Book reviews

Figuring Sport

Graeme Cohen and Neville de Mestre
UTS, 2007, ISBN 978-0-7331-0023-9

There can be no better way to begin this review than by quoting a paragraph from the Foreword in this book given by legendary athlete Ron Clarke MBE:

> *Figuring Sport* by Graeme Cohen and Neville de Mestre brings many of the former mysteries into light of contemporary knowledge through both text and equations, ensuring the learning process suits both verbal and analytical learners. And most of all, it's a very interesting read.

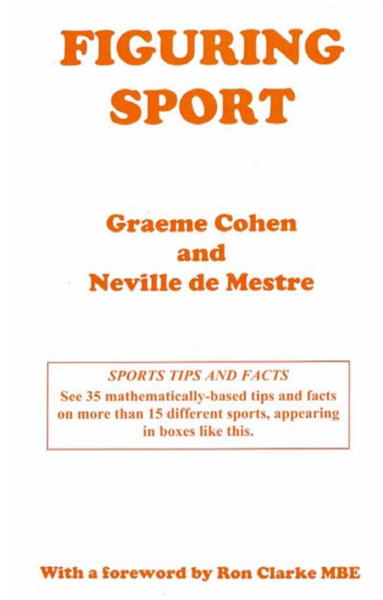
I found *Figuring Sport* to be a friendly monograph which serves as a window to mathematics in sports and clearly demonstrates that it is possible for a monograph to be both popularly written and easily understandable with minimal prior knowledge of the subject.

This brief monograph is intended for suitably well-prepared high-school mathematics students, and others with an elementary background in differential calculus and probability theory. There are 170 pages of basic text of seven chapters including exercises, projects and references at the end of each chapter. There is a decent smattering of exercises throughout which are at an appropriate level. There are no solutions provided, which can sometimes be a drag for students wanting some positive reinforcement.

Chapter 1 deals with tennis and other racquet sports like squash and badminton. The primary objective here is to identify the winning chances in these sports using elementary probability theory.

Chapter 2 analyses the various facts and figures of weightlifting, golf, speed records and darts. Initially, a simple linear model is proposed to compare the performance of winners in several classes of an event. In the case of weightlifting, the authors conclude that the linear model is not suitable as it is not based on physical theory. The authors then propose two other models, namely a power law model and the Siff model, and then compare these three models to derive some important conclusions. Also discussed are the mathematical aspects of golf, speed records and darts. Most of the material in this chapter has been published earlier by the authors in *Sports Engineering* in 2002.

The mathematics of cricket is discussed in Chapter 3. Using projectile theory in conjunction with the effect of drag, the authors show how best to hit a ball in the air beyond the boundary and score a six. The chapter also describes how individual bowlers' strike rates may be used to rate a team's bowling attack. Also included is the story of the controversial run-out of England's famous cricketer Colin Cowdrey and the mathematical analysis of this incident by Marice Brearley (at that time a senior staff member of the Department of Mathematics at the University of Adelaide) using actual photographs. I found this chapter quite interesting to read. However, one important aspect of bowling is missing in this chapter, that is, the type of spin bowling called 'Doosra' of the great Sri Lankan bowler Muttiah Muralitharan. This has been the subject of significant biomechanical analysis of spin bowling technique.



Chapter 4 deals with field events and in particular, the shot-put, long jump and high jump, and involves aspects of the theory of projectile motion. Knowledge of a first course on differential calculus and basic trigonometry is required to understand the presentation in this chapter. The sensational long jump and high jump records in the Mexico City Olympic games in 1968 by Bob Beamon and Richard Fosbury of the USA are also explained with sufficient reasonings although some questions still remain unanswered!

Chapter 5 discusses some aspects of snooker, billiards and pool ball whereas Chapter 6 is more focused on the organisation of sport than playing the sport. Tournament design and the right selection of the position for a player are also discussed using elementary graph theory and operations research at first-year undergraduate level.

The last chapter deals with four distinct sports, namely bowling, cycling, rowing and football. From a biomechanical point of view, it is indeed an interesting and useful chapter.

Overall I found this book a neat and minimalist treatment, strongly based on elementary knowledge of algebra, calculus, geometry and probability theory. The style is friendly. As an applied mathematician myself with interest in biomechanics and biomedical engineering I did not expect to learn a great deal from such popularisation, but was pleasantly surprised to have my expectations dashed. There are many gems to be found in this book that I have not seen elsewhere, and we owe a great deal to the formidable erudition of the two authors. It was a great pleasure to read this book by two of Australia's distinguished mathematicians.

I strongly recommend the inclusion of this book in the personal library of those who love sports and want to know more about the science behind sports.

