, 0)$ if $n \geq 2$. The case when n is even is easy. In this case one easily sees that it suffices to consider the paths given by $t \mapsto (t\cos(\pi/n), t\sin(\pi/n))$ and $t \mapsto (t, 0)$. Next, suppose n is odd and $n > 1$. In this case, f is negative in the sectors

$$\frac{\pi}{2n} < \theta < \frac{\pi}{n} \quad \text{and} \quad -\pi < \theta < -\pi + \frac{\pi}{2n},$$

whereas f is positive in the sectors

$$-\frac{\pi}{2n} < \theta < 0 \quad \text{and} \quad \pi - \frac{\pi}{n} < \theta < \pi - \frac{\pi}{2n}.$$

With this in view, we see that if γ_1 is the parabolic path given by

$$t \longmapsto \left(-t\cos\frac{\pi}{2n} + t^2 \sin\frac{\pi}{2n}, t\sin\frac{\pi}{2n} + t^2\cos\frac{\pi}{2n} \right),$$

then $f(\gamma_1(t)) < 0$ for $t \neq 0$ with $|t|$ small, while if γ_2 is the parabolic path given by

$$t \longmapsto \left(t\cos\frac{\pi}{2n} - t^2 \sin\frac{\pi}{2n}, t\sin\frac{\pi}{2n} + t^2\cos\frac{\pi}{2n} \right),$$

then $f(\gamma_2(t)) > 0$ for $t \neq 0$ with $|t|$ small. Also, γ_1 and γ_2 intersect transversally at $(0, 0)$. Thus, f has a strict saddle point at $(0, 0)$.

(v) Let $n \in \mathbb{N}$ and $g\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $g(x, y) := \mathrm{Im}(x + iy)^n$. Note that the surface $z = g(x, y)$ is parametrically given by $x = r\cos\theta$, $y = r\sin\theta$ and $z = r^n \sin n\theta$, where $r \geq 0$ and $-\pi < \theta \leq \pi$. By arguments similar to those in (iv) above, it can be proved that g does not have a saddle point at $(0, 0)$ if $n = 1$, while g has a strict saddle point at $(0, 0)$ if $n \geq 2$.

Remark 3. Due to the nature of our definition of a saddle point, it is not entirely trivial to show that a specific point is a saddle point of a function. The discriminant test can help in many cases, but if it fails, then one has to painstakingly construct regular paths with the desired properties. To this end, it helps to look at the level curves and to know how the function behaves near the point in question, but still some guessing is needed. Further, when Corollary 1 is not applicable, it becomes even more challenging to show that a point is *not* a saddle point of a given function. For this, we need to logically rule out the existence of regular paths satisfying the properties stated in Definition 1.

Remark 4. As is common in introductory texts on multivariable calculus, we have restricted to functions of two variables. But it is clear that many of the notions and results discussed here extend readily to $\mathbb{R}^n$ in place of $\mathbb{R}^2$. For instance, the notions of paths, regularity, transverse intersections, indefiniteness of a quadratic form, and the Hessian form admit straightforward generalisations. If one uses transversally intersecting regular paths in $\mathbb{R}^n$ to define a saddle point of a function of n variables exactly as in Definition 1, then Proposition 2 continues to hold and Theorem 1 admits an analogue with the condition '$\Delta f(\boldsymbol{p}) < 0$' replaced by '$f_{x_i x_i}(\boldsymbol{p}) f_{x_j x_j}(\boldsymbol{p}) - f_{x_i x_j}(\boldsymbol{p})^2 < 0$ for some i, j with $i \neq j$'. However, the analogue of Proposition 1 for $\mathbb{R}^n$ is not valid if $n > 2$. [Consider, for example, $f\colon \mathbb{R}^3 \to \mathbb{R}$ defined by $f(x_1, x_2, x_3) := x_1 x_2 + x_3$.] In other words, a saddle point is not automatically a critical point. For this reason, a straightforward analogue of Definition 1 is not very satisfactory. A better option may be to define a real-valued function f on an open subset D of $\mathbb{R}^n$ to have a saddle point at $\boldsymbol{p} \in D$ if there are submanifolds of D whose tangent spaces at $\boldsymbol{p}$ span $\mathbb{R}^n$ such that f has a local maximum at $\boldsymbol{p}$ on one and a local minimum at $\boldsymbol{p}$ on another. Returning to the case $n = 2$, if one wants to consider surfaces more general than those defined by graphs of functions of two variables, another plausible definition for $\boldsymbol{p}$ to be a saddle point of a surface S in $\mathbb{R}^3$ could be that there is a plane P passing through $\boldsymbol{p}$ such that $P \cap S$ is like a 'graph' for which $\boldsymbol{p}$ is a vertex of degree ≥ 4.

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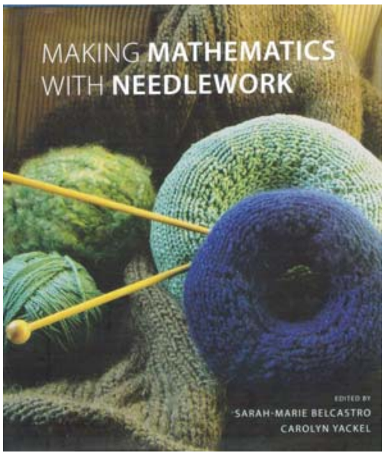
Book reviews

Making Mathematics with Needlework

Sarah-Marie Belcastro and Carolyn Yackel (eds)
A.K. Peters Ltd, 2008, ISBN: 978-1-56881-331-8

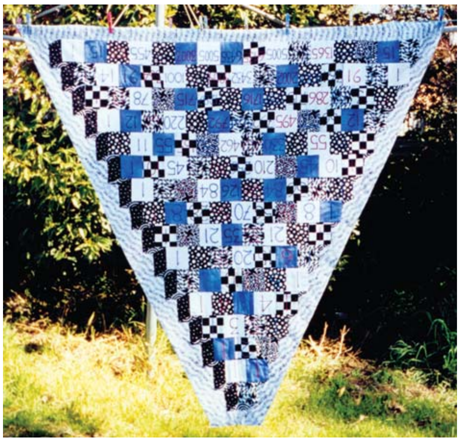
It is 150 years since August Ferdinand Möbius discovered the now famous Möbius strip or band. Almost all readers of this review will have made a Möbius strip out of paper at some stage. You could make a quilt in the same way. A quilted Möbius strip would be very warm around the shoulders especially with the twist in the front. Your scarf would be a work of art and the talk of the party. A colleague pointed out that, in addition to being useful and artistic, the quilted Möbius band would save time — you would have to iron only one side.



