Maths matters

It is the perception that counts, not the reality!

Bob Anderssen*

My parents would not countenance me going to university to simply study mathematics. For them, they had no doubts. Mathematics and science were the corner stones of knowledge. ‘But, Robert, if you only study mathematics and science, you will never get a (good) job.’ They pointed to the daughter of close friends who had studied science and now worked as a chemist in a jam factory.

Initially, I studied engineering before switching to science and then to mathematics.

Naturally, my parents, like all good parents, wanted to maximise the opportunities for their children, and were willing to make the necessary sacrifices. Given their and their friends’ backgrounds and the time when they grew up, they had a perception that was not reality. Just reflect for a moment on the family dynamics that must have occurred in situations where one of the children wanted to be an artist, a write, a poet, an actor.

The basic perception still persists — mathematics is important, fundamental, special, but not essential to guaranteeing a good job. For me, the challenge for the mathematics profession is to turn this around so that the perception becomes ‘mathematics is essential to guaranteeing a good and secure job’, no matter what might be the chosen profession. This has always been true, but its reality is not widely understood or appreciated. Interestingly, from a community perspective, it is accepted that engineers study mathematics, while for other professions, such as chemistry and biology, mathematics is not involved.

In my view, engineers, chemists, biologists and others are utilising brain circuitry similar to that used by mathematicians when involved with creative activities like planning an experiment, modelling or pattern recognition. This is a matter which will be illustrated and pursued below. The scientific understanding of how the brain works has progressed to the point where it is now appreciated that, in one way or another, similar circuitry in the brain is being used for quite different activities. The most widely know example is the high that one gets during long distance running or jogging, and as the initial effect induced by a recreational drug.

For myself and other colleagues, this false perception about mathematics is the result of a general lack of understanding in the wider community about the role, importance and practice of mathematics in everyday applications, such as tomography (based on the mathematics of Johann Radon), GPS and TV. Consequently,

*CSIRO Mathematical and Information Sciences, GPO Box 664, Canberra, ACT 2601.
E-mail: Bob.Anderssen@csiro.au

my overall goal is to give some examples of why the role and importance of mathematics is not as widely appreciated as it should be, to make some suggestions as to how this can be changed, to stress the growing importance of mathematical modelling and to make some comments about challenges that will impact on the future development of mathematics.

Why does this false perception exist?

In collaborative research, the biology (engineering, industrial, or other) colleague is only interested in the results that the mathematician collaborator generates to answer the matter under examination. The technical mathematical details by which the results have eventually been derived are, at most, of marginal interest, even though they give the mathematicians involved a feeling of enhanced self-esteem or result in new insight about mathematics itself. The fact that the results could be derived and validated mathematically in different ways, might have a wider applicability or may be of interest mathematically in their own right is only of interest to the mathematicians.

For example, in cereal science, the plant breeder wants simple techniques, such as NIR (near infrared) spectroscopy, to distinguish between wheat varieties that make good breads, pastas, cakes or biscuits. The breeder is not interested in any of the technical details about the calibration-and-prediction protocols used to define how to segregate the wheats on the basis of their NIR spectra, or some other experimental protocol. The only proof required is validation that the segregation is consistent with the breeders', growers', millers' and customers' expectations. It is interesting how, in such situations, the proof (validation) is not the mathematical details but the response and approval from higher authorities. Painful as it might be for the mathematician or statistician, this is the reality of the situation. It is here that our perception is incorrect — though mathematics has played a crucial role, the validation and acceptance comes from the higher authority related to the problem context.

In the study of pattern formation in plants, the biologist likes to reference publications about the reaction-diffusion modelling of pattern formation without appreciating that such models only see the macroscopic structure of the signalling, communication and switching within and between the particular set of cells of the plant involved with the patterning. The relevant comment that summarises the challenge that underlies the utilisation of mathematical models to solve biological problems is that of Willard Gibbs:

> One of the principal objectives of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity.

In a way, the challenge of finding the appropriate 'point of view' represents a good way to explain and illustrate to a wider audience why mathematical modelling is not as easy and as simple as it is often seen and sometimes lampooned.