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A History of Abstract Algebra

Israel Kleiner
Birkhäuser Verlag, 2007, ISBN-13: 978-0-8176-4684-4

This is a wonderful little book! If only it had existed when I taught ring theory and the history of algebra to my honours classes, or linear algebra, group theory and Galois theory to undergraduates, or better still, when I first studied abstract modern algebra myself.

From my first introduction to modern algebra I took to it like a duck to water. But some six or so years later the dream started to sour somewhat for me and I turned towards the history of mathematics and specifically the history of algebra. My main motivation for this was that, with the exception of Galois theory, abstract algebra seemed to exist in a vacuum: it provided a range of definitions, structure theorems and techniques, but to what end? I felt as if I had a range of answers, but to what questions? From what perspective could I say that some theorem was important while another was trivial? I still remember being at a group theory conference in the 1960s where someone was proving a range of isomorphism and other theorems for some sort of left (I think) generalisation of a semigroup The results struck me as pointless and I was delighted to hear Graeme Higman, from the audience, ask sarcastically, 'Do I suppose that the next thing to do is to prove the same results for the right?' I found that Higman, at least, had a perspective from which to assess these results so there must be, somewhere, a range of questions to which all this algebra provided some sort of answers. So I started reading history of mathematics, an interest which became easier to pursue when the *Archive for the History of Exact Sciences* and, later, *Historia Mathematica* started to be published.

Kleiners's *History*, while not quite comprehensive, covers the main trends up to about 1930, a little earlier in the case of group theory. This takes it beyond the span of historical compendia such as those by Kline [1], Grattan-Guiness [2] or Katz [3] and even beyond Novy [4]. At various times, while reading Kleiner, I would think, 'Ah, but he hasn't thought of (whatever) angle', and then, a few pages further on, that very angle would often surface. At 170 pages the book is quite concise: no wading through large amounts of text to find what you want.

The book is geared to the practicing mathematician and teacher of mathematics rather than to the specialist historian. Its references are to secondary, rather than to primary sources. But a specialist historian can easily quarry the secondary sources provided to get to a deeper level.

A History of Abstract Algebra shows its origins as a series of separately published articles. While this leads to a certain amount of repetition (not much, given the brevity of the book) it facilitates the reading of single chapters. But it results in the links between modern and classical algebra not being drawn as clearly as they might have been. Its style, while concise, is not particularly engaging. At times its terseness leads to minor inaccuracies, for example in relation to Lagrange's work on resolvents (p. 19).

One can quibble with a few other bits, such as '... to the Pythagoreans the side of a square of area 2 was nonexistent ... ' (p. 7), referring to the unknown in equations as 'variables' (p. 8) or to Viète's insistence on homogeneity as a 'shortcoming' (p. 9), the absence of any discussion of the relevance of coordinate geometry in relation to Descartes' algebra, and the intuitive guess that every polynomial of odd degree has a root.

Let us now look at the contents a bit more closely. The book starts with a very abbreviated history of classical algebra from Babylon to Braunschweig (Gauss's birthplace), followed by one chapter on each of group theory, ring theory, field theory and linear algebra, then one on the pivotal role of Emmy Noether, a discursive chapter and a chapter on six major contributors to abstract algebra, again including Noether. The discursive intermezzo chapter outlines a way in which a course in abstract algebra can be founded on a series of five historically based problems.

Before Kleiner's book there already existed a very comprehensive book on the history of group theory, namely Wussing's [5] text, but ring theory has not so far been very well served, and Kleiner's contribution is particularly welcome.

Not surprisingly, Kleiner notes early on in his chapter on ring theory that 'Noncommutative ring theory began with attempts to extend the complex numbers to various number systems' (p. 42). He comments that Hamilton's invention of the quaternions was 'conceptually groundbreaking' (p. 42) as, indeed it was, since it involved abandoning the implicit assumption that, in any such extension, multiplication would need to be commutative. However, Kleiner fails to stress the simultaneous approach of Weierstrass [6] who, while insisting on commutativity, was willing to include zero divisors (although not nilpotent elements) into his 'generalised arithmetic'. He showed that this would necessarily lead to what he considered a trivial structure, namely a finite direct sum of copies of $\mathbf{R}$ and $\mathbf{C}$. It was this very formulation by Weierstrass that set up the framework for future structure theory, notably that of Wedderburn.

Kleiner takes us through the conceptual and terminological contributions of Lie, Dedekind, Cartan, Frobenius, Artin and others to noncommutative ring theory, and then addresses the roots and development of commutative ring theory up to the contributions of Noether and Artin.

Chapter 4 deals with field theory, nicely tracing it from Galois theory, algebraic number theory, algebraic geometry and Galois' polynomial congruences modulo p.