Chapter 1 of this very interesting book deals with the quilted Möbius band. The chapter opens with a description of how to make a Möbius strip out of paper and presents a few experiments that you can do with the surface. Then the mathematical aspects of the Möbius strip are explained using concepts from algebraic topology. A few graph colouring problems set in the context of the Möbius strip are outlined. This leads to a description of imaginative ideas for using in the classroom at all levels. The chapter concludes with instructions on how to make a quilted Möbius band. The bibliography for this chapter includes books from mathematics, mathematics education, quilting and history of fashion. The chapter is illustrated with delightful colour photos and figures. The other chapters, on different topics, have a similar structure.

From a craft perspective, this is a charming book. It includes precise and detailed instructions for projects both practical (knitted beanies and socks) and whimsical (quilted Möbius strips, a knitted torus and a class exercise on the construction of a Fortunatus' purse). The mathematics applied to knitting is particularly useful in the construction of knitted garments, providing simple formulae for ensuring that stitches are picked up evenly. The analysis of all possible variations in basic stocking stitch is fascinating, and the innovative ideas for the construction of knitted garments encourages further experiments in producing garments without seams. Like the authors, we find sewing sleeves into jumpers tedious. The chapter on knots, cables and braids is of great interest to those who indulge themselves in knitting Aran jumpers and the Algebraic Socks are fun. Although the mathematics involved in the construction of the Wearable Pants is complex for a non-mathematical person, the pants themselves are great.

Using the Sierpinski triangle and carpet to generate designs for crocheted shawls is interesting, but the resulting design does not appear very special to someone who is unaware of its significance. As an exercise in learning how to do filet crochet, however, it is easy and logical, and the resulting shawl is an excellent beginner's project. A more advanced project could be generated from the Koch snowflake, but it would need more advanced crochet skills.


Quilted Pascal's triangle by Sabine Wilkens

The chapters on symmetry in cross-stitch patterns and the construction and working of patterns in Spanish Blackwork are very well written, and encourage one to try them out. If only we had the time!

We are grateful to our colleague Dr Sabine Wilkens for allowing us to publish the photo of her quilted Pascal's triangle to accompany this review.

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Geometric Puzzle Design

Stewart Coffin
A.K. Peters, 2007, ISBN: 978-0-19-857043-1

Mathematicians are often captivated by puzzles. This includes the puzzles that Stewart Coffin writes about, two- and three-dimensional puzzles that are often assembled from pieces made of wood. There are many books on solving puzzles, but Coffin's book is different. It is one of the few books on the design and making of puzzles.

For most of the puzzles in this book the problem is to assemble the puzzle from a collection of pieces, and for some of the three-dimensional puzzles disassembly is part of the problem. The popular seven-piece tangram, whose pieces can be assembled into many shapes including a square, is an example of a two-dimensional puzzle. The beautiful symmetric wooden puzzles that appear to be made of rods or sticks, and which come apart if only one finds the right piece to move, are examples of three-dimensional puzzles.

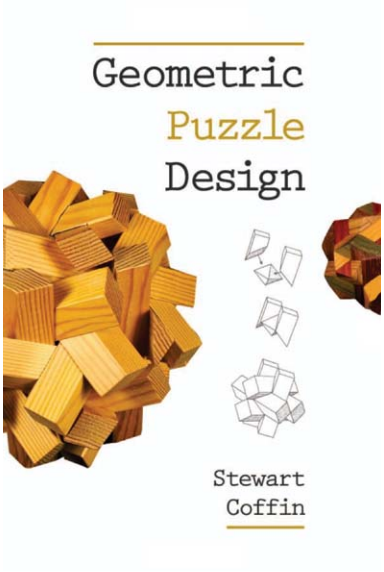
Stewart Coffin is very well known for his many years of puzzle creation. Jerry Slocum and Jack Botermans in their book *Puzzles New and Old: How to Make and Solve Them* (p. 84) say of Coffin:

> Quite simply, Stewart Coffin is the most outstanding designer and maker of interlocking puzzles that the world has even seen.

Coffin successfully gives the reader an insight into how a puzzle designer thinks; one can begin to see how he has come up with the ideas for his puzzles.

The book is not a mathematics book, though Coffin does pose a few mathematical questions for the reader. It is likely to appeal to those who want to know how puzzles are designed, how they are made or how one might think up some of these lovely geometric shapes. It should also interest those who want to know more so as to be able to solve geometric puzzles as it gives some insight into how they work. At the very least, it will inform the reader about the variety of puzzles that have been created.

Not all these puzzles are difficult to solve. The attraction of some is their beauty and symmetry, and their seeming simplicity.



Although the bulk of the book is about three-dimensional puzzles, it begins with puzzles in two dimensions. There are dissections, puzzles where the pieces are all made from a single shape and where the aim is to assemble these pieces into one or a variety of shapes. For example, pentominoes in which all pieces are made from five squares. There are puzzles which mislead, by unusual angles or a space where not expected. These are all described from the point of view of a designer, so one can see how they might be altered to make other puzzles.

We see block puzzles in which one just has to find a way to stack or put together the various pieces. Other puzzles have interlocking pieces, many constructed from notched sticks, and in some the order of assembly is important. There are sets of pieces which can be put together in various ways to form different symmetrical shapes, there are puzzles where the pieces must all move into place together rather than one at a time and there are puzzles with unlikely construction and disassembly, designed to mislead the solver.

