Book reviews

The mind of the mathematician

Michael Fitzgerald and Ioan James
The John Hopkins University Press, 2007, ISBN 978-0-8018-8587-7

It goes without saying that mathematicians have minds — my two university-educated daughters may disagree with that, but of course we know better. So why write a book about the mind of mathematicians? Well the first point to note is that one of the authors, Ioan James, is a mathematician — naturally. He may be known to some as he is the editor of the Topology journal. The second author, Michael Fitzgerald, is a psychiatrist and psychoanalyst and this fact gives away the emphasis of the book. Prior to this book the only comment I had read about a mathematician from a psychoanalytic point of view was from the famous psychoanalyst Eric Idle, who once wrote ‘And Rene Descartes was a drunken fart, I drink therefore I am’. So I was interested to read this book given my singular lack of information about delving into the heart of the mathematician — their mind — and hence their personality.

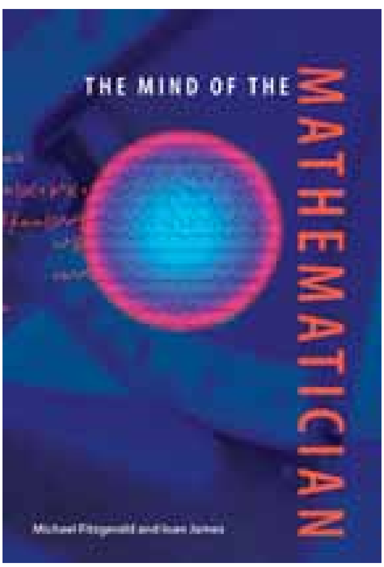
The book is divided into two parts. The first tries to delve into the mind of the mathematician in just three short chapters. While the second looks at the lives of 20 mathematicians, but not really from a point of their mathematics.

The first chapter is a bit of a hodge-podge of ideas as the authors consider what is the attraction of mathematics. They seem to trot out the standard answers, quoting from people like G.H. Hardy, A. Whitehead and Elie Cartan, mentioning the old joke ‘[a] mathematician is a scientist who neither knows what he is talking about nor whether whatever he is indeed talking about exists or not’ and left me wanting more. Perhaps the section should have been called ‘a summary of the literature giving a definition of mathematics’. I would have liked an attempt at an explanation of what it is that drives people to pursue maths. Why do these people spend their time thinking about primes, topology or combinations? Perhaps an examination of whether we have carried over from childhood a natural inquisitiveness. Do we really differ from physicists, or biologists or chemists or …? So much here went wanting. It didn’t have to be an in depth exposition; just a summary would have sufficed.

In the section about ‘Mathematicians and the Arts’, the authors consider music, literature and the visual arts. They cover these fields in just over three pages, with nearly two pages devoted to musical mathematicians. This part is a much better summary and there is a nice counter-example of Mozart being very interested in maths (so he just went up in my estimation). Again they direct to the literature, making this section a good basis for building a more detailed approach for, say, teachers. By and large the most developed section in the first chapter is ‘Savant

Skills and Other Phenomena' which is to be expected given one author's field of expertise. They cover the lightning calculators, the child prodigy, and how Asperger Syndrome makes its impact — a theme picked up and developed in a later chapter. At the end of the chapter I felt it was a mixed bag and was asking myself: who was it written for? Certainly it might be of benefit to teachers, but I'm not sure who else.

Chapter 2 looks at mathematical ability and here the book became more purposeful and specific. The first section of the chapter 'Making of a Mathematician' considers cognitive stimulation in early childhood, the position within the birth sequence which translated to the firstborn having a distinct advantage, and the education of a close family member or mentor. The authors looked into whether a mother or father had more influence in their child's mathematical ability. They give specific examples of these cases from well-known mathematicians from the 18th and 19th centuries. It would have been nice to compare with some stats from last century. They do consider briefly the curricula in American childcare centres from recent times and other probably more obscure areas like myopia, allergies, right- or left-handedness and result of ability.