Another possible explanation for the false perception that mathematics is not essential in getting a good job is the widespread lack of appreciation of the role and importance of basic mathematical knowledge. A good illustrative example of the importance of basic mathematics comes from the acute sensitivity that occurs regularly in real-world situations such as in exploration geophysics, as is given by the Railroad Rail Problem [1, pp. 3–4 and Chapter 2]. This is a good example where the derivation of a rigorous estimate is quite involved technically, whereas an indicative estimate based on a right-angle triangle approximation gives insight about the apparent intuitive contradictory nature of the solution.

Rules-of-thumb play a key role in industrial decision making on a daily basis. Scientific facts and mathematical/statistical modelling have played and will continue to play a crucial role in their formulation and in determining their domain of validity. Their importance economically should never be underestimated — they allow quick representative decisions to be made which reduces the risk of disaster in challenging situations. They represent an excellent way of motivating the interest of students in the role played by mathematics in applications. However, it is important for all of us to remember and acknowledge that they are based on modelling. The following observation, attributed to George Box [2, p. 424], is an important quotation to remember:

> All models are wrong, but some are useful.

If mathematics is used to solve a problem and fails to give a satisfactory answer, the mathematics is often the only thing that is called into question. The flaw often relates to the assumptions on which the modelling is based, rather than the subsequent mathematical manipulations.

Finally, there is a failure to make people proud of the mathematics that they can do. A common comment from non-mathematical friends, acquaintances and colleagues is that they are 'not good at mathematics'. (A good example of this response is discussed in the recent *Gazette* article by Peter Pleasants [4, p. 90].) However, when the matter is pursued, one finds that it is the last mathematics that has been learnt that is the reason for the claim. The fact that they can solve simple algebraic and geometric problems, or they know and understand Pythagoras' theorem, or can differentiate algebraic relationships, or solve ordinary differential equations is taken for granted. We all find it challenging to learn new mathematics. The higher the level we learn mathematics, the more mathematics we will know intuitively with skill and confidence. It is important to make this fact clear at all stages in the learning of mathematics from pre-school, to primary, to secondary school, to university, and in the daily discussion of quantitative concepts. Some suggestions, on how this might be done, include:

- Guessing: treat the mistake by a student or a colleague as a good guess and build on that.
- Mathematics should not be competitive but cultural. We all enjoy activities like music, solving puzzles, jokes, paintings and gardening. Mathematics should be treated as a similar cultural pursuit in terms of a search for

patterns. A good example is the current popularity of Sudoku and the historic and continuing popularity with crossword puzzles, chess, jigsaw puzzles, etc.

- Understand why others see things differently. Take a rectangular sheet of paper and show young children that it can be folded into two different cylinders, and then ask them which contains the larger volume. The answer is often that they are the same because the areas are the same. For their age and level of mental development, it is an excellent guess. Understanding the reasons why an incorrect answer is given will help make mathematics to be viewed more as a cultural, rather than a competitive, activity.
- The importance of mistakes is that they represent progress. George Polya is reported to have made something like the following comment to a student who thought that George must never make mistakes: 'I regularly make mistakes. The difference between you and me is that I find my mistakes much faster than you find yours.'

An easy way to make people feel self-esteem about their mathematical knowledge is to use illustrative examples of the role of that mathematics in applications. Such advice is hidden in the following comment in the Preface of the book *Methods of Mathematical Physics* by Jeffreys and Jeffreys [3]:

> We think that many students whose interests are mainly in applications have difficulty in following abstract arguments, not on account of incapacity, but because they need to 'see the point' before their interest can be aroused.

In fact, in all levels of research, the challenge and struggle is to 'see the point' that allows new patterns to be identified and explained.

The importance of modelling: the impact of applications on mathematics

> The profound study of nature is the most fertile source of mathematical discoveries. J.P.J. Fourier

In my view, one of the key doorways to stimulating a greater awareness of the reality of mathematics is modelling. It is the cornerstone on which collaboration is based. The model identifies the question the non-mathematical colleague is investigating and to which mathematics can be applied. It is the structure that the mathematician solves, and is a framework from within which an answer is generated and to which the colleague then gives an interpretation.

The same model and solution can have arisen in a completely different situation. The only difference between these two situations is the interpretation that is applied to the solution. The diffusion equation is an excellent example. Its solutions can answer questions about heat conduction, the uptake of nutrients by plants, the infiltration of water into the soil, etc.

It is crucial to understand that much successful modelling is being performed without the assistance of a mathematician or statistician, and in many situations the non-mathematician is unaware that modelling is being implicitly performed. But, in some form or other, mathematics is being utilised.