A Catalan solid is the dual of an Archimedean solid. A Catalan solid is convex, its group of symmetries is face transitive but not vertex transitive, and the faces are not regular polygons. Two of the thirteen Catalan solids give rise to some aesthetically pleasing puzzles. The first to make an appearance in Coffin's book is the rhombic dodecahedron, which has 12 rhombic faces. Several chapters are based partly or wholly on this solid. There is a symmetrical arrangement of 12 sticks which totally encloses a rhombic dodecahedron, but this arrangement is not stable. Coffin discusses a few ways in which this unstable construction can be used to create something that can be assembled and which will stay together. The 12 sticks that surround the rhombic dodecahedron can have pieces added to them so that they become interlocking. The cross section can be altered to form sticks that can be notched, creating a puzzle that does not fall apart. The sticks can be split, so that there are 24 pieces. These 24 pieces can then be joined to different pieces to again form a puzzle that is stable. The ends of the sticks can be changed to form various stellations. The symmetries of these puzzles allow for the use of different coloured woods to good effect. A well-made wooden rhombic dodecahedral puzzle is very beautiful.

The second Catalan solid mentioned by Coffin is the rhombic triacontahedron. It has 30 rhombic faces, 60 edges and 32 vertices. One might think that the many puzzles based on the rhombic dodecahedron would transfer to the rhombic triacontahedron but this is not so. However, it is the basis for some puzzles, and Coffin shows some of what can be done.

The design and making of geometric puzzles is a puzzle in itself. As Coffin writes '... there is often no clear dividing line as to where the design process stops and the solution begins ...'.

For some of us the hardest part would be the woodwork. There are woodworking tips throughout the book and the final chapter gives advice on woodworking and choice of tools.

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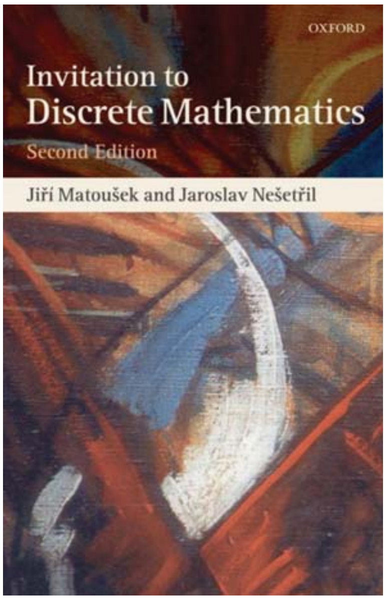
Invitation to Discrete Mathematics (second edition)

Jiří Matoušek and Jaroslav Nešetřil
Oxford University Press, 2008, ISBN 978-0-19-857043-1

Matoušek and Nešetřil give us a really enjoyable and mathematically appealing introduction to the major themes and problems in discrete maths, specifically combinatorics and graph theory, in this book. They delve into some interesting topics, with considerable rigour and detail for the capable reader. However, I fear it may be at too high a level for a first year Discrete Mathematics course in Australia, requiring too much mathematical maturity and interest for the average undergraduate in such a course.

Later topics include a chapter on probabilistic proofs and Ramsey's theory (both motivated and explained beautifully in my opinion), a chapter on generating functions (not quite as nice as Wilf's *generatingfunctionolgy* [2]), and the final chapter entitled 'Applications of linear algebra' which includes block designs (of interest to my colleagues at The University of Queensland I'm sure) and some more probabilistic algorithms (fast probabilistic checking of matrix multiplication, and of associativity of binary operations on a set).



The exercises throughout the book are all interesting and will be a challenge for most students. The text avoids lots of repetitive 'apply the definition to the following'-type exercises and gets straight to the good stuff. There are hints in the back, which still leave lots of work to do.

In several places the authors present a number of alternative proofs for the same results, which I think is really nice to show students (and remind the rest of us) that there can be many approaches to mathematics, each having its strengths and detractions. They prove Cayley's formula for the number of trees on n vertices in at least four different ways, for example. It is not always clear which one comes from 'The Book' [1]!

I don't expect this text to replace any of the usual bulky and colourful Discrete Maths texts currently in use for first year and CS-leaning courses in Australia, but I think as a resource for novel and accessible approaches to sometimes difficult topics and interesting extensions, it is well worth taking a look. For later level students who've taken a standard discrete math course, I think Chapters 7–13, the second half of the book, would make a great basis for an interesting course.

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AMSI News

Philip Broadbridge[*]

Energy and water are the primary necessities around which settlements are built. However we have often been reminded that clean water and energy supply cannot be taken for granted. A lot of interest is being shown in the AMSI-MASCOS-UNESCO industry workshop and short course, ‘Future Models for Energy and Water Management in a Regulated Environment’, to be held at Queensland University of Technology from 20 to 22 July. We look forward to hosting 10 UNESCO delegates and a range of invited and contributing speakers. Information for this event as well as a list of other sponsored workshops can be accessed at http://www.amsi.org.au/workshops_conf.php.

After seven years of operation the sponsored educational activities of AMSI and ICE-EM are well known, so student applications for support are becoming increasingly competitive. Over summer, AMSI funded 20 undergraduate vacation scholarships at a number of universities. In February, the vacation scholars also attended the CSIRO-managed Big Day In, held at Macquarie University, where along with a larger number of CSIRO Vacation Scholars, they presented summaries of their projects. The talks were of a very high standard.

For the past two years, AMSI-supported Australian PhD students have contributed well to the PIMS-MITACS Graduate Industrial Mathematics Modelling Camp and the Industrial Problem Solving Workshop. This year, six Australian students will be attending the next event to be held in Calgary late in May. As a benefit of our reciprocal agreement with MITACS, the students will be provided with free accommodation. We are able to send such a number because in some cases, the students’ home institutions have been willing to share the expenses.

From the outset of the National Curriculum Board, AMSI has facilitated dialogue (a better word would be ‘trialogue’) among university mathematicians, education experts and the NCB. Following the February meeting of the Australian Council of Heads of Mathematical Sciences, both that body and the Education Advisory Committee of AMSI submitted commentary to NCB on the draft mathematics shaping document. I am very pleased to report that our own Michael Evans, manager of the ICE-EM Mathematics program, is one of a small number to be selected from over 100 applicants to be on the NCB Mathematics Curriculum writing team.

