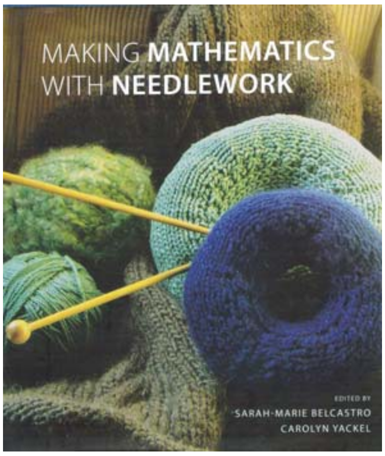
Book reviews

Making Mathematics with Needlework

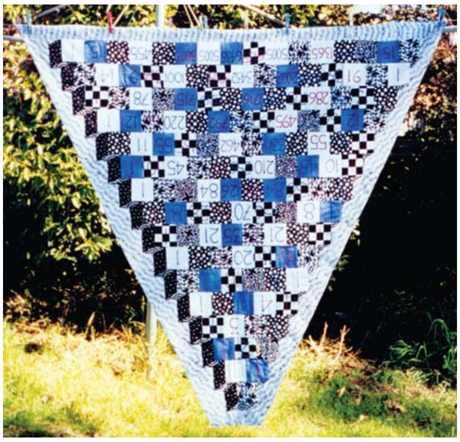
Sarah-Marie Belcastro and Carolyn Yackel (eds)
A.K. Peters Ltd, 2008, ISBN: 978-1-56881-331-8

It is 150 years since August Ferdinand Möbius discovered the now famous Möbius strip or band. Almost all readers of this review will have made a Möbius strip out of paper at some stage. You could make a quilt in the same way. A quilted Möbius strip would be very warm around the shoulders especially with the twist in the front. Your scarf would be a work of art and the talk of the party. A colleague pointed out that, in addition to being useful and artistic, the quilted Möbius band would save time — you would have to iron only one side.



Chapter 1 of this very interesting book deals with the quilted Möbius band. The chapter opens with a description of how to make a Möbius strip out of paper and presents a few experiments that you can do with the surface. Then the mathematical aspects of the Möbius strip are explained using concepts from algebraic topology. A few graph colouring problems set in the context of the Möbius strip are outlined. This leads to a description of imaginative ideas for using in the classroom at all levels. The chapter concludes with instructions on how to make a quilted Möbius band. The bibliography for this chapter includes books from mathematics, mathematics education, quilting and history of fashion. The chapter is illustrated with delightful colour photos and figures. The other chapters, on different topics, have a similar structure.

From a craft perspective, this is a charming book. It includes precise and detailed instructions for projects both practical (knitted beanies and socks) and whimsical (quilted Möbius strips, a knitted torus and a class exercise on the construction of a Fortunatus' purse). The mathematics applied to knitting is particularly useful in the construction of knitted garments, providing simple formulae for ensuring that stitches are picked up evenly. The analysis of all possible variations in basic stocking stitch is fascinating, and the innovative ideas for the construction of knitted garments encourages further experiments in producing garments without seams. Like the authors, we find sewing sleeves into jumpers tedious. The chapter on knots, cables and braids is of great interest to those who indulge themselves in knitting Aran jumpers and the Algebraic Socks are fun. Although the mathematics involved in the construction of the Wearable Pants is complex for a non-mathematical person, the pants themselves are great.



Quilted Pascal's triangle by Sabine Wilkens

Using the Sierpinski triangle and carpet to generate designs for crocheted shawls is interesting, but the resulting design does not appear very special to someone who is unaware of its significance. As an exercise in learning how to do filet crochet, however, it is easy and logical, and the resulting shawl is an excellent beginner's project. A more advanced project could be generated from the Koch snowflake, but it would need more advanced crochet skills.

The chapters on symmetry in cross-stitch patterns and the construction and working of patterns in Spanish Blackwork are very well written, and encourage one to try them out. If only we had the time!

We are grateful to our colleague Dr Sabine Wilkens for allowing us to publish the photo of her quilted Pascal's triangle to accompany this review.