The authors devote over three pages to considering if there is a gender difference in maths ability and consider studies from 1988, 2003, 2005 and a book from 1997 *Women in Mathematics*, so the reader wanting to know more has numerous sources to research. They highlight how women have been ignored in many books in the past, as well as the fact that if they had the ability there was very little opportunity for development — hopefully an issue rectified today, but I do wonder if this is so. Next, maths prodigies are considered, with the observation made that they are often firstborn males. In this section, they give the examples of prodigies Norbert Wiener and William James Sidis (which is an interesting read) and how one was successful and the other not, despite the fact that both had similar abilities. The last aspect examined is of age and achievement. The adage of 'maths is a young person's pursuit' finds support from quotes by Einstein, Hardy and a study by Lehman who found that, for mathematicians, the peak years are from 34 to 40. It seems to me that there needs to be a new Fields Award for those over 40 producing original work, rather than supporting a forgone conclusion.

Chapter 3 examines the dynamics of mathematical creation, an area I found fascinating. Consideration is given to different cognitive styles like visual or verbal thinkers and intuitive or logical thinkers, but all this seems to be an ongoing debate, as there is no survey to conclude one way or the other. Next the authors look at autism. Fortunately some people who have this ability (or disability) have left a written record of their thinking, like Temple Grandin's *Thinking in Pictures*. They also consider various ways mathematicians work or think, like Galois having difficulty expressing himself verbally because he worked just about exclusively in his head, while Poincaré was an auditory thinker and David Hilbert used gardening as his thinking tool (this delightful reminisce about Hilbert came from Courant). In the section on the unconscious mind, their main data comes from the reflections of Poincaré, Kolmogorov and Gauss with a little appertiser from Einstein. By far the largest section of the chapter is devoted to developmental disorders like schizophrenia and Asperger syndrome — there are some interesting gems here worth reading.

The second part, Chapters 4–8, is devoted entirely to the 20 mathematicians, of whom two were female: Ada Byron (Countess of Lovelace) and Sonya Kovalevskaya. Fortunately, it isn't a rehash of what has appeared in other books that expound the biographies of mathematicians. Here was at least an attempt to look at the human side of them with a very brief look into their characters as best they could. It is as best they could do because much of the character analysis is drawn from biographical books about the mathematicians. The birth dates of these people cover from around the mid-18th century (1736) to very early in the 20th century (1906). So, as can be appreciated, there will be large gaps in our knowledge of the earlier ones covered. Still, the authors make a valiant attempt at analysing the mathematicians and it is interesting to see what they come up with, but of course that is partly because of their sample set. I'll let you read for yourself who has Asperger syndrome and who doesn't, as well as many of the issues covered in the first three chapters. The thing that struck me most when reading their biographies is that many had such tragic lives. I hope that isn't representative of mathematicians! Another aspect that struck me was how young so many were when they died. Some 20% didn't make it to 40 years of age, while 45% didn't make it past 60! Again, I hope that isn't representative.

I'm not sure who is the intended audience of the book; perhaps teachers and students, or just the general reader interested in this topic. If it is the former group, there is a lot here to work with as well as to follow up in the literature. For mathematicians there isn't much new, though Chapters 2 and 3 will provide some interesting reading. If it is for their families then it may be very insightful. I remember Dr Paull from The University of Queensland saying to me once that he thought there was a little bit of Asperger's syndrome in all mathematicians. So being the analytic-analytics that we are I suspect many would have psychoanalysed their own make up by now. While the suggestion of the book is that many mathematicians have these attributes, I suspect that it is a small group, and that the great majority of us are a little more down-to-earth than what is portrayed. Or, as my friend Eric put it, 'I drink therefore I am'. That seems sensible to me.

