Book reviews

Hypatia of Alexandria: mathematician and martyr

Michael A.B. Deakin
Prometheus Books, 2007, ISBN 978-1-59102-520-7

I found *Hypatia of Alexandria* a well-written, engaging and informative book which clearly demonstrates that it is possible for a book to be both scholarly and popularly written. The author achieves this partly by the device of completely avoiding footnotes or references in the text, and relegating notes, as well as mathematical details and source materials, to appendices which constitute almost half of the 230-page book. Deakin's subject was a romantic figure who has captured people's imaginations for nearly two millennia. These imaginations have yielded many quite fanciful reconstructions of the events of her life and death, and from the various fictions Deakin has managed to untangle a large amount of probable fact.

Briefly, Hypatia was a woman mathematician and philosopher, a Pagan in then largely Christian Alexandria, born in the fourth century of our era and killed by a group of Christians in 415 or 416. She was the major mathematician of her era as her father, Theon of Alexandria, had been before her.

Alexandria was a city in ferment in the fourth and fifth centuries, with tensions not only between Christians, Jews and Pagans, but also between various Christian groupings vying for orthodoxy. Deakin compares Alexandrian factionalism in the fifth century with that of modern Malaysia. Since the earlier drafts of this book this parallel has become rather unfamiliar, and the world has unfortunately come to experience more apposite parallels in the many violent faction-ridden cities of Iraq, Afghanistan and elsewhere.

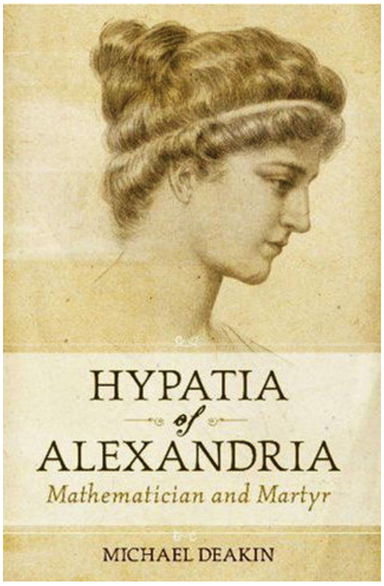
Compounding the religious factionalism was the decidedly negative PR image mathematicians enjoyed during the fifth century in some Christian circles (pp. 63, 64). This was expressed among others by the Council of Laodicea, John of Nikiu, Augustine of Hippo and the Theodosian code. We mathematicians were perceived to practice divination, to 'have made a covenant with the devil to darken the spirit and to confine man in the bonds of hell' (Augustine), to teach pernicious doctrines, and to deserve summary execution (Theodosian code). Who would aspire to be a mathematician in such an environment?

It is possible that this image may owe its existence to the writings of late Pythagoreans such as Iamblichus (ca. 250 – ca. 330). Since the fifth century BCE Pythagoreans had been split into two groups — the Akousmatikoi and the Mathematikoi (see e.g. [1]). Even if the latter were not all necessarily mathematicians in our sense we have to concede that they invented the term and were entitled to apply it to themselves. The secretive and mystical components of Pythagoreanism, such as Iamblichus' theory that certain magical rituals were necessary to liberate the soul from the body (cf. [2]) and the conflation of the notions of mathematics with those of Pythagoreanism, may have made Christians feel threatened by mathematicians in general, perceiving them to be a rival religious sect.

Deakin certainly presents a heroic picture of his subject:

> Hypatia's case then was this. She lived in a time when her intellectual heritage, a 700-year-old tradition, was crumbling. The supports that had once seemed so secure — the Museum and the libraries — had all been swept away by the swell of ignorant dogmatism. Almost alone, virtually the last academic, she stood for the intellectual values, for rigorous mathematics, ascetic Neoplatonism, the crucial role of the mind, and the voice of temperance and moderation in civic life .... But that tide of opinion which knows no possibility of doubt ... all this was against her; and her life became forfeit to the bloodlust of those who would claim, with the ironic certainty of unconscious self-refutation, their access to a higher morality. (Page 66)

I add that the tradition she came from was actually even longer than 700 years: as a Neoplatonist and Mathematician, her tradition went back to Athens (Plato, Eudoxus), Chios (Hippocrates) and Miletus (Thales), a tradition of some 900 years, even longer than our European mathematical tradition since Gerard of Cremona, Fibonacci and the first universities. It sharpens our perspective when we reflect that the Academy, founded by Plato, lasted over 900 years, longer than the University of Oxford has so far.