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Geometric Puzzle Design

Stewart Coffin
A.K. Peters, 2007, ISBN: 978-0-19-857043-1

Mathematicians are often captivated by puzzles. This includes the puzzles that Stewart Coffin writes about, two- and three-dimensional puzzles that are often assembled from pieces made of wood. There are many books on solving puzzles, but Coffin's book is different. It is one of the few books on the design and making of puzzles.

For most of the puzzles in this book the problem is to assemble the puzzle from a collection of pieces, and for some of the three-dimensional puzzles disassembly is part of the problem. The popular seven-piece tangram, whose pieces can be assembled into many shapes including a square, is an example of a two-dimensional puzzle. The beautiful symmetric wooden puzzles that appear to be made of rods or sticks, and which come apart if only one finds the right piece to move, are examples of three-dimensional puzzles.

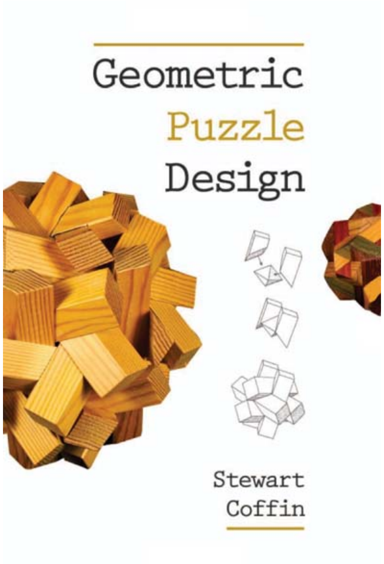
Stewart Coffin is very well known for his many years of puzzle creation. Jerry Slocum and Jack Botermans in their book *Puzzles New and Old: How to Make and Solve Them* (p. 84) say of Coffin:

> Quite simply, Stewart Coffin is the most outstanding designer and maker of interlocking puzzles that the world has even seen.

Coffin successfully gives the reader an insight into how a puzzle designer thinks; one can begin to see how he has come up with the ideas for his puzzles.

The book is not a mathematics book, though Coffin does pose a few mathematical questions for the reader. It is likely to appeal to those who want to know how puzzles are designed, how they are made or how one might think up some of these lovely geometric shapes. It should also interest those who want to know more so as to be able to solve geometric puzzles as it gives some insight into how they work. At the very least, it will inform the reader about the variety of puzzles that have been created.

Not all these puzzles are difficult to solve. The attraction of some is their beauty and symmetry, and their seeming simplicity.



Although the bulk of the book is about three-dimensional puzzles, it begins with puzzles in two dimensions. There are dissections, puzzles where the pieces are all made from a single shape and where the aim is to assemble these pieces into one or a variety of shapes. For example, pentominoes in which all pieces are made from five squares. There are puzzles which mislead, by unusual angles or a space where not expected. These are all described from the point of view of a designer, so one can see how they might be altered to make other puzzles.

We see block puzzles in which one just has to find a way to stack or put together the various pieces. Other puzzles have interlocking pieces, many constructed from notched sticks, and in some the order of assembly is important. There are sets of pieces which can be put together in various ways to form different symmetrical shapes, there are puzzles where the pieces must all move into place together rather than one at a time and there are puzzles with unlikely construction and disassembly, designed to mislead the solver.

A Catalan solid is the dual of an Archimedean solid. A Catalan solid is convex, its group of symmetries is face transitive but not vertex transitive, and the faces are not regular polygons. Two of the thirteen Catalan solids give rise to some aesthetically pleasing puzzles. The first to make an appearance in Coffin's book is the rhombic dodecahedron, which has 12 rhombic faces. Several chapters are based partly or wholly on this solid. There is a symmetrical arrangement of 12 sticks which totally encloses a rhombic dodecahedron, but this arrangement is not stable. Coffin discusses a few ways in which this unstable construction can be used to create something that can be assembled and which will stay together. The 12 sticks that surround the rhombic dodecahedron can have pieces added to them so that they become interlocking. The cross section can be altered to form sticks that can be notched, creating a puzzle that does not fall apart. The sticks can be split, so that there are 24 pieces. These 24 pieces can then be joined to different pieces to again form a puzzle that is stable. The ends of the sticks can be changed to form various stellations. The symmetries of these puzzles allow for the use of different coloured woods to good effect. A well-made wooden rhombic dodecahedral puzzle is very beautiful.