Director of AMSI since 2005, Phil Broadbridge was previously a professor of applied mathematics for 14 years, including a total of eight years as department chair at University of Wollongong and at University of Delaware. His PhD was in mathematical physics (University of Adelaide). He has an unusually broad range of research interests, including mathematical physics, applied nonlinear partial differential equations, hydrology, heat and mass transport and population genetics. He has published two books and 100 refereed papers, including one with over 150 ISI citations. He is a member of the editorial boards of four journals and one book series.

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E-mail: phil@amsi.org.au

University of
South Australia

The 53rd Annual Meeting of the Australian Mathematical Society

28 September – 1 October 2009

University of South Australia, City West Campus, Adelaide

Featuring the 2009 Clay-Mahler Lecturer Terence Tao

Plenary Speakers

Werner Ballmann
(Bonn/Max Planck Institute)

Michael Eastwood (ANU)

Dennis Gaitsgory (Harvard)

Ezra Getzler (Northwestern)

Kerry Landman (Melbourne)

William Moran (Melbourne/DSTO)

Jacqui Ramagge (Wollongong)

Clay-Mahler Lecturer Terence Tao
(UCLA)

ANZIAM Lecturer Peter Taylor
(Melbourne)

Early Career Lecturer
Akshay Venkatesh (Stanford)

Special Sessions

- Algebra and number theory
- Analysis and PDEs
- Applied differential equations
- Combinatorics
- Computational mathematics
- General session
- Geometric analysis
- Geometry and topology
- History and philosophy of mathematics
- Interpolation in mechanics
- Logic
- Mathematics education
- Mathematical physics
- Mathematical psychology
- New applications of mathematics
- Noncommutative geometry and operator algebras
- Optimal control
- Optimisation: theory and methods
- Probability: theory and applications
- Random processes
- Statistics
- Topological and symbolic dynamical systems
- Topological groups

Director: Vladimir.Ejov@unisa.edu.au
Homepage and Registration: http://www.unisa.edu.au/austms2009/
Conference email: Austms09@unisa.edu.au

News

General News

ANZIAM: Nominations for 2010 ANZIAM Medal

A search is underway to identify nominees for the 2010 ANZIAM Medal, and nominations should be forwarded in confidence to the Chair of the Selection Panel, Professor Graeme Wake by the end of October 2009 at g.c.wake@massey.ac.nz.

This is the most prestigious award for Industrial and Applied Mathematics awarded by ANZIAM. Nominees should have given outstanding service to the profession of Applied Mathematics in Australia and/or New Zealand through their research achievements and through activities enhancing applied or industrial mathematics or both. The person nominated must be a long-term member and valuable contributor to ANZIAM and/or its predecessor, The Division of Applied Mathematics of the Australian Mathematical Society. A full curriculum vitae of the nominee should be sent along with a supporting statement.

The first award was made at the 1995 ANZIAM conference. Previous winners of the ANZIAM Medal are: 1995 Professor Renfrey B. Potts (University of Adelaide); 1997 Professor Ian H. Sloan (University of New South Wales); 1999 Professor Ernest O. Tuck (University of Adelaide); 2001 Associate Professor Charles E.M. Pearce (University of Adelaide); 2004 Professor Roger H.J. Grimshaw (University of Loughborough, previously Monash University); 2006 Professor Graeme C. Wake (Massey University, Auckland); 2008 Professor James M. Hill (University of Wollongong).

More information is on http://www.anziam.org.au/The+ANZIAM+medal.

ANZIAM Award for outstanding new researchers: J.H. Michell Medal

Nominations are called for the award of the J.H. Michell Medal for 2010, for ANZIAM outstanding new researchers. Nominees must be in their first 10 years of research on 1 January 2010 after the award of their PhD, and be members of ANZIAM for at least three years.

Nominations close on 30 September 2009. Further information can be obtained from http://www.anziam.org.au/Medals/michell.html.

The Chair of the Selection Panel for the 2010 award is Dr Mark Nelson, School of Mathematics and Applied Statistics, The University of Wollongong, Australia. E-mail: mnelson@uow.edu.au.

Nominations can be made by any member of ANZIAM other than the nominee. A nomination should consist of a brief CV of the nominee, together with the nominee's list of publications and no more than a one-page resume of the significance of the nominee's work. Nominations should be forwarded to the Chair of the Selection Panel, in confidence.

Please note that, where necessary, the Selection Panel will consult with appropriate assessors concerning evaluation of any nominee's research.

National Strategy Document

The link for the Strategy document written by Hyam Rubinstein in consultation with the Australian Council of Heads of Mathematical Sciences is:
http://www.amsi.org.au/pdfs/National_Maths_Strategy.pdf.
It has generated considerable interest especially from local media wanting to know about the local supply of teachers. Please do what you can to publicise the call for a national strategy and, if the media contact you, please give them all the help you can with local contacts.

Members of ACHMS have been sent at least one copy. If anyone has a good use for a paper copy, e.g. a good industry contact, please contact enquiries@amsi.org.au.

IMU: Call for nominations for the offices of President and Secretary of IMU (IMU AO Circular Letter 2/2009)

The IMU Nominating Committee is now complete. It will consist of the following persons: David Mumford (chair); László Lovász; Anthony Afuwape; Nigel Hitchin; C.S. Seshadri; Ian Sloan; Frances Yao.

The IMU Nominating Committee is chaired by David Mumford who will initiate further correspondence and receive the nominations submitted by the Adhering Organizations. The Nominating Committee will review the nominations received and make suggestions for IMU offices.

The main point of this letter is that, now that the Nominating Committee is complete, the Adhering Organizations and Committees for Mathematics are invited to submit names for the offices of the IMU President and the IMU Secretary for the next term (2011–2014). The Adhering Organizations' and Committees for Mathematics' submissions must include a candidate's declaration of willingness to serve if elected and a CV. Please send your nominations by 15 June 2009 via your official Adhering Organization's/Committee for Mathematics' contact address (e-mail, fax, post) to David Mumford, the Nominating Committee chair, whose contact information is:

E-mail: David_Mumford@brown.edu
Fax: +1 401-863-1355

Postal address:
Professor David Mumford,
Division of Applied Mathematics,
Brown University, Box F,
Providence RI, 02912, USA.

sigma prizes for outstanding contribution

sigma wishes to award two personal prizes to individuals who have made outstanding contributions in the field of mathematics and statistics. The personal prizes (of £1000 each) will be awarded to an individual employed by a UK higher-education institution and an individual employed by a higher-education institution based outside the UK.