In his examination of the intellectual, social and religious situation in Alexandria around the year 400, Deakin outlines some of the theological differences current between various Christian groups concerning the Trinity and the nature of Jesus. These differences might seem trivial to us but they apparently stirred up strong emotions and even violence between the various sects. He notes that Neoplatonism was compatible with some of these theologies so that there were Christian as well as Pagan Neoplatonists in Alexandria.

The various Christian factions, as well as the Jews and the Pagans, were held together somewhat tenuously by Orestes, the local Prefect of the Eastern, Greek-speaking Roman Empire, the capital of which was Constantinople (= Byzantium). While Orestes, a Christian, was relatively tolerant, the same could not be said for the Christian Archbishop Cyril — later proclaimed Saint Cyril of Alexandria — who was appointed in 412. In a series of tit-for-tat actions, Christians prevailed on Orestes to reduce dancing displays popular among the Jews, some Jews retaliated by killing a group of Christians, Christians burned synagogues and expelled all the Jews, and Cyril called on a band of armed monks for support. One of these monks attacked Prefect Orestes and was tortured to death in punishment. It seems that Hypatia was then attacked and murdered by Cyril's supporters because she was a supporter of Orestes. And she was a soft target.

Deakin does an excellent job in teasing out the scope of Hypatia's mathematics from the limited primary source material. Unfortunately this material does not permit definitive conclusions about Hypatia's specific contributions to editions of Diophantus' *Arithmetic* or to her father's commentary on the *Almagest*. Some of Deakin's incidental remarks are a bit simplistic: it goes too far to say that 'Euclid's masterpiece rendered obsolete all that had preceded it' (p. 87). In no way did the *Elements* supersede, for instance, Hippocrates' work on lunes or Archytas' brilliant duplication of a cube.

Some minor quibbles: There is one time when I consider that Deakin's methods to wring the last drop of information about his subject from the evidence go beyond the bounds of good scholarship. This occurs when he infers, from the writings of her pupil Synesius, what Hypatia's own philosophical views might have been: '[i]f we want to learn of her philosophy we will do best by examining his ... we may be sure that it represents a strain of philosophical thought close to Hypatia's own.' (pp. 78, 79). This is tricky business. Philosophers take care, for instance, not to infer too much about Socrates' *views* (as distinct from his methods) from those of his pupil Plato. Indeed one might well highly regard a supervisor who supports a PhD student who is writing a case for a wildly different position or theory from that of the supervisor. It denigrates Hypatia to suggest that Synesius could not have continued to be her friend or to value her comments on his work if their views had diverged.

There is an inconsistency related to the book's subtitle: 'Mathematician and Martyr'. On p. 72 Deakin discusses the death by torture of Ammonius, a military monk allied with Cyril (and opposed to the more tolerant Christian Prefect Orestes). Cyril apparently declared Ammonius to have been a martyr. Deakin comments that Ammonius could not have been a martyr since he was never asked to renounce his faith. Although in common parlance the term 'martyr' is used far less precisely, Deakin is entitled to restrict its denotation in his book. But then he is hardly entitled to refer to Hypatia as mathematician and martyr. After all she was never asked to renounce her Pagan faith.

These are minor quibbles indeed and do not prevent me from thoroughly recommending this book to anyone interested in the first well-documented woman mathematician or who thinks we are having it tough now with funding cuts and growing workloads.

References

[1] Burkert, W. (1972). *Lore and Science in Ancient Pythagoreanism.* Harvard University Press, Cambridge MA. (Translated by E.L. Minar Jr.)

[2] Riedweg, C. (2002). *Pythagoras: His Life, Teaching and Influence.* Cornell University Press, Ithaca and London. (Translated by S. Rendall.)

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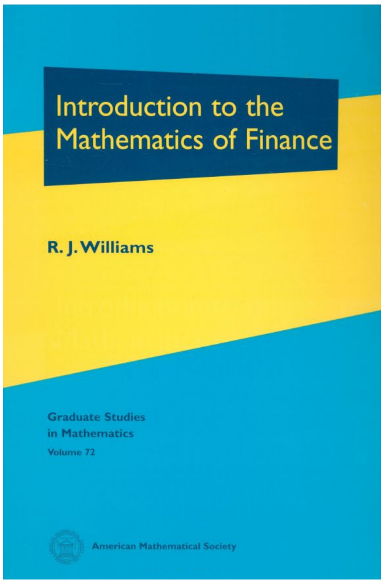
Introduction to the mathematics of finance

R.J. Williams
American Mathematical Society, 2006, ISBN 978-0-8218-3903-4

This is a brief introductory text intended to introduce suitably well-prepared mathematics students, and others with an equivalently strong background in probability theory, to the basic ideas of pricing and hedging of financial derivatives. There are 121 pages of basic text plus four appendices and references adding up to 150 pages in total.

