The Australian Mathematical Society

Gazette

Jan de Gier and S. Ole Warnaar (Editors)  E-mail: gazette@austms.org.au
Department of Mathematics and Statistics    Tel: +61-(0)3-834-49682
The University of Melbourne              Fax: +61-(0)3-834-44599
VIC 3010
Australia

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The Gazette seeks to publish items of the following types.

• Mathematical articles of general interest, particularly historical and survey articles
• Reviews of books, particularly by Australian authors, or books of wide interest
• Classroom notes on presenting mathematics in an elegant way
• Items relevant to mathematics education
• Letters on relevant topical issues
• Information on conferences, particularly those held in Australasia and the region
• Information on recent major mathematical achievements
• Reports on the business and activities of the Society
• Staff changes and visitors in mathematics departments
• News of members of the Australian Mathematical Society

Local correspondents are asked to submit news items and act as local Society representatives. Material for publication and editorial correspondence should be submitted to the editors.

Notes for contributors

Contributions to the Gazette should be sent to gazette@austms.org.au. Submissions should be fairly short, easily read and of interest to many of our readers. Technical articles are refereed. Authors are encouraged to typeset technical articles in \LaTeX, \LaTeXe, or variants. A special Gazette classfile gazette.cls can be downloaded from http://www.austms.org.au/Gazette or may be obtained by email from the editors. In exceptional cases other formats may be accepted.

Other contributions should preferably also be typeset in \LaTeXe or variants, but may also be submitted in other editable electronic formats such as plain text or Word documents.
294 Editorial

295 President’s column
   P. Hall

297 Math matters
   I. Roberts

306 Mathellaneous
   N. Do

313 The style files
   T. Roberts

315 Bilateral average intelligence increase through tourism
   A. Conway

316 2006 ICE-EM Graduate School
   H. Grey, A. Garnadi, C. Goddard, L. Gow and K. Lock

321 Obituary Alex Rubinov
   S. Morris

323 50th AustMS meeting
   W. Chen

326 Exactly one hundred nontrivial composites
   A. van der Poorten

328 AMSI News
   P. Broadbridge

330 Quadrance graphs
   L.A. Vinh

333 Trisecting the equilateral triangle with rational trigonometry
   N.J. Wildberger

338 A counting function for the sequence of perfect powers
   M.A. Nyblom

344 Book reviews

349 News

354 AustMS
After three years and fifteen issues of the Gazette our term as editors has come to an end, and the new editorial team of Birgit Loch and Rachel Thomas, assisted by Eileen Dallwitz, will take over at the end of this year. We are sure the new editors will enjoy their job as much as we have and hope they can help to raise the Gazette’s standard and reputation further.

The Gazette (albeit much more interesting) is no Woman’s Weekly or Men’s Health, written by one group of people for consumption by another group. Rather the Gazette is for its users by its users, — or so it should be — and therefore only as relevant and interesting as we, members of the AustMS, make it. As resigning editors we appeal to all AustMS members to continue to support the Gazette by making interesting submissions — technical and otherwise — and by acting as referees, reviewers or local correspondents. Together with its technical journals the Gazette is the face of the Australian Mathematical Society and, more broadly, Australian mathematics. We all share a responsibility to make this face respectable and presentable.

No one has done more in this respect than Norman Do in his Mathellaneous column. This issue’s contribution will be Norman’s last and we wish to thank him for his tireless efforts in writing such wonderful columns. We also wish to thank May Truong, Administrative Officer of the AustMS, for her help in dealing with numerous behind-the-scene issues over the past three years. We are grateful to our regular columnists, local correspondents and book reviewers, as well as to John Bamberg, Ray Booth, Nathan Clisby, Neville de Mestre, Mike Eastwood, Peter Forrester, William Hart, Mike Hirschhorn, Le Anh Vinh, Vicky Mak, Brendan McKay, Lawrence Reeves, Ken Sharpe, John Stillwell, Rudolf Vyborny, Ray Watson, Norman Wildberger, Panlop Zeephongsekul and Sanming Zhou for their help and contributions in 2006.