Nominations are now being sought for the sigma Outstanding Contributors 2009. Full details can be found at http://www.sigma-cetl.ac.uk/index.php?section=100.

For the purposes of the sigma prizes, mathematics and statistics support refers to activities and/or resources which are provided to support and enhance students' learning of mathematics or statistics (in any one or more disciplines, at any level of higher education) and which are provided separately and in addition to traditional lectures, tutorials, examples classes, personal tutorial sessions etc.

Completed PhDs

Monash University

- Dr Ailie Gallant, *Trends in extremes of the Australian climate*, supervisor: Professor Michael Reeder.

University of Melbourne

- Dr Abbey Trewenack, *Reaction-diffusion models for dispersing and settling populations in biology*, supervisor: Kerry Landman.
- Dr Geoffrey Decrouez, *Generation of multifractal signals with underlying branching structure*, supervisors: Owen Jones and Kostya Borovkov.

University of Queensland

- Dr Asrul Sani, *Stochastic modelling and intervention of the spread of HIV/ AIDS*, supervisor: Dirk Kroese.

University of Sydney

- Dr Richard Finlay, *The variance gamma (VG) module with long range dependence*, supervisor: Professor Eugene Seneta.

Awards and other achievements

2009 Australian Fulbright Scholars

The 2009 Australian Fulbright Scholars have recently been announced and we are pleased to announce that Dr Peter Jarvis has been made Fulbright Tasmania Scholar (Senior Scholar).

For the full list of scholars, visit http://www.fulbright.com.au/scholars/current-australian-scholars.html and for details about Peter, see http://www.fulbright.com.au/scholars/australian-scholars/current/Jarvis.html.

Australian National University

Professor Xu-Jia Wang has been elected to the Academy of Science. The Academy states that Xu-Jia Wang is distinguished for his work on non-linear partial differential equations and applications, including geometry and optimal transportation.

Professor Wang is also a recipient of the Australian Mathematical Society Medal, awarded in 2002.

Monash University

- Professor John Lattanzio has been elected by the Astronomical Society of Australia to be a lifetime Honorary Fellow.
- Professor Christian Jakob has been awarded another three-year grant from the Department of Energy in the United States.

University of Ballarat

Prabhu Manyem has been invited to spend seven months as a Visiting Associate Professor in the Mathematics Department at National Cheng-Kung University (NCKU), one of Taiwan's top universities. He has been at NCKU since this January. He will return to Ballarat in July. Prabhu is collaborating with Professor Ruey-Lin Sheu of NCKU, on duality related problems in optimisation.

University of Sydney

Professor Norman Dancer has been awarded the Hannan Medal of the Australian Academy of Science. He has also been awarded a Leverhulme Visiting Professorship to the UK for parts of 2009 and 2010.

University of Western Australia

In 2008, Dr Bob Sullivan was voted 'Lecturer of the year' at UWA and 19th best across all universities in Australia by students in his first-year calculus course (enrolment 540) which is provided mainly for engineering and science students. (See http://lectureroftheyear.com.au/ and click on '2008 Results'.)

Appointments, departures and promotions

University of Melbourne

- Dr Anthony Fernando has been appointed Research Fellow as of 9 February 2009.

- Ms Qun Lin has been appointed Research Fellow as of 23 March 2009.
- Professor Philip Broadbridge, AMSI Director, left on 20 February 2009.

University of New South Wales

- Dr Denis Potapov has joined UNSW as a Reseach Associate/Associate Lecturer working with Fedor Sukochev. He was formerly at Flinders University.
- Dr Josef Dick has joined UNSW as a Lecturer. He was formerly a Research Associate at UNSW.
- Ms Atiya Zaidi has joined UNSW as an Associate Lecturer. She is working with Dr Chris Tisdell and is in charge of the School's new learning centre, known as the Student Support Scheme.
  http://www.maths.unsw.edu.au/students/current/help/sss.html

University of Western Sydney

- Dr Peter O'Brien and Dr Than Pe have left the university.

University of Wollongong

- Dr Anne Porter has been promoted to Associate Professor.

New Books

Swinburne University

Chakraborty, A. (2008). *A Numerical Study of Biological Problems in a Predator–Prey System.* VDM Verlag.

University of Adelaide

Greenfield, T. and Metcalfe, A. (2007). *Design and Analyse Your Experiment Using Minitab.* Wiley-Blackwell UK.

Morris, K. *Goodness-of-fit Tests for Continuous Distributions Based on Characterisations of the Uniform Distribution.* Monograph accessible from http://www.maths.adelaide.edu.au/kerwin.morris (accessed 1 April 2009).

University of Melbourne

Borovkov, A.A., and Borovkov, K.A. (2008). *Asymptotic Analysis of Random Walks: Heavy-Tailed Distributions.* Cambridge University Press.

Koliha, J.J. (2008). *Metrics, Norms and Integrals: An Introduction to Contemporary Analysis* World Scientific, Singapore.

University of New England

Professor Yihong Du (UNE), Professor Hitoshi Ishii (Waseda University, Japan) and Professor Wei-Yueh Lin (Providence University, Taiwan) (2009). *Recent Progress on Reaction-Diffusion Systems and Viscosity Solutions.* World Scientific

Publishing. This book contains a carefully selected collection of articles representing the recent progress of some important areas of nonlinear partial differential equations. The authors of the articles are international experts from Australia, Canada, China, France, Japan, Mexico and the USA.

University of New South Wales

Deng, B., Du, J., Parshall, B. and Wang, J. (2008). *Finite Dimensional Algebras and Quantum Groups*. Mathematical Surveys and Monographs, Vol. 150. The American Mathematical Society.

Conferences and Courses

Conferences and courses are listed in order of the first day.