Frank de Hoog, in his Maths Matters contribution [5] reminds the reader of the fact that

> ... only a tiny proportion of all the application of mathematics is performed by those who consider themselves to be a mathematician ... even if we limit ourselves to sophisticated applications of mathematics.

One can surmise that, in such situations, the biologist (engineer, economist, etc.) is using similar circuitry in the brain to that used by the mathematician. Consequently, many of the tasks being performed on a daily basis by the bulk of the community might have a greater neural affinity with mathematics than is currently appreciated. It is therefore an aspect that might assist people in the community to have a greater self-esteem about their problem-solving abilities. There are many smart people out there in the community who are not mathematicians, but are using mathematical skills. Some, such as engineers and computer scientists, have strong mathematical backgrounds and are directly exploiting the circuitry in the brain where that knowledge is stored. Others, like the biologist, are exploiting implicitly the circuitry that evolution has built into our brains to perform (quantitative) pattern recognition tasks.

If one believes, as I do, that the essence of mathematical creativity is seeing new patterns, then many people, such as experimentalists and artists, are using mathematical-related circuitry in their brains.

These ideas are not new!

However, it is a classical situation where the 'obvious must be said over and over again before it is fully understood'. An example of where obvious things must be said over and over again before they are fully understood is global warming. The scientific reality of the increasing average-temperature-evidence was there but not the perception. Now that the perception is there, it is interesting how rapidly community attitudes are changing. Consequently, there is hope for changing the community's perception about mathematics.

In his Math Matters contribution, John Henstridge [6] explained with considerable skill how academic mathematics, and the mathematics profession in Australia, has become a captive of the community's perception that mathematics is important but not essential for the education of professionals. Among other good points, John stresses how important marketing is to the survival of his consulting company. I am in full agreement with his conclusion that the mathematics community must '... recognise and support all our graduates and all the work that they do' and that 'It will be a challenge [for the mathematical community] to do so with one voice ... '.

In the 'Mathematics graduates are highly employable' brochure that the Australian Mathematical Society published and circulated in the mid-1980s and early 1990s, one finds the following quotation by Ross Gittins, Economics Editor, Sydney

Morning Herald:

> . . . if possible do Maths . . . (it) is the single most useful ability to have in your kit-bag to equip you for any eventuality.
>
> In any case employers set a lot of store by mathematical ability and are more likely to hire someone with a good background in Mathematics.

As a profession we must strongly market this point of view. Not only must we do it, even more importantly we must find champions like Ross Gittins who will also do it. We must learn to take a back seat as far as receiving credit for the idea. We must not market it as our idea but the idea of various champions.

Academic perceptions

It is ironic that, when universities need to save funds, they prune mathematics, the least expensive of the disciplines. Money could be saved by pruning the more expensive disciplines. Why don't they? Those more expensive disciplines have strong lobby groups and the perception is that they are essential for the image of the university. The strongest disciplines in this regard are the one who are perceived to be able to guarantee jobs for their graduates.

For me, it is unbelievable that some mathematicians try to justify their existence on the basis that their work might be important in two hundred years. Surely, a wiser, more understanding and less superior attitude would be that they are contributing to the body of mathematical knowledge that is used to solve practical problems, and that the greatest pleasure would be to learn that a recent contribution is already being utilised.

At the ICIAM Congress in 1995 in Hamburg, V.I. Arnold, in his plenary talk, commented in the following manner, exploiting the duality in meaning of both 'pure' and 'applied'

> My best pure mathematics was done in applied mathematics and my best applied mathematics was done in pure mathematics.

Future challenges

I believe many changes are happening now that will have a strong influence on how the future will unfold for the mathematics disciplines and professions:

- Applications will have an increasingly important impact on how mathematics is pursued and funded in the future. A good example is the embracing of biology by various mathematical disciplines when seeking research funding, such as algebraic biology initiatives.
- The great success of modern mathematics is the huge spectrum of results that it has formalised which are available for the solution of practical problems. Because of this pool of knowledge, there is an increasing opportunity and speed with which mathematics can now be found which resolves many new practical problems.

