Exactly one hundred nontrivial composites

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Fine advice John Conway once gave me on instant factorisation of three-digit numbers should be better known. The difficulty is that doing it seems to require one to memorise the 168 primes less than a thousand. “Surely you’re not saying that’s a problem, Alf?” John said askance to me. However, not to worry. There are only, and indeed exactly, one hundred (100) *nontrivial composites* less than a thousand.

Nontrivial composites

Usually one thinks of positive integers as being one of 1, prime, or composite. John recommends the more refined partition: 1, prime, trivially composite, or nontrivially composite.

Here, a composite integer is *trivially* composite if it is divisible by 2, 3, or 5.

Exercises

(a) List the nontrivial composites less than a thousand. Learn them. Annoy your friends by factorising every three digit number that comes your way.

(b) Let S denote the set $\{1, 2, 3, \ldots, 100\}$. As usual $|S|$ denotes $\#S$, the number of elements in the set (in this case $\#S = 100$, of course). Let

$$S_n = \{a \in S \colon n \vdash a \ (n \text{ divides } a)\}.$$

Compute

$$|S| - (|S_2| + |S_3| + |S_5|) + (|S_6| + |S_{15}| + |S_{10}|) - |S_{30}|\,,$$

and, noting that there are just three nontrivial composites, namely $49 = 7^2$, $77 = 7 \cdot 11$, and $91 = 7 \cdot 13$, less than one hundred, find the number of primes less than 100.

(c) Similarly, now take $S = \{1, 2, 3, \ldots, 1000\}$. Given that there are exactly one hundred nontrivial composites less than a thousand, find the number of primes less than 1000.

John Conway was saddened to find when I last met him (Calgary, June 2006, at a meeting celebrating the 90th birthday of Richard K. Guy) that I had not recently redone exercise (a) and could not instantly report $871 = 13 \cdot 67$. He points out that exceeding our grasp by aiming to factorise not just three digit but four digit integers is the best way to internalise the first hundred nontrivial composites; the three bonus factorisations below promote that cause.

The First Hundred Nontrivial Composites

$49 = 7^2$	$301 = 7 \times 43$	$497 = 7 \times 71$	$679 = 7 \times 97$	$841 = 29^2$
$77 = 7 \times 11$	$319 = 11 \times 29$	$511 = 7 \times 73$	$689 = 13 \times 53$	$847 = 7 \times 11^2$
$91 = 7 \times 13$	$323 = 17 \times 19$	$517 = 11 \times 47$	$697 = 17 \times 41$	$851 = 23 \times 37$
$119 = 7 \times 17$	$329 = 7 \times 47$	$527 = 17 \times 31$	$703 = 19 \times 37$	$869 = 11 \times 79$
$121 = 11^2$	$341 = 11 \times 31$	$529 = 23^2$	$707 = 7 \times 101$	$871 = 13 \times 67$
$133 = 7 \times 19$	$343 = 7^3$	$533 = 13 \times 41$	$713 = 23 \times 31$	$889 = 7 \times 127$
$143 = 11 \times 13$	$361 = 19^2$	$539 = 7^2 \times 11$	$721 = 7 \times 103$	$893 = 19 \times 47$
$161 = 7 \times 23$	$371 = 7 \times 53$	$551 = 19 \times 29$	$731 = 17 \times 43$	$899 = 29 \times 31$
$169 = 13^2$	$377 = 13 \times 29$	$553 = 7 \times 79$	$737 = 11 \times 67$	$901 = 17 \times 53$
$187 = 11 \times 17$	$391 = 17 \times 23$	$559 = 13 \times 43$	$749 = 7 \times 107$	$913 = 11 \times 83$
$203 = 7 \times 29$	$403 = 13 \times 31$	$581 = 7 \times 83$	$763 = 7 \times 109$	$917 = 7 \times 131$
$209 = 11 \times 19$	$407 = 11 \times 37$	$583 = 11 \times 53$	$767 = 13 \times 59$	$923 = 13 \times 71$
$217 = 7 \times 31$	$413 = 7 \times 59$	$589 = 19 \times 31$	$779 = 19 \times 41$	$931 = 7^2 \times 19$
$221 = 13 \times 17$	$427 = 7 \times 61$	$611 = 13 \times 47$	$781 = 11 \times 71$	$943 = 23 \times 41$
$247 = 13 \times 19$	$437 = 19 \times 23$	$623 = 7 \times 89$	$791 = 7 \times 113$	$949 = 13 \times 73$
$253 = 11 \times 23$	$451 = 11 \times 41$	$629 = 17 \times 37$	$793 = 13 \times 61$	$959 = 7 \times 137$
$259 = 7 \times 37$	$469 = 7 \times 67$	$637 = 7^2 \times 13$	$799 = 17 \times 47$	$961 = 31^2$
$287 = 7 \times 41$	$473 = 11 \times 43$	$649 = 11 \times 59$	$803 = 11 \times 73$	$973 = 7 \times 139$
$289 = 17^2$	$481 = 13 \times 37$	$667 = 23 \times 29$	$817 = 19 \times 43$	$979 = 11 \times 89$
$299 = 13 \times 23$	$493 = 17 \times 29$	$671 = 11 \times 61$	$833 = 7^2 \times 17$	$989 = 23 \times 43$

$1001 = 7 \times 11 \times 13$	$1003 = 17 \times 59$	$1007 = 19 \times 53$

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