Finally, we could not have done without our assistant editors Maaike Wienk and Celestien Notschaele. Maaike and Celestien not only took care of us, but handled most of the production and daily running of the Gazette.

Jan de Gier and Ole Warnaar
The past and the future

This is my first column as the Society’s President, and I want to begin by paying tribute to my predecessor, Michael Cowling, for the wisdom and energy that he has used to guide the Society during the last two years. I’ve learned a great deal at Michael’s hand during my all-too-brief period as Incoming President. If I can supply a reasonable approximation to his skill during my own term, I shall feel pleased.

With characteristic foresight, Michael began a number of important initiatives during his two-year term. Where these have not yet come to fruition, I’ll continue them. I’ll discuss one of them below — the combined issue of the health of our journals and the Society’s income. I’ll take up other matters, and new initiatives, in future columns.

I wish also to pay tribute to the services that Jan Thomas made to Society, before stepping down in September as our Executive Officer. She worked tirelessly to publicise our achievements, and to lobby both federal and state governments for support for mathematics. The Australian Mathematical Sciences Institute, and the International Centre of Excellence for Education in Mathematics, are just two of the major initiatives that she carried to fruition.

As we embark on our 51st year we can look back on many achievements, and glories, in Australian mathematics. These are discussed in depth in Graham Cohen’s remarkable history, Counting Australia In, published to coincide with the Society’s first half-century. I urge all members of the Society to purchase a copy! An order form can be found at the web page: http://www.austms.org.au/Counting.

Graham’s astonishingly detailed book is still warm from the presses. Indeed, our most recent achievement, and our grandest — Terry Tao’s Fields Medal, awarded in August — is included. It is particularly appropriate that Terry’s extraordinary achievements were honoured on the cusp of our semi-centennial; they show what we are capable of, and suggest directions for our future.

However, not all the challenges that lie ahead are so uplifting. Across the country, many of our colleagues are facing very difficult working conditions, caused by poor funding environments for the mathematical sciences. Some of our colleagues are uncertain about their futures, and this concerns me greatly. The ARC research review of the mathematical sciences will be launched by the end of the year, and at that point we’ll have to redouble our efforts to boost the fortunes of mathematics in Australia. There will be more about this in later columns, I’m sure.

Our journals, and the Society’s income

At the Annual General Meeting in Sydney in September our Treasurer reported on the challenges facing us in connection with our journals. These were once significant sources of income for the Society, but in recent times the sign of that income has reversed. The Treasurer noted that, during the last year, some of the drains on our resources could fairly be viewed as unique. However, in the difficult times that lie ahead we can expect further financial obstacles.

In scholarly terms, too, our journals are losing ground. To address these issues the
Society has put together a “Publications Committee”, or working party, chaired by Michael Cowling. In the first instance the committee will consider whether it is in the Society’s interests to use a not-for-profit publisher, for example one of the UK university presses or a mathematical society much larger than our own, to help us reduce production costs and get our journals into more libraries than at present. It is expected that the committee will make recommendations by February next year. I anticipate that the committee will then turn its mind to other issues that impact on the strength and vitality of our publishing enterprises, and will become something of a “think tank” for this side of our activities.

Indeed, the challenges currently faced by our journals are certain to be joined by others in the years ahead. For example, we are finding it increasingly difficult to locate the editors and other assistants that the Society needs to keep its publishing activities afloat. I’m sure the reasons are obvious to all of us: Especially in our universities, our members are facing unprecedented demands on their time and energy. They are experiencing increasing difficulty finding opportunities, in their hectic working schedules, for providing general support to the discipline. The severe pressures on Australian universities and government laboratories today are impacting not just on us as individuals, but on the professional societies that play a vital role in providing peer support for the nation’s scientists and scholars.

As a result, the time may come when we’ll have to financially assist our editors and their assistants for work which, in the past, we all took for granted as part of the many volunteer efforts that mathematicians make for their profession. Of course, we shan’t pay wages to our editors, but we may have to provide money for relief from teaching or other duties. For all these reasons, I’d like to mention a suggestion that was raised at the AGM in September: that we consider an above-inflation increase in the Society’s annual subscription. How necessary, and how high, any increase might be