Chapter 5, on linear algebra, locates the modern study of systems of linear equations as starting with Leibniz' determinants and their subsequent systematic study by Cauchy and others. Most of Kleiner's treatment in this chapter is fairly standard.

As mentioned above, this section of the book winds up with a chapter on Emmy Noether and the advent of abstract algebra. It is hard for us, brought up on the abstract approach first popularised by van der Waerden's two volume *Modern Algebra* [7] (largely based on Noether's work), to really appreciate the seismic shift that occurred, as a result of Noether's contributions, in mathematicians' understanding of what algebra should be about.

All this so far in 102 pages! The remainder of the book is quite different. First there is a chapter outlining a course for high-school teachers of mathematics, taught by Kleiner, which presents the major ideas of abstract algebra to the students via five problems and attempts to solve or otherwise address them over the past half millenium. Kleiner says that the students did not follow this course with any other on abstract algebra, but I imagine they had already encountered some abstract algebra previously in their undergraduate years. The problems include 'What are the integer solutions of $x^2+2=y^3$?' (problem II) and 'Can we solve $x^5-6x+3=0$ by radicals?' (problem IV), as well as problems on the square root of -1, angle trisection and multiplying triples. Kleiner's motivating questions are well chosen.

When teaching ring theory, Galois theory and the like I have long used a genetic approach; that is, one in which I kept the human dimension, particularly the reformulation of questions, in focus. Consequently, I was fascinated by this chapter of Kleiner. What rich veins he has tapped into across the whole gamut of abstract algebra. Pursuing such threads would certainly save abstract algebra courses from being mere sequences of axiomatic systems with seemingly arbitrary axioms. On the other hand I think that the huge range of ideas engendered by Kleiner's problems would be too bewildering for undergraduate students in a first course.

After this intermezzo chapter come short biographies of six of the main players in the development of abstract algebra, namely Cayley, Dedekind, Galois, Gauss, Hamilton and again Noether, in this lexicographical order. These sections are nicely written but cover well-trodden ground. However, if you have any interest in abstract algebra and its beginnings I strongly recommend reading at least the first 100-odd pages of this book.

References

[1] Kline, M. (1972). *Mathematical Thought from Ancient to Modern Times*. Oxford UP, New York.
[2] Grattan-Guiness, I. (1998). *The Norton History of the Mathematical Sciences*. Norton, New York.
[3] Katz, V. (1998). *A History of Mathematics*, 2nd edn. Addison Wesley, New York.
[4] Novy, L. (1973). *Origins of Modern Algebra*. Noordhoff, Leyden.
[5] Wussing, H. (1969). *Die Genesis des abstrakten Gruppenbegriffes*. VEB, Berlin.
[6] Weierstrass, K. (1884). Zur Theorie der aus n Haupteinheiten gebildeten complexen Grössen. *Nachr. Kön. Ges. Wiss. Göttingen* 1884, 395–410.
[7] van der Waerden, B.L. (1931). *Moderne Algebra*, 2 volumes. Springer, Berlin.

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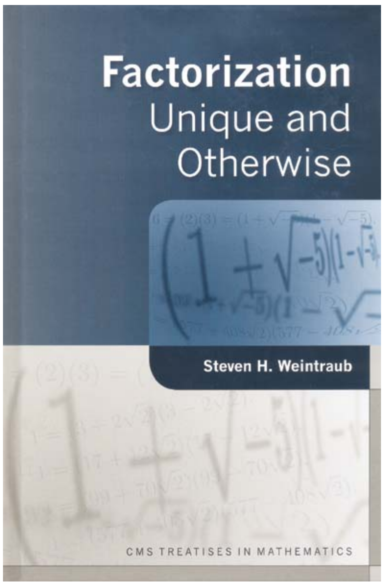
Factorization Unique and Otherwise

Steven H. Weintraub
A.K. Peters Ltd, 2008, ISBN: 978-1-56881-241-0

Factorisation is an ubiquitous theme in mathematics: think of rational and algebraic integers, polynomials, matrices, groups, modules and topological spaces. In spite of its intriguing title, Weintraub's book deals with only a small part of one of these. A more accurate title would be 'Factorization of integers in quadratic number fields'.

He begins by defining integral domains and quadratic fields in a way suitable for a student who knows about real and complex numbers but is unfamiliar with abstract algebra, and then continues in the same vein to describe Euclidean and unique factorisation domains, introducing the concept of factorisation into irreducible elements. Some mathematical notions, such as induction and congruence of natural numbers, are needed but their definitions and properties are relegated to appendices.

By this stage, we are already up to p. 60 of a 250-page book, suggesting that what we have is a gentle introduction to commutative algebra for undergraduates. But here, in an abrupt change of style, the author launches into a detailed analysis of factorisation in the ring of integers of a quadratic number field.



