Maths matters

Opportunities for the Mathematical Sciences?

Frank de Hoog*

I've always found it easy to agree to deliver in the distant future. Sadly though, the distant future has a habit of maturing into a pressing deadline and that has certainly been the case for this piece.

The writing instructions were to voice an opinion, preferably one that is provocative, which will stimulate debate. While it's flattering to be asked about one's opinion and it's easy to be provocative, I found the requirement to be stimulating rather more difficult and therefore turned to the past 'Maths Matters' columns in the *Gazette* for inspiration.

However, I found that these articles, with a few notable exceptions, paint a rather gloomy picture about the future prospects for our profession. Issues such as declining student numbers and funding within universities are clearly extremely serious and are already impacting on our ability to recruit quality mathematical scientists. Whilst the recent increases for funding of the mathematical sciences announced in the budget will help considerably, rebuilding these activities will take place from quite a low base in many universities. In addition, the imminent introduction of measures to quantify productivity will no doubt include citation rates and impact factors, which are both low for mathematics when compared to other disciplines.

Nevertheless, I believe that the long-term potential for the mathematical sciences is extremely high. The challenge for the profession will be to successfully exploit this potential. That is the subject of this 'Maths Matters'.

Biases

Before continuing, I need to declare some biases and introduce some nomenclature. In terms of biases, let me give a bit of background. My entire career has been concerned with the application of the mathematical sciences. Initially this was in the analysis and development of algorithms but now includes industrial process modelling in the minerals and manufacturing sectors, biological modelling and financial mathematics. My biases then are firmly at the applied end of the scale. In terms of nomenclature, the boundaries between the various disciplines such as applied and pure mathematics, statistics and operations research are often quite blurred when dealing with applications, so I'll use the term 'mathematics' in a generic sense to include all of the various disciplines that make up the mathematical sciences.

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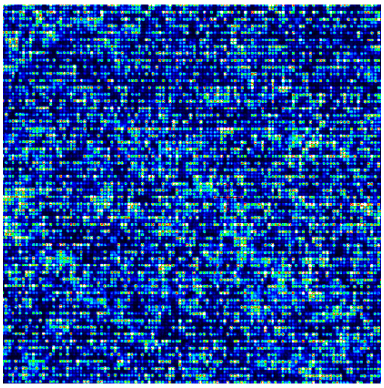
Potential

Whether we like it or not, the way we do science is changing in a number of ways. Improvements in instrumentation, data management, robotics, and communication technologies have resulted in huge productivity gains in the experimental sciences. In molecular biology, for example, the cost of analysing genomic data is predicted to exceed the cost of producing it within the next year or two. According to Szalay and Gray [8] this 'data explosion' will continue for some time. Specifically:

> Data volumes are doubling every year in most areas of modern science and the analysis is becoming more and more complex, ... Many predict dramatic changes to the way science is done, and suspect that few traditional processes will survive in their current form by 2020.



Statistical analysis of gene expression data generated by microarrays has identified biomarkers associated with more aggressive brain tumours (copyright CSIRO Australia)

Theoretical progress has been equally impressive. Many processes are now understood to the point where the underlying theory has been captured in software which is then used as a predictive tool. Such software tools are now common for physical or chemical processes such as stress analysis and reaction kinetics. Another example that we now take for granted is software for weather prediction. There are similar trends in the environmental and biological sciences where there is an increasing reliance on 'in silico' experimentation. A result of this is that future infrastructure planning for research organisations involves fewer wet labs, research stations and other experimental facilities and more information and communication related infrastructure.

These trends will impact substantially on the skills required to participate effectively in science. The 2020 Science Group [2], for example, believes that mathematical and computer sciences need to be completely integrated into science. They assert:

> Scientists will need to be completely computationally and mathematically literate, and by 2020, it will simply not be possible to do science without such literacy.

The final change I want to highlight is that scientific institutions are increasingly targeting investigations that deliver high impact through understanding at a system, rather than a component level. This invariably involves multi-disciplinary teams, which are often quite large, that work across disciplines and need to incorporate enabling technologies such as the mathematical sciences. The reasons for this are summarised succinctly by Donald J. Lewis [6], a former Director of the Division of Mathematical Science at the National Science Foundation (US), who states that:

> Today's challenges faced by science and engineering are so complex that they can only be solved through the help and participation of mathematical scientists. All three approaches to science, observation and experiment, theory, and modeling are needed to understand the complex phenomena investigated today by scientists and engineers, and each approach requires the mathematical sciences.

