Book reviews

Master Classes in Mathematics*

W.M. Stephens (Ed.)
The Mathematical Association of Victoria, 2006, ISBN: 978 1 87694 932 7

In the volume 'Master Classes in Mathematics' Max Stephens has assembled a record of nine selected talks given to the Mathematical Association of Victoria (MAV) by active teachers and mathematicians during the mid- to late-1950s. For some of the reproduced talks the original notes of the author have been typeset: for others, handwritten notes taken by an MAV member present at the talk have enabled the reconstruction.

The volume provides an interesting snapshot of the Mathematical Association of Victoria at the middle of its one hundred year history. In his 'Foreword' to the volume, Max Stephens states: 'One reason for the decision to publish this selection is that they show the vitality and energy of the MAV at a specific period of time. The nine papers selected for publication in this book were chosen from a much larger set of talks and presentations given to the MAV ...'. Max also writes: 'The whole collection (of talks) shows that the MAV's current goal of Valuing Mathematics in Society was seen as fundamentally important fifty years ago'. The transcripts of the talks themselves very much support Max's assertion, which is indeed a remarkable observation because 50 years ago the MAV did not have even a room as a dedicated office yet alone a publications base.

A very brief description of each of the talks follows. As is obvious from their titles, the talks ranged from the mathematically quite technical to observations about mathematics teaching and usage.

Professor M.H. Belz (Statistics and Genetics, April 1954): Maurice Belz demonstrated the calculation of the probability of a genetic characteristic being passed on, and talked about the propagation of haemophilia. The talk is remarkable for its insight at a time when the development of the science of genetics was gathering pace.

A.E. Schruhm (The Teaching of Algebra, June 1954): Bert Schruhm advocated a shift away from a junior secondary algebra curriculum designed 'as a training for the benefit of the mathematical specialist', and some of his proposed changes did occur decades later. In this talk, he gave several examples of practical problems and their algebraic formulation.

K.S. Cunningham (Mathematics as a Subject in the School Curriculum, April 1955): Taking a different approach from Schruhm, Ken Cunningham reviewed the history of mathematics in the school curriculum and argued 'that we need to pass under rigorous scrutiny almost all that we do in the teaching of mathematics, that

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our approach is still too formal'. Apparently the time allocated to mathematics in the curriculum was an issue then, as it is today.

R.T. Leslie (A Career in Statistics, August 1955): Rupert Leslie talked about employers of statisticians, and the nature of statistical endeavours in each of the chemical industry, CSIRO and other industries, giving examples of sampling required (for example, in testing early TV tubes), and gave an optimistic picture of employment prospects for statisticians. His talk pre-dated the growth in the Australian Bureau of Statistics.

M. Lester (Some Mathematics of Orchestral Wind Instruments, March 1957): Attendees at Meg Lester's talk on orchestral wind instruments would have been required to have an undergraduate mathematics background including a working knowledge of partial derivatives and Fourier series. The talk must have been entertaining, with a 'supporting act' musician demonstrating tuning, overblowing and harmonics.

Professor T.M. Cherry (Mathematics in the International Geophysical Year, April 1958): There was much classical applied mathematics in this talk, which a well-prepared Year 12 Specialist Mathematics student could appreciate in 2006. Satellite orbits were analysed: Tom Cherry showed his audience how they might calculate the radius and period of a satellite orbit with the aid of a rotary clothes hoist.

Professor E.R. Love (Infinity, August 1959): Russell Love's talk on Infinity (1959) was clearly aimed at Year 12 students. He gave a nice treatment of the paradox of rotating the curve $y = 1/x$ about the x-axis, leading to a 'trumpet' of finite volume but infinite surface area. His treatments of arithmetical processes and infinite series are still very relevant for Year 11/12 students in 2006.

B.I. Merz (Notation, June 1959): Blanche Merz's talk was really about the history of mathematical notation (for example, the origins of the plus and minus signs), from the ancient to the relatively modern notations of the last three or four centuries. Blanche observed: 'Many symbols are made, few are chosen. The list of discarded mathematical signs would fill a volume'. Given that almost no new notation has been popularly adopted in elementary school mathematics in the last fifty years, this talk is quite fresh for present day readers.

J. Bennie (Mathematicians in Industry, October 1959): James Bennie pondered the prospects for mathematicians to be employed in large industrial and business organisations. His remarks are of considerable historical interest: in the decades that followed his talk, the use of non-trivial mathematics in business, economics, networks and communications increased dramatically.