Let D be a square free integer and $\mathcal{O}(\sqrt{D})$ the ring of integers of the quadratic number field $\mathbb{Q}[\sqrt{D}]$. It is known (and stated but not proved in this book) that $\mathcal{O}(\sqrt{D})$ is a unique factorisation domain for exactly nine values of $D < 0$, namely –1, –2, –3, –7, –11, –19, –43, –67 and –163. It is unknown for which positive values of D $\mathcal{O}(\sqrt{D})$ is a unique factorisation domain, although vast numbers of examples are known, and it is conjectured (by Gauss no less) that there are infinitely many. It is also known that there are infinitely many D for which $\mathcal{O}(\sqrt{D})$ is not a unique factorisation domain. Clearly, there is much interesting and intricate mathematics involved in sorting out these results.

Weintraub does a commendable job in doing part of this in an elementary fashion. For example, he shows that $\mathcal{O}(\sqrt{D})$ has unique factorisation for $D =$ –1, –2, –3, –7, and –11; and fails to have unique factorisation for $D < -7$, and $D \equiv 1, 2, 3, 6$, or 7 (mod 8) as well as D composite and $\equiv 5$ (mod 8). Similarly, he shows that for positive D, $\mathcal{O}(\sqrt{D})$ has unique factorisation for $D = 2, 3, 5, 7, 11, 13, 17, 21$ and 29, and fails to have unique factorisation for D satisfying various divisibility conditions and congruence relations modulo 8.

This section of the book is followed by a copious list of exercises which involve the student getting his or her hands dirty finding or confirming many factorisations and checking whether the factors are irreducible.

There follow two chapters dealing with number theoretic applications of factorisations: one on the Gaussian integers including Fermat's theorem on integers which can be expressed as the sum of two squares, and the other on the so-called Pell's Equation and the more general problem of representing integers by binary quadratic forms. Once again, the author gets down and dirty with six pages of tables of solutions of Pell's equation $a^2 - b^2D = 1$ for various values of D. There is also a lengthy discussion of the useful tool of composition of quadratic forms, as well as calculation of units in $\mathcal{O}(\sqrt{D})$.

The final chapter of 40 pages is an overview of algebraic number theory. Here the author finds it necessary to abandon the elementary approach used up to this point. We are now on the more familiar ground of abstract commutative algebra, algebraic integers, ideal theory, Dedekind domains, and class field theory. With these tools, the author can return to the quadratic case with detailed investigations of generators of ideals, class groups and unit groups in $\mathcal{O}(\sqrt{D})$. Unfortunately, here the proofs are far from complete, replete with phrases of the type 'It is a theorem that ... ' with no reference.

The book concludes with three appendices. Two are of an elementary nature, one on mathematical induction and the other on congruences and their applications to Fermat's Little Theorem, Wilson's Theorem and Quadratic Reciprocity. The final appendix contains the details of some of the more complicated proofs that for various square free integers D, $\mathcal{O}(\sqrt{D})$ is a Euclidean domain.

The author suggests two ways in which the book can be used. The first is for a first course in number theory, in which the first two appendices should be covered as an integral part of the course. I believe that the book is not suitable for this purpose. Firstly, a large number of topics and applications normally covered in such a course are missing, replaced by much information unlikely to interest beginners, such as extensive tables of class numbers, solutions to Pell's equations and so on. Secondly, although many of the basic definitions and theorems are there, treated in great detail for an audience unfamiliar with abstract and linear algebra, several which are needed to understand the text are absent, or mentioned briefly in cavalier style. These include factor rings, Abelian groups and modules, polynomial rings, homomorphisms, fields of fractions of an integral domain, field extensions and Galois theory. Thirdly, while several topics considered unimportant by the author, such as Euclidean norms other than the complex modulus, principal ideal domains which are not Euclidean and the properties of free Abelian groups, are mentioned or used without proof, and the author provides no references to texts which explain them. Finally, while there are over 300 problems suitable for students, there is no indication of their relative importance for understanding the text, and no solutions.

A second way suggested by the author to use the book is for an advanced course in number theory, specialising in the topics covered in the book. Here again I have grave doubts of its suitability, this time because of the unsatisfactory treatment of general algebraic number theory and the lack of references, both within the text and as a bibliography. The only references in the text are to *Number Theory* by Borevich and Shaferevich (for an extensive table of class numbers of quadratic number fields), and a 1985 *Bull. Amer. Math. Soc.* review article (for the history of the class number problem).

It seems to me that Weintraub has written in a lively discursive style on matters which interest him and should interest other mathematicians. But his book is not a research monograph and it is unsuitable for his suggested audience.

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