The changes to the way that science is being done, as described above, all indicate that there will be a sustained increase in the application of the mathematical sciences. Research institutions such as CSIRO (the Commonwealth Scientific and Industrial Research Organisation) have recognised this and have been actively seeking to increase their capability in these and related disciplines. Our experience in recruitment at CSIRO shows that career options for quality researchers in a number of areas of the mathematical sciences are substantial. Furthermore, history suggests that new applications will pose new theoretical questions, thereby reinvigorating the discipline. All of these suggest to me that the long-term potential for the mathematical sciences is extremely high.

Challenges

While I believe that the potential for the mathematical sciences is high, exploiting the potential presents a number of challenges. These are summarised in the following dot points and addressed in greater detail below.

- Mathematical scientists need to become better at expressing their 'value proposition'.
- The changes to the way that science is done will require much more domain knowledge and therefore incur a much greater initial cost for participation by mathematical scientists.
- Science and engineering is a competitive business and there are many scientists and engineers who will aggressively pursue new opportunities in the mathematical sciences.

Unless we effectively address these challenges, it is unlikely that those who consider themselves to be mathematicians will be able to realise the potential.

The easiest and perhaps most compelling argument for support of the mathematical sciences is the linkage with applications in the real world. This was certainly argued strongly, apparently with good effect, in the recent review of mathematical sciences research in Australia and at the forum at the Shine Dome in February. However the reality is more complicated.

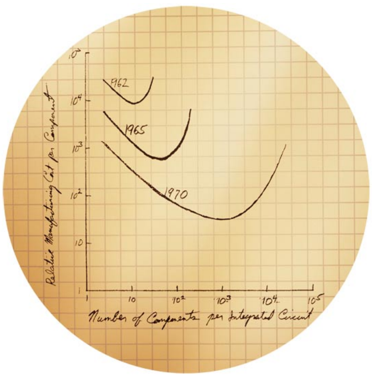
My suspicion is that many, perhaps the majority, of mathematicians have little experience in applying mathematics and consequently have unrealistic views about the paths to adoption. There is some support for this in the recent review of statistics [7], specifically the statement:

> Many teachers of mathematics in schools and universities clearly have had little experience of, knowledge of, or direct contact with, the vast range of applications of quantitative methods that have opened up in recent years in government and industry and in other academic disciplines.

There is further support in Larry Forbe's [3] experience that there is resistance to the idea that research in pure mathematics is necessarily a supporting role to the application of the mathematical sciences to real-world problems.

This should not come as a surprise. The fact is that applications are not the only drivers for developments in mathematics. Indeed, the links with applications for much of mathematics is tenuous or non-existent, as is to be expected from any mature discipline that deals with abstraction. Arguably, most of the drivers come from the discipline itself and many 'dialects' have been developed that can only be understood by the cognoscente. Whilst these specialised languages are absolutely essential to make progress in the various sub-disciplines of the mathematical sciences, they are also a barrier to communication. It is therefore very difficult for any individual to develop an in-depth understanding of the mathematical sciences. Consequently, many known results that are required for the solution of real-world problems are reinvented.

I am not trying to argue here that abstract mathematical results do not play a crucial role in the application of mathematics. There are many examples where they have played a crucial role. Nevertheless, most pieces of abstract mathematics will not be useful for solving applied problems and arguments that the results will be crucial in fifty or even a hundred years are at best optimistic and, at worst, dilute our credibility. It also dilutes the contributions that are not associated with applications.



Gordon Moore's original graph from 1965 (copyright Intel Corporation)

I am also not suggesting that mathematics does not play an important supporting role. Just one example of the importance of this role is the development and analysis of algorithms, much of which is based on earlier fundamental results in functional analysis. This has had at least as much influence on what is computationally feasible as Moore's law. What I am saying is that we need to be more discerning when articulating the value of the mathematical sciences to applications in order to increase our credibility. One size does not fit all!

As an aside, while I am enthusiastic about 'the mysterious process between theory and applications' [1], [3], [9], I don't believe that the only, or even primary, role of fundamental research in pure mathematics is to support the application of mathematics. Such a role is far too restrictive and, as far as I'm aware, there are not a lot of mathematicians who spend their time sitting by the phone in the hope that I, or someone else with a practical problem, will call about an application that will be solved by the theory they have developed. A lot of mathematics is done that is not intended to support anyone. I'm not suggesting that Hardy's toast — 'To Pure Mathematics, may it never be of use to anyone' — resonates with a large section of the mathematical community. Most mathematicians would be very pleased to find their work applied. It's simply that the drivers for research in mathematics are much broader than the applications of mathematics. We need to be much more assertive and proactive in articulating appropriate value propositions for all research in the mathematical sciences which should then be judged on its own merits.