Workshop on stochastics and special functions

Date: 22 May 2009
Venue: The University of Queensland
Web: http://www.maths.uq.edu.au/MASCOS/Orthogonal09.html
Contact/enquiries: Phil Pollett (pkp@maths.uq.edu.au)

This one-day workshop is devoted to the theory and applications of special functions with particular emphasis on how they arise in stochastic processes. There are two main speakers: Erik van Doorn, who will speak on birth–death processes and extreme zeros of orthogonal polynomials, and Peter Forrester, who will speak on the connection between the zeros of the Riemann zeta function and eigenvalues of random matrices. Additionally, there will be several shorter invited presentations.

Registration is free, but registration is essential. Morning and afternoon teas and lunch will be provided to all participants.

IMST 2009-FIM XVII and IMST 2009-FIM XVIII

During 2009, the Forum for Interdisciplinary Mathematics (FIM) will be sponsoring two International conferences (17th and 18th) on Interdisciplinary Mathematical and Statistical Techniques (IMST). Below are dates, web addresses and some information concerning these conferences. If you are interested in attending, presenting a paper or organising a session, please contact the organisers.

IMST 2009-FIM XVII
Date: 23–26 May 2009
Venue: University of West Bohemia, Plzen, Czech Republic
Web: http://home.zcu.cz/~pgirg/IMST2009/

IMST 2009-FIM XVIII
Date: 2–4 August 2009
Venue: Jaypee University of Information Technology, Waknaghat/ Shimla, India
Web: http://www.juit.ac.in/IMSTFIM2009/imst_fim_2009.htm

Organiser: Satya Mishra, Department of Mathematics and Statistics, University of South Alabama, Mobile, AL 36688, USA. Office Phone: 251 461 1642; Office FAX: 251-460-7969; E-mail: mishra@jaguar1.usouthal.edu

Second International Conference on Mathematical Modelling and Computation (MMC09) and Fifth East Asia SIAM Conference (EASIAM5)

Date: 8–11 June 2009
Venue: Universiti Brunei Darussalam, Brunei
Web: http://mmc09.ubd.edu.bn
Contact: mmc09_easiam5@hotmail.com

Deadline for abstracts and early registration: 15 May 2009.

NSDS09

Date: 22–27 June 2009
Venue: Sevilla, Spain
Web: http://congreso.us.es/nsds09

One-day mini-symposium

Date: 3 July 2009
Venue: University of Melbourne
Web: http://www.ms.unimelb.edu.au/~degier/DiscrCI_SLE_Int09.php

A one-day mini-symposium on discrete conformal invariance, SLE and integrable lattice models will be held at the University of Melbourne on Friday 3 July 2009.

1st PRIMA Congress

Date: 6–10 July 2009
Venue: University of New South Wales, Sydney
Web: http://www.primath.org/prima2009
Contact: Alejandro Adem (adem@pims.math.ca)
Local Arrangements Committee (prima2009@maths.unsw.edu.au)

AMSI-ANU Workshop on Spectral Theory and Harmonic Analysis

Date: 13–17 July 2009
Venue: The Australian National University, Canberra
Web: http://wwwmaths.anu.edu.au/events/SpectralTheory09/

The workshop will bring together leading international researchers together with top Australian mathematicians in two rapidly developing, and increasingly intertwined, fields of analysis: spectral theory and harmonic analysis. The workshop will include both research seminars and expository lectures, the latter designed for graduate students and presented by renowned expositors Jan van Neerven and Hart Smith.

Future models for energy and water management under a regulated environment

Date: 20–22 July 2009 Venue: Queensland University of Technology, Brisbane
Web: http://www.amsi.org.au/energy.php

AMSI invites all professionals with an interest in modelling water and energy management to join colleagues in this three-day event co-sponsored by AMSI, MASCOS, UNESCO and MITACS.

The event presents a unique opportunity to:

- attend short courses from Shahbaz Khan (UNESCO, Paris), Graham Weir (IRL NZ) and Elliot Tonkes (Energy Edge P/L);
- hear keynote addresses by eminent local and international practitioners including Ian Rose (ROAM Consulting), David Swift (ESPIC South Australia), Mukand S. Babel (Asian Institute of Technology, Thailand) and Michael O'Sullivan (University of Auckland);
- participate in workshops, offering industry professionals a rare opportunity to share their experience, expertise and discuss problems;
- network with specialists in your field.

If you or your staff would like to deliver a 20-minute talk on any of the topic mentioned on the above webpage please submit an abstract (max 300 words) before 15 June 2009 to ews@amsi.org.au.

We look forward to your participation.

Mini Winter School on Geometry and Physics

Date: 20–22 July 2009
Venue: Institute for Geometry and its Applications, University of Adelaide
Web: http://www.iga.adelaide.edu.au/workshops/winterschool2009.html

Three days of expository lectures aimed at third-year and honours students interested in postgraduate studies in pure mathematics or mathematical physics.

Financial support for travel and accommodation is available. The closing date for applications is 10 June 2009, but we would like to hear from interstate participants as soon as possible so we can book accommodation in time.

Groups St Andrews 2009 in Bath

Date: 1–15 August 2009
Venue: University of Bath, Bath, UK
Web: http://www.groupsstandrews.org/2009/

Main speakers: Gerhard Hiss (RWTH, Aachen, Germany); Volodymyr Nekrashevych (Texas A&M, USA); Eamonn O'Brien (Auckland, New Zealand); Mark Sapir (Vanderbilt, Nashville, USA); Dan Segal (All Souls College, Oxford).

Program: The speakers above have kindly agreed to give short courses of lectures in the first week of the conference. In the second week of the conference there will be three theme days: an ‘Engel groups day’, a day to celebrate the birthdays of John Cannon and Derek Holt, and a ’B.H. Neumann day’. In addition there will be a program of one-hour invited lectures and contributed talks.

The organisers are pleased to announce that full registration is now open. Please visit the conference website for further details and registration instructions.

Please book early! We look forward to seeing you in Bath in August.

The organising committee: Colin Campbell (St Andrews), Edmund Robertson (St Andrews), Colva Roney-Dougal (St Andrews), Martyn Quick (St Andrews), Geoff Smith (Bath), Gunnar Traustason (Bath).