The second Catalan solid mentioned by Coffin is the rhombic triacontahedron. It has 30 rhombic faces, 60 edges and 32 vertices. One might think that the many puzzles based on the rhombic dodecahedron would transfer to the rhombic triacontahedron but this is not so. However, it is the basis for some puzzles, and Coffin shows some of what can be done.

The design and making of geometric puzzles is a puzzle in itself. As Coffin writes '... there is often no clear dividing line as to where the design process stops and the solution begins ...'.

For some of us the hardest part would be the woodwork. There are woodworking tips throughout the book and the final chapter gives advice on woodworking and choice of tools.

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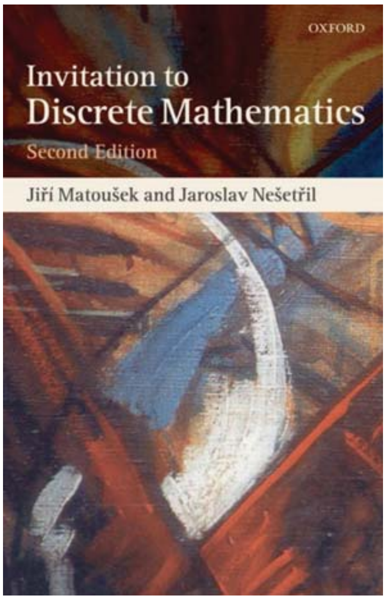
Invitation to Discrete Mathematics (second edition)

Jiří Matoušek and Jaroslav Nešetřil
Oxford University Press, 2008, ISBN 978-0-19-857043-1

Matoušek and Nešetřil give us a really enjoyable and mathematically appealing introduction to the major themes and problems in discrete maths, specifically combinatorics and graph theory, in this book. They delve into some interesting topics, with considerable rigour and detail for the capable reader. However, I fear it may be at too high a level for a first year Discrete Mathematics course in Australia, requiring too much mathematical maturity and interest for the average undergraduate in such a course.

Later topics include a chapter on probabilistic proofs and Ramsey's theory (both motivated and explained beautifully in my opinion), a chapter on generating functions (not quite as nice as Wilf's *generatingfunctionolgy* [2]), and the final chapter entitled 'Applications of linear algebra' which includes block designs (of interest to my colleagues at The University of Queensland I'm sure) and some more probabilistic algorithms (fast probabilistic checking of matrix multiplication, and of associativity of binary operations on a set).



The exercises throughout the book are all interesting and will be a challenge for most students. The text avoids lots of repetitive 'apply the definition to the following'-type exercises and gets straight to the good stuff. There are hints in the back, which still leave lots of work to do.

In several places the authors present a number of alternative proofs for the same results, which I think is really nice to show students (and remind the rest of us) that there can be many approaches to mathematics, each having its strengths and detractions. They prove Cayley's formula for the number of trees on n vertices in at least four different ways, for example. It is not always clear which one comes from 'The Book' [1]!

I don't expect this text to replace any of the usual bulky and colourful Discrete Maths texts currently in use for first year and CS-leaning courses in Australia, but I think as a resource for novel and accessible approaches to sometimes difficult topics and interesting extensions, it is well worth taking a look. For later level students who've taken a standard discrete math course, I think Chapters 7–13, the second half of the book, would make a great basis for an interesting course.

References

[1] Wikipedia (2009). Paul Erdös. http://en.wikipedia.org/wiki/Paul_Erdos (accessed 31 March 2009).

[2] Wilf, H.S. (2006). *generatingfunctionology*, 3rd edn. A.K. Peters Ltd, Wellesley, MA.

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