It is a neat and rather minimalist treatment, strongly based on martingale theory and equivalent martingale measures. The prerequisites, however, are quite formidable and the treatment is certainly not self-contained.



The origins of the text as classroom material are still visible and a useful small set of exercises augments each of the chapters. After an introductory chapter explaining the ideas of financial markets and derivatives the binomial market model is introduced and pricing of European and American contingent claims are treated. Next comes a good discussion of the finite market model, including the first and second fundamental theorems of asset pricing, and some discussion of incomplete markets. After this the story moves to continuous time and the geometric Brownian motion model, described herein as the Black–Scholes model. The principal part of the remaining text treats this model first in the one dimensional case, and then the multi-dimensional one. The brief appendices sketch the ideas for Conditional Expectation, Discrete and Continuous Time Stochastic Processes and Brownian Motion and Stochastic Integration.

In some places the author has coined her own terminology; for example describing markets as viable rather than using the more evocative usual description of having no arbitrage opportunities.

The Index could have benefited from more attention. For example the separating hyperplane theorem has its proof referenced (p. 51) but not the application thereof (pp. 36, 37) for which it was introduced.

For students who are able to accept the considerable mathematical challenge of this material, it is an excellent introduction.

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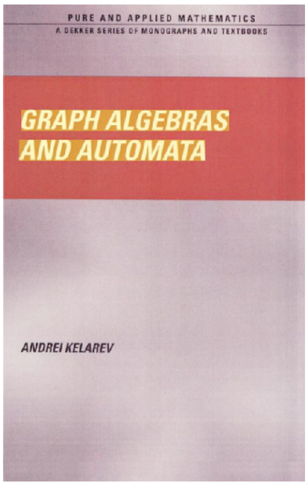
Graph algebras and automata

Andrei Kelarev
Marcel Dekker, 2003, ISBN 0-8247-4708-9

The study of graph algebras falls neatly within the intersection of several fields, including computer science, combinatorics, graph theory, operations research and universal algebra. Automata are used to model formal languages and machines and are particularly amenable to mathematical treatment and algebraic techniques and form the basis for many applications of mathematics to theoretical computer science. A graph algebra is a device for turning a graph into an algebra, and abstract graphs are closely related to the underlying diagrams of automata. The definitions are relatively straightforward but the mathematics and connections are subtle and lead to many difficult open problems.

This is the first extended text incorporating graph algebras, and indeed the choice of related topics is broad and ambitious. There are many existing books on automata and languages overlapping in material with Chapters 3 to 6 of this book. These and the earlier Chapters 1 and 2 generally deal with standard background material, but always the author finds new and interesting examples and angles to present the ideas and give the reader practice.



Chapters 1, 2 and 3 comprise one-third of the text and are liberally sprinkled with exercises. Most of the results are quoted without proof, but the author carefully provides references so that the interested reader can easily locate proofs elsewhere.

From Chapter 4 onwards, the style of writing changes and more attention is given to developing topics through proofs, rather than quoting results and giving the reader practice through exercises.

The later Chapters, 7 to 12, are quite specialised and bring together many results by the author and others in an impressive spectrum of collaborations. The final chapter lists a number of research problems. There is an extensive bibliography with over 250 entries, including a very large number by the author, and a comprehensive index and glossary of notation.

Kelarev's book is a very worthwhile addition to the literature and should be acquired at least as a reference work by all mathematics libraries and a number of specialists. It is unusual in combining so many topics that would normally be taught in disjoint courses in a second-, third- or fourth-year pure mathematics undergraduate curriculum, culminating in a cascade of chapters close to the frontier of research.

As a pedagogic tool this book would be especially useful for talented-student programs that are looking for something unusual, bridging several fields and combining a number of classical topics with recent progress on the research front. It would be especially useful for potential honours students seeking suitable topics for developing essays or minor theses. The book could be a springboard for potential masters or doctoral students with interests in algebra, combinatorics and theoretical computer science.

A naive undergraduate student, however, may need guidance, since many important results are listed without narrative. In my opinion, the book could be improved if some of the routine or repetitive exercises were deleted in favour of providing more explanations that connect results or provide context. It may help also if the author used his experience to indicate which results are particularly deep or surprising. There are also a number of typos and minor innacuracies that could hold up an inexperienced reader. However, the reader who perserveres will find in this book a wealth of ideas and resources.

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