- Practitioners, in collaborations with mathematicians, will increasingly demand relevant and useful interpretations of the mathematics that is being proposed to solve their practical problem. The users of mathematics will become increasingly sceptical of complex mathematical constructs which appeal to the mathematician but tell them little about the processes supposedly being investigated.
- There will be a greater use of computers to perform complex calculations the results of which will be used to make decisions that will impact on our daily lives. The growing importance of preventive medicine is a positive illustration, if the solution of the underlying inverse problem has been solved with sufficient accuracy.
- The great success of technology in the last decade or so has been the development of computer-controlled instrumentation. An obvious daily example is the bar-coding of goods and commodities for electronic checkout. An important scientific example is the various instruments designed to perform DNA sequencing. However, the plethora of data that such instruments are generating, is spawning a need for new paradigms for their analysis and interpretation.
- With the wider use of computers to perform more and more basic tasks, the stage is set for major cultural changes. In this regard, the computer and, hence, the underlying mathematics have a crucial role to play in exploring computationally various scenarios for the solution of global warming challenges, the efficient management of water and petroleum resources and the protection of biodiversity. The use of the computer to first explore alternatives for the solution of such problems will generate an increasing demand for scientists, engineers, biologists, etc. with strong mathematical backgrounds.

In one way or another, the above items will be responsible for more and more people wanting to perform their own mathematical modelling and simulation. This is a clear target group for the mathematics professions to encourage, mentor and support. The technology is there for more people to be owners of their mathematical modelling and simulations. They are not going to stop having such ownership simply because some mathematician or statistician says that they do not have the sophistication to do so. What must be done is to give them the confidence to turn to mathematics professionals for enhancing their mathematical expertise and viewing them as potential collaborators.

Conclusions

As already mentioned above, the professions that have the greatest clout politically and the strongest impact on the community and potential students are the ones that can guarantee (good) jobs for their graduates. This fact plays a key role in behind-the-scenes negotiations and decisions about future directions and funding in academia and the allocation of research funds. The position for the mathematical sciences in such negotiations will be considerably improved once we, speaking with one voice, convince parents, teachers, mentors and students of the new reality that

> ... mathematical knowledge and skill has been, is and will continue to be essential to guaranteeing a good and secure job in whatever profession a person decides to pursue.

This, in my view, will only be achieved once there is a greater appreciation in society of the role and importance of mathematics at all levels of daily activities. The challenge is to reverse the situation encapsulated in Edward E. David's comment that 'the importance of mathematics is not self-evident'. Reverse it by creating a situation where all become aware of the role of mathematics in their daily activities no matter how humble they are.

The responsibility for doing this rests not only with the mathematical sciences professionals like you and me, but also with creating situations where champions of the non-mathematician community, like journalists, media professionals, and TV presenters and stars, trumpet this new reality loud and clear.

For me, for example, it will be clear that mathematics has become a more essential part of our cultural heritage when the TV weather commentator says something about the mathematics that sits behind the changing temperature and pressure dynamics that are displayed each evening, perhaps explaining that close contour lines in atmospheric pressure imply that the winds there will have a higher velocity than elsewhere.

References

[1] Acton, F. (1997). *Numerical Methods That (usually) Work*. Mathematical Association of America.
[2] Box, G.E.P. and Draper, N.R. (1987). *Empirical Model-building and Response Surfaces*. Wiley.
[3] Jeffreys, H. and Jeffreys, B.S. (1972). *Methods of Mathematical Physics*. Cambridge University Press.
[4] Pleasants, P. (2008). Maths in the media: On the couch with Queensland's *Courier Mail*. *Gaz. Aust. Math. Soc.* **35,** 90.
[5] de Hoog, F. (2007). Maths matters: Opportunities for the mathematical sciences? *Gaz. Aust. Math. Soc.* **34,** 137.
[6] Henstridge, J. (2006). Maths matters: If math matters, why don't we see more of it? *Gaz. Aust. Math. Soc.* **33,** 231.

Bob Anderssen was born in Brisbane on January 13, 1939. He completed BSc and MSc degrees, which tended to focus on applied and computational mathematics, at the University of Queensland. His PhD in mathematics, on the numerical performance of variational methods, was obtained at the University of Adelaide, where he was also a tutor in mathematics. In 2008, he received a honorary DSc (honoris causa) from La Trobe University. He has been president of the Australian Mathematical Society and chair of the National Committee for Mathematics. He has been awarded the George Szekeres and Joe Moyal medals, given the G.S. Watson Annual Lecture at La Trobe University in Bendigo and is a Fellow of the Australian Mathematics Society. His hobbies include gardening, hiking and classical music.