Readers of 'Master Classes in Mathematics' will be surprised both by things that have changed since the 1950s, and by other things that have not. In collecting these talks, Max Stephens has produced a gem that is a very appropriate commemoration of 100 years of MAV activities.

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Measure Theory, Volumes 1 and 2

V.I. Bogachev
Springer, 2007, ISBN 978-3-540-34513-8

The art of writing book reviews is changing with technology. I assume that you too can find this book on the publisher's web site, obtain the basic facts about the book, browse through the table of contents, and examine sample pages. So, in this review I will concentrate on providing extra information.

This is a two-volume work on measure theory. Volume 1 deals with classical measure theory and Volume 2 deals with more recent ideas in the subject. Many ideas in Volume 1 are contained in Halmos' classical *Measure Theory* or other works on real analysis or probability. On the other hand there are relatively few text books that focus on the ideas in Volume 2. In keeping with modern scholarship, there is more emphasis on probability in Bogachev's work than you would tend to find in books on real analysis, and also more emphasis on topology, set theory, and functional analysis than you would tend to find in books on probability. I liked the balance.

A quick look at MathSciNet will establish the author's standing as a distinguished scholar. This book suggests that V.I. Bogachev is also a dedicated teacher.

I enjoyed tips from Bogachev the teacher. Here is an example. It's only natural that a student will want to have a mental picture of a typical element of a σ-algebra generated by a given class of sets. On page 5, the author warns us about the futility of this exercise.

> One should not attempt to imagine the elements of a σ-algebra generated by the class $\mathcal{F}$ in a constructive form by means of countable unions, intersections or complements of the elements in $\mathcal{F}$. The point is that the above-mentioned operations can be repeated in an unlimited number of steps in any order. For example, one can form the class $\mathcal{F}_\sigma$ of countable unions of closed sets in the interval, then the class $\mathcal{F}_{\sigma\delta}$ of countable intersections of sets in $\mathcal{F}_\sigma$, and continue this process indefinitely. One will be obtaining new classes all the time, but even their union does not exhaust the σ-algebra generated by all the closed sets (the proof of this fact is not trivial; see Exercises 6.10.30, 6.10.31, 6.10.32 in Chapter 6).

When I first read this paragraph, I imagined that I was in Bogachev's class in snow-covered Moscow and he turned around from writing on the blackboard and gave us this warning. And, when something is not immediately clear to me, I love to be told that 'this fact is not trivial' — it's so reassuring.

The structure of the book contributes to its accessibility. Each section of each chapter is only a few pages — about the length of a lecture. Thus I could approach reading a section without being daunted by the task in front of me.

At the end of each chapter, there is a section 'Supplements and exercises'. In the supplementary remarks, the author sketches different directions in which the ideas in the chapter have been extended. These remarks whet my appetite for more.

There are many, many exercises. I did not notice any that I would describe as drill exercises. On the other hand exercises seem to be manageable, especially the many exercises that are specifically recommended for the student reader. If you solved all these problems, you'd know a lot about measure theory. The author has generously offered hints to most of the 850 exercises. Again we see the careful writing of a good teacher.

There is an appendix entitled 'Bibliographic and Historical Comments'. For each chapter of the text, there are several pages of such comments. Without wanting to spoil your reading pleasure, let me say that it appears that Bogachev enjoyed writing these sections. They are scholarly, extensive, sometimes humorous. Many comments here might well be a springboard for a research project in some aspect of the history of measure theory.

The bibliography is immense. It has the helpful feature that each item in the bibliography is accompanied by the list of those pages where the item is cited. I wish that more authors did this.

In another appendix, Bogachev gives us an outline of his course on real analysis in Moscow and its connections with sections of the text. As a teacher, I particularly enjoyed this glimpse of how a teacher with Bogachev's expertise approaches his subject.

Overall, Bogachev's *Measure Theory* is an impressive work of scholarship.

The cover states that 'the target audience includes graduate students seeking to acquire deeper knowledge of measure theory'. However, the price is likely to be beyond the means of most Australian university students. One cannot buy the volumes separately. On the other hand, it is good value for money in terms of dollars/page. The work comes in a well bound, hardcover edition and each volume has a lovely blue ribbon as a book mark. A nice touch by Springer!

This is a big book and demands time for enjoyment and careful study. There is no point in writing a book, unless there are readers. In this sense, readers are as important as authors. And the sorts of people who can read this book are people like us, readers of the *Gazette*. Unfortunately, academic life is dominated by 'sciometrics' that encourage us to write, write, write. Imagine having the time to read, read, read — 'wouldn't it be lovely'?