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Euclid and his twentieth century rivals: diagrams in the logic of Euclidean geometry

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For over two thousand years, Euclid's *Elements* was regarded as the gold standard in mathematical rigour. By the end of the 19th century various gaps had been noticed — for example where we are asked to consider a point of intersection of two circles, but none of the axioms guarantee that the circles intersect. Hilbert's *Foundations of Geometry*, first published in 1899, provided an account meeting modern standards of rigour. It was however, rather more abstract, and rather less diagrammatic. The book under review sets out to provide a rigorous diagrammatic foundation for plane geometry, and in fact to formalise of the arguments of the first four books of Euclid, by showing that Euclid's proofs can be translated into proofs within a formal system. It also discusses a computer system, called CDEG (computerised diagrammatic Euclidean geometry), which is able to do various calculations within the formal system.

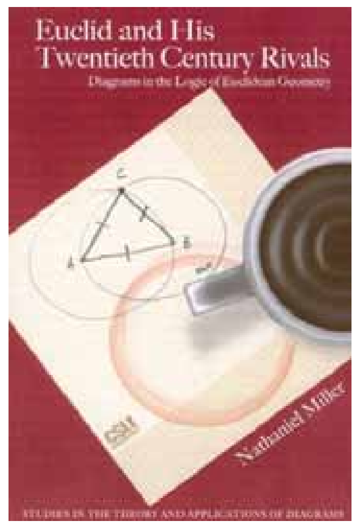
The diagrams that form the basis of the system are somewhat abstract, and contain only topological rather than geometric structure. So, for example, a line segment can be stretched or bent without changing the diagram. A result of this is that we are prevented from making unwarranted assumptions about a diagram as a result of incidental features. For example, if we have a line segment and two points not on the line segment, then we may draw a line segment connecting the two points. But we cannot say, without further information, whether the two line segments will intersect; or if they do whether it is at one of the endpoints or in the interior. As a result case analyses are unavoidable, and the formal system mostly works not with individual diagrams but with *diagram arrays*: sets of diagrams, usually thought of as representing different cases. The formal system then consists of various rules which allow a given diagram array to be replaced by a new one. Some of these increase the number of diagrams (for example, when performing a geometrical construction which involves various cases), while some reduce them (for example, removing a case which turns out to be impossible, or removing an identical copy of an existing diagram).

Chapter 1 contains a very readable introduction to the area. Chapter 2 describes the abstract notion of a diagram array (the syntax) and the configurations in the plane they are supposed to represent (the semantics). Chapter 3 describes the various rules for manipulating diagram arrays; it is these rules which are used to construct diagrammatic proofs. Also in Chapter 3 are brief discussions of the extensions needed to deal with arc lengths and with areas, and of the implementation CDEG. Chapter 4 contains some meta-mathematical results about the system.

Overall, the book is written informally. For example most of the time the author deals not with the fully abstract diagrams called *diagram graph structures*, but with a partially abstract notion called *nicely well-formed primitive Euclidean diagrams.* The latter are actual geometric objects in the plane; but some features are described abstractly, such as which line segments are supposed to be part of the same line. It was not explained how to translate statements about these nicely well-formed primitive Euclidean diagrams into statements about diagram graph structures; indeed even the axioms were only given in the partially abstract language. In fact the author writes

> ... given the complexity of the definitions, a skeptic might not be sure that this system is completely formal until seeing a computer implementation.
>
> [p. 53]

It is therefore disappointing that the implementation CDEG is not, as far as I can see, publicly available. Aside from the question of whether the system is completely formal, it also seems to me that without some computer implementation the system may not be widely taken up. I suspect that on the whole the book will appeal more to those interested in logic and formal systems than those interested in geometry. However it is generally well written, contains a good overview of the topic, and I found the idea of a formal approach to diagrammatic reasoning to be interesting.



Finally, a note on the title: it comes from an 1879 book *Euclid and his modern rivals* by Charles Dodgson (also known as Lewis Carroll).

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