Dresden 2009 Conference

Date: 11–17 September 2009
Venue: Dresden, Saxony, Germany
Web:
www.informatik.htw-dresden.de/~paditz/SecondAnnouncementDresden2009.doc

For all further information, please email: alan@rogerson.pol.pl.

Third Japanese/Australian workshop on real and complex singularities

Date: 15–18 September 2009
Venue: The University of Sydney (Medical Foundation Auditorium)
Web: http://www.maths.usyd.edu.au:8000/u/laurent/RCSW

AustMS 2009 Early Career Researchers Workshop

Date: 27 September 2009
Venue: Mt Lofty House, Adelaide Hills
Web: http://www.unisa.edu.au/austms2009/earlycareer/
Organisers: Bronwyn Hajek (Bronwyn.Hajek@unisa.edu.au), Anthony Henderson (anthonyh@maths.usyd.edu.au)

53rd AustMS Annual Conference, 2009

Date: 28 September – 1 October 2009
Venue: University of South Australia, City West Campus, Adelaide
Web: http://www.unisa.edu.au/austms2009/
E-mail: Austms09@unisa.edu.au

Registration is now open via the conference website.

The Program Committee consists of Konstantin Borovkov (Melbourne), Regina Burachik (UniSA), Vladimir Ejov (UniSA, Conference Director), Catherine Greenhill (UNSW), Alex Molev (Sydney), Amnon Neeman (ANU), Garry Newsam (DSTO), Jacqui Ramagge (Wollongong), Bevan Thompson (Queensland), Mathai Varghese (Adelaide), and Lesley Ward (UniSA).

The Core of the Local Organising Committee consists of Regina Burachik (UniSA), Julea Crea (Event Coordinator), Vladimir Ejov (UniSA, Conference Director), C. Yalcin Kaya (UniSA), and Lesley Ward (UniSA).

9th Engineering Mathematics and Applications Conference (EMAC2009)

Date: 6–9 December 2009
Venue: University of Adelaide, South Australia
Web: http://www.maths.adelaide.edu.au/emac2009/
Contact: Andrew Metcalfe (andrew.metcalfe@adelaide.edu.au

EMAC2009 is organised by the Engineering Mathematics Group, a special interest group with the Australian and New Zealand Industrial and Applied Mathematics Division of the Australian Mathematical Society. The meeting is held biennially.

Conference Chair: Charles Pearce (University of Adelaide). Co-Chairs: Fred Bowden (DSTO), Alan Branford (Flinders University), Julia Piantadosi (University of South Australia).

Key dates: Abstracts may be submitted until 30 October 2009. Talks will be accepted on the basis of the abstract. Full written papers are due by 30 November, and will be available at the conference. Papers will be reviewed as they are received, and provided they meet, or are revised to meet, the standard, will be published in a special issue of the ANZIAM electronic journal.

AMSI workshop: new directions in geometric group theory

Date: 14–18 December 2009
Venue: The University of Queensland, Brisbane
Web: http://sites.google.com/site/ggtbrisbane/

Organisers: Murray Elder (University of Queensland, m.elder@uq.edu.au); Lawrence Reeves (University of Melbourne, L.Reeves@ms.unimelb.edu.au); Stephan Tillmann (University of Melbourne/University of Queensland, S.Tillmann@ms.unimelb.edu.au).

NZIMA/NZMRI Summer Workshop

Date: 3–10 January 2010
Venue: Hanmer Springs, New Zealand
Web: http://www.math.auckland.ac.nz/wiki/2010_NZMRI_Summer_Workshop

The theme of the 2010 NZIMA/NZMRI Summer Workshop is ‘Groups, Representations and Number Theory’. It is organised by Ben Martin and Eamonn O’Brien.

The principal speakers are: Martin Bridson (Oxford); Michel Broue (Universite Paris VII); Persi Diaconis (Stanford); Roger Howe (Yale); Gus Lehrer (Sydney); Marcus du Sautoy (Oxford).

The meeting will take place in Hanmer Springs, about 90 minutes north of Christchurch. Arrival is 3 January; departure is in the afternoon of 10 January.

Graduate students based in Australia who are student members of the Australian Mathematical Society are eligible to receive a subsidy of up to AUD$300 each, which will cover your local expenses. This support is generously provided by the Australian Mathematical Society.

For further details on the location and format of the meeting, please visit the web page.

This workshop is funded by the New Zealand Institute of Mathematics and its Applications (http://www.nzima.auckland.ac.nz).

AMSI ICE–EM 2010 Summer School

Date: 11 January – 5 February 2010 Venue: La Trobe University Contact: Grant Cairns, Director G.Cairns@latrobe.edu.au

Please let prospective honours and postgraduate students know the dates. Details of the courses and lectures will be made available later in the year.

Call for Contributions:
Educational Interfaces between Mathematics and Industry

Date: 19–23 April 2010
Venue: Lisbon, Portugal
Web: http://www.cim.pt/eimi/
Contact: Gail FitzSimons (gail.fitzsimons@education.monash.edu.au)

The joint study, organised by the International Commission on Mathematical Instruction (ICMI) and the International Council for Industrial and Applied Mathematics (ICIAM), on Educational Interfaces between Mathematics and Industry is designed to enable researchers and practitioners around the world to share research, theoretical work, projects descriptions, experiences and analyses. It will consist of two components: the Study Conference and the Study Volume. Submissions are due by 15 September 2009.