Let me conclude with this simple recommendation. If your university library contains Halmos' *Measure Theory*, then it should also contain Bogachev's *Measure Theory*.

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Leonhard Euler

Emil A. Fellmann
Birkhäuser Verlag 2007, ISBN 978-3-7643-7538-6

In 1995, a biography of Leonhard Euler by the Basel historian of science Emil A. Fellmann appeared as a paperback in the series 'Rowohlts Monographien'. This copiously illustrated, small volume, written in German and directed to a general readership, has since gone through enormous success and, hence, not surprisingly, is out of print. In 2002 Springer Tokyo published a Japanese translation. The book was translated into English by Erika and Walter Gautschi of Purdue University. At last, occasioned by Euler's tercentenary, Birkhäuser Basel brought it out as hardcopy in December 2006. To my knowledge, it is the first monograph on Euler available in English.

Euler's life can be 'naturally' divided into four periods and, accordingly, the book is organised in four chapters: 1. Basel 1707–1727; 2. the first Petersburg period 1727–1741; 3. the Berlin period 1741–1766; 4. the second Petersburg period 1766–1783. What is remarkable, even amazing, is the wealth of information on Euler's life and work that is covered by less than 200 pages while, at the same time, the book makes pleasant reading. Citations from his autobiographical writings and from sources intimately close to him convey a lively picture of Euler's personality. Thus, the first chapter opens with Euler's curriculum vitae as of 1767 that he dictated to his first-born son Johann Albrecht. Then we learn of his family background, his early years and the important role Johann I Bernoulli played in his mathematical upbringing.

Chapter 2 first looks at the founding of the Petersburg Academy by Peter I and Catherine I, the appointment of numerous Germanophone scholars and scientists as academicians, including the Basel mathematicians Jakob Hermann and Bernoulli's sons Niklaus II and Daniel, and Euler's 'first years in the tsardom'. Three of Euler's great, and big, books from these years, *Mechanica*, *Scientia navalis* and *Tentamen novae theoriae musicae* ..., receive special attention. And we obtain first-hand knowledge, directly from Euler's pen, about his first marriage and start of a family and the circumstances under which he left Petersburg for Berlin.

Chapter 3 deals with Euler's role as the Director of the Mathematical Class of the Royal Prussian Academy under its president Maupertuis. During his quarter-century in Berlin, Euler produced one compendious work after another; the biography focuses upon: *Methodus inveniendi ...* (with his 'calculus of variations'), *Neue Grundsätze der Artillerie* (ballistics), where Euler recognises Benjamin Robins' achievements, *Introductio in analysin infinitorum* (the beginnings of function theory), which preceded his fundamental works on differential calculus and on integral calculus, his largely natural philosophical *Lettres à une Princesse d'Allemagne* and his optics culminating in his *Dioptricae*. One section is devoted to Euler's interest in chess (he had been taught by a Jewish chess expert) and to his subsequent occupation with the knights move, another to the enlargement of the Euler family (he fathered thirteen children). Also discussed are his not always easy relations with King Frederick II and the circumstances of his departure from Berlin.

The fourth chapter depicts Euler's second Petersburg period, during which about 50% of his opus was made ready by him for the press. Still being "the best introduction into the realm of algebra for a 'mathematical infant' ", Euler's *Algebra* and its origination is briefly mentioned as are a few of his astronomical accomplishments, including his lunar theory that earned him a prize from the Royal Society. Perhaps little known are the somewhat dramatic events surrounding the aging Euler's second marriage, which are narrated in one section by Gleb Mikhailov. The chapter concludes with Euler's rather well-documented last days of his life. In an epilogue, impressive statistics of Euler's enormous productivity, compiled by

Adolf Yushkevich, is displayed; it has been rated as not ranking behind that of Voltaire, Goethe, Leibniz and Telemann.

Fellmann, in the prologue to the book, attributes Euler's phenomenal productivity to three factors: his possibly unique memory (he still knew protocols of the Academy meetings by heart 10 years), his ability to concentrate (creating his immortal works with 'a child on his knees and a cat on his back'), and simply his routine of steady and quiet work. What makes this brief biography so readable and likeable is that at no point does it degenerate into a hagiography, which in the case of Euler is quite a feat. As the author uses original sources, Euler's correspondence in particular, with careful consideration, we gain a lively impression of Euler's character, activities and ways of thinking. Moreover, certain myths that have developed over the last two-and-a-half centuries, for example various stories about Euler's eye problems, are deservedly debunked.