I believe that changes in the way that science is done will also change the way that mathematics is applied. Much of my own work has been done on manufacturing processes that have been around for a long time. These are generally quite robust processes as this was all that manufacturing facilities a century or more ago could cope with. This robustness generally means that the process of interest is, at worst, only weakly coupled to its environment and that a reductionist approach, using simple mathematical models, will work well. Examples include rolling processes, painting, crystal growth, lens design and so on.

It's easy to participate in these problems because the models are relatively simple and well developed, and the context is easy to understand. Some domain knowledge is required but, by collaborating with a domain expert, it's possible to start making substantial contributions in a few months. However, the contributions that the mathematical sciences make to investigations that seek to deliver high impact through understanding at a system level are quite different. As mentioned previously, these investigations are tackled by multi-disciplinary teams that work across disciplines, and participation is usually as a team member rather than as an individual. Furthermore, the components of such systems are often strongly coupled and the analysis of individual components does not provide insight into the behaviour of the system. This means that the domain knowledge required to participate effectively can be very considerable.

Mathematics is ubiquitous in science and engineering. Everything that needs to be quantified needs mathematics at some level of sophistication and only a tiny proportion of all the application of mathematics is performed by those who consider themselves to be a mathematician. This is true even if we limit ourselves to sophisticated applications of mathematics. Much of science and engineering is about the application of mathematics and there are many scientists and engineers who will aggressively pursue new opportunities and directions.

The mathematical sciences have a history of being late adopters. Friedman [4] for example, argues that there are a number of useful methodologies that had seminal beginnings in Statistics which were, for the most part, subsequently ignored by statisticians. These include pattern recognition, neural networks, machine learning, graphical models (Bayes nets), chemometrics and data visualisation. Arguably, one might add data mining to the list. In addition, our infrastructure for dissemination of information is cumbersome. As a rule, the time required to publish a research finding in the mathematical sciences is at least a year, the impact factors of our journals are low and our web presence is usually minimal.

Competitive advantage

Despite the fact that applications are not the only drivers of innovation in the mathematical sciences, there is little doubt that they have been an important driver.

Larry Forbes [3] puts it well in a previous 'Maths Matters' column: 'Mathematics is the language of technology, and during the course of its history, much of it was invented precisely for that purpose'. Furthermore, this language can be stunningly powerful as exemplified by the phrase 'the unreasonable effectiveness of mathematics'. This phrase was originally coined by Wigner [10] and later expanded upon by Hamming [5]. A recurring theme here is that mathematics is precisely the right language for applications as evidenced by the seemingly irrational fact that the same mathematics turns up in many completely different applications.

Another concept that was considered to be fundamental was the notion of invariance. I am constantly amazed at how much insight can be gained simply by abstracting the key elements of a practical problem and then performing a dimensional analysis. Sometimes, this is all that is required! Mathematical scientists have the advantage of an overview of the mathematical sciences that enable them to exploit 'the unreasonable effectiveness of mathematics' and thereby provide the understanding and insight that is the hallmark of the application of mathematics at its best.

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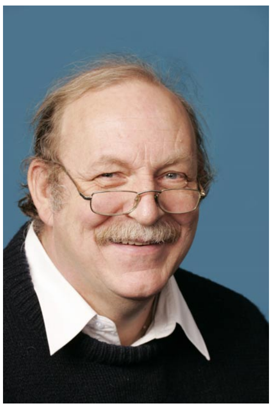

Frank de Hoog is a Mathematical Scientist with over 30 years research and teaching experience. His role as Research Director in CSIRO Mathematical and Information Sciences is to help set the directions and implement frontier and strategic research.

Frank has lectured at UCLA and ANU on Computational and Applied Mathematics and has made a number of research contributions in these areas. Specifically, he has contributed to topics in the numerical solution of differential and integral equations, numerical transform techniques, computational linear algebra, solid mechanics, vibration of structures, stress analysis, heat and mass transfer and rheology. These have been documented in over 100 refereed journal papers and conference proceedings.

Since joining CSIRO in 1977, Frank has also worked on applying mathematics to industrial problems. Projects on which he has worked include modelling of blast furnaces, gravity separation, alumina precipitation, mill modelling, roll coating, structural vibrations, coil handling and financial risk. As part of this work, he has undertaken a number of secondments to industry.