Visiting mathematicians

Visitors are listed in alphabetical order and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

Mr Tarje Bargheer Arklint; University of Copenhagen, Denmark; 16 February to 13 July 2009; –; UMB; Dr Craig Westerland
Dr Peter Brooksbank; –; 12 June to 10 July 2009; MAGMA; USN; J.J. Cannon
Dr Nils Brun; –; 3 June to 12 July 2009; MAGMA; USN; J.J. Cannon
Jose Burillo; Universitat Politècnica de Catalunya, Barcelona, Spain; 20 July 2009 to 20 August 2009; geometric group theory, Richard Thompson's groups, amenability; UQL; Dr Murray Elder

Ms Corrie Jacobien Carstens; University of Amsterdam, The Netherlands; 23 February to 15 August 2009; –; UMB; Dr Craig Westerland

Prof Suhyoung Choi; KAIST, Dept of Mathematical Sciences; 28 February to 31 August 2009; –; UMB; Craig Hodgson

Dr Florica Cirstea; University of Sydney; 14 July 2008 to 14 July 2011; applied and nonlinear analysis; ANU; Neil Trudinger

Dr Robert Clark; University of Wollongong; 1 July 2008 to 1 July 2011; statistical science; ANU; Alan Welsh

Renato da Costa; Pontifical Catholic University, Rio de Janeiro, Brazil; until July 2009; –; CUT; –

Dr Alexei Davydov; –; 5 January to 30 June 2009; –; USN; A.I. Molev

Prof Zengji Du; School of Mathematical Sciences, Xuzhou Normal University; 22 October 2008 to 22 October 2009; differential equations; UNSW; Chris Tisdell

Zhao Guohui; Dalian University of Technology, China; until August 2009; –; CUT

Dr Rainer Hollerbach; Applied Mathematics, Leeds; 9 February to 28 August 2009; nonlinear simulation of magneto-shear tachocline instabilities in the sun; MNU; Prof Paul Cally

Prof Tien-Chung Hu; National Tsing Hua University; 24 February to 31 July 2009; Convergent results for sums of dependent random variables, with possible applications to V-Statistics; USN; N.C. Weber

Wei Jin (student); Central South University, China; 20 September 2008 to September 2010; –; UWA; Cheryl Praeger

Dr Gwenael Joret; Universite Libre de Bruxelles; 30 March to 24 July 2009; –; UMB; David Wood

Dr Philip Kokic; ABARE; 8 July 2008 to 7 July 2009; statistical science; ANU; Alan Welsh

Ko Wang Kyung; Anyang University, Anyang Korea; until July 2009; –; CUT; –

Mr Gye-Seon Lee; KAIST, Korea; 26 February to 1 September 2009; –; UMB; A/Prof Craig Hodgson

Prof C.C. Lindner; Auburn University, USA; 11–25 July 2009; combinatorics; UQL; Elizabeth Billington

Kek Sie Long; University Tun Hussein Onn, Malaysia; May 2009 to November 2009; –; CUT; –

Cheng Longsheng; Nanjing University of Science and Technology, Nanjing, China; until June 2009; –; CUT; –

Mr Li Luo; Chinese Academy of Sciences; 1 January to 30 June 2009; algebra; USN; R. Zhang

Dr James McCoy; University of Wollongong; 1 January to 30 June 2009; applied and nonlinear analysis; ANU; Ben Andrews

A/Prof Marc Mehlman; University of New Haven; 15 April 2009 to 20 June 2009; statistics; USN; M.S. Peiris

Prof James Meiss; University of Colorado at Boulder; 22 June to 8 August 2009; applied maths; USN; H. Dullin

Hu Ming; Jiangsu University of Science and Technology, Zhenjiang, China; May 2009 to October 2009; –; CUT; –

Prof Christine Mueller; University of Kassel, Germany; 15 October to 24 December 2009; –; UMB; Prof Richard Huggins

Dr Mary Myerscough; University of Sydney; 10–12 June 2009; mathematical biology; UQL; Prof Phil Pollett

Prof Helen Perk; Oklahoma State University; 1 August 2008 to 31 May 2009; mathematical physics; ANU; Murray Batchelor

Prof Jacques Perk; Oklahoma State University; 1 August 2008 to 31 May 2009; mathematical physics; ANU; Murray Batchelor

Prof Steve Rosenberg; Boston University; 1 March to 1 July 2009; analysis and geometry; ANU; Alan Carey

Mr Ege Rubak (student); Aalborg Uni, Denmark; April to November 2009; –; UWA; Adrian Baddeley

Dr Lionel Siess; Inst. Astronomy & Astrophysics, Brussels; 1 February to 30 June 2009; evolution and nucleosynthesis of super AGB stars; MNU; Prof John Lattanzio

Dr Alan Stapleton; University of Michigan; 22 June to 21 August 2009; invariant theory, cellularity and geometry; USN; G.I. Lehrer

Dr Damien Stehle; Ecole Normale Superieure, Lyon; 19 July 2008 to 18 July 2009; computational aspects of lattices; USN; J.J. Cannon

Dr Sixi Su; Beijing University of Posts and Telcomms, China; 15 January to 14 July 2009; –; UMB; Dr Sanming Zhou

Bijan Taeri; Isfashan Uni Technology, Iran; November 2008 to September 2009; –; UWA; Cheryl Praeger

Dr Kilkothur Tamizhmani; Pondicherry University; 9 October to 7 November 2009; applied maths; USN; N. Joshi

Moorum Theeraech; Mahidol University, Thailand; until October 2009; –; CUT; –

Dr Guillaume Therien; University of British Columbia, Canada; 5 January to 30 June 2009; –; UMB; Dr Andrew Robinson

Dr Helena A. van der Waal; –; 3 June to 30 July 2009; MAGMA; USN; J.J. Cannon

Prof Erik van Doorn; University of Twente; 19–25 May 2009; stochastic processes; UQL; Prof Phil Pollett

Valentina Vulcanov; Freie Universität Berlin; until mid-July 2009; –; UWG; –

Dr John Voight; University of Vermont; 12 to 31 May 2009; MAGMA; USN; J.J. Cannon

Dr Jared Weinstein; University of California; 20 April to 15 May 2009; MAGMA; USN; J.J. Cannon

Prof Robert Wilson; –; 27 April to 5 June 2009; MAGMA; USN; J.J. Cannon

Dr Dan Yasaki; University of North Carolina; 22 April to 19 May 2009; MAGMA; USN; J.J. Cannon

Ms Martha Yip; University of Wisconsin-Madison, USA; 9 March to 20 August 2009; –; UMB; Prof Arun Ram

Dr Zhitao Zhang; Chinese Academy of Science, Beijing; 15 September to 15 November 2009; nonlinear analysis; USN; E.N. Dancer

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