There is considerable demand for a definitive work biography of Euler. Yet such an undertaking would, as the author muses, be synonymous with preparing a universal history of the mathematical sciences in the entire 18th century. One may therefore suspect that some time will pass for this to occur. Of course, everybody engaging in serious Euler research must go to the sources. The huge edition *Leonhardi Euleri Opera Omnia*, begun in 1911, is now, after a century of compilation, nearing its completion. Meanwhile, for further reading, the present book offers an excellent bibliography.

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Colossus: The Secrets of Bletchley Park's Code-breaking Computers

Jack Copeland
OUP, 2006, ISBN 978-0192840554

Colossus: Bletchley Park's Greatest Secret

Paul Gannon
Atlantic, London, 2006, ISBN 978-1843543305

The two books being reviewed have considerable merit. Although both were published 61 years after the end of WW2, they are equal first in presenting reasonably complete accounts of the determination of the structure of the Lorenz SZ40 Geheimschreiber encrypted teleprinter that was used for high level German communications from 1941 to 1945. A contemporary report (1 December 1944) to the American Army cryptologic agency summarised the situation: 'Daily solutions to Fish (the SZ40) messages at GC&CS (the British cryptologic agency) reflect a background of British mathematical genius, superb engineering ability and solid common sense'.

The Colossus and associated machinery should not be confused with the bombe electro-mechanical devices used to handle Enigma traffic. In fact the work on the Enigma was Bletchley Park's second biggest secret and is not covered in either book. Credit for earlier work (1932–1939) on Enigma must be awarded to three mathematicians employed by the Polish Security Bureau. Arguably the third biggest secret consisted of the insecurities detected in the JN-25A cipher used as a general operational communications system by the Japanese Navy in 1939–1940. Its successors JN-25B, JN-25C, etc, maintained the principal weakness, but that is another story.

To return to Fish, the reconstruction of the device was due to Bill Tutte, a chemistry undergraduate working at Bletchley Park in 1942. He used some work by the senior cryptologist John Tiltman — who played a major role in the work on JN-25A — and Gerry Morgan, head of the Bletchley Research Section. Tutte's account 'FISH and I' (1998) is available in at least one book as well as various web sites. Both books give other versions of it. The Report of the Director of the GC&CS for 1942 describes 'the elucidation and reconstruction of the German teleprinter from scratch' as the 'most sensational feat' of the Bletchley Research Section.

The SZ40 was known to use the standard Baudot code to represent letters by quintuples of what we would now call binary digits. One of the 32 quintuples was used for 'shift to digits mode' and another for 'reverse the shift'. The machine encrypted by generating a long apparently random sequence of quintuples that was added (mod 2 vector addition) to the message sequence. The other side committed the gross error of sending two virtually identical messages with the same starting point in the apparently random sequence of additives. This gave Bletchley Park around 4,500 consecutive additives. Tutte eventually worked out that one component in the construction of the first element of the quintuples was periodic of period 41. Similar components with periodicities 31, 29, 26 and 23 were then found for the second, third, fourth and fifth elements. Even though the complementary components were appreciably more complicated they could be and were understood. And so the SZ40 stood revealed.

The reconstruction can be sensibly compared only with the analogous achievement (1940) against the Japanese diplomatic cipher machine now generally called 'Purple' after the colour used for folders containing files on it. Here the credit goes to William Friedman, Frank Rowlett and others of the US Army cryptologic unit. Some aspects of the Polish work on Enigma may well be close to the level of the SZ40 and Purple breaks.

The early cryptanalytical work on the SZ40 had been assisted by the practice of transmitting 'indicators' — instructions specifying the initial setting of the machine — with each message. This practice was discontinued in 1943 with indicators being sent by courier in sealed parcels in advance instead. Some special tricks had to be invented to exploit statistical features of the plain text of the traffic. Jack Good, the resident statistician, played a key role. Given this, long messages could be attacked, but the process needed massive calculation. And here Max Newman came in. He was involved in designing first the non-electronic 'Robinson' devices and then the Colossus. Various others deserve great credit for the 'superb engineering', particularly the communications engineer Tom Flowers. Colossus was at least partly electronic and so technology had passed into the computer era.

Thus General Eisenhower was able to launch the invasion of France in June 1944 confident that the other side expected it would happen near Calais rather than in Normandy. Systematic deception about the location was not enough: there had to be knowledge that the deception had worked.

The handling of some later JN-25 cipher systems and certain others did produce some problems similar to those involved in handling SZ40 output. However the calculations involved were decimal rather than binary. This was just too difficult to process electronically in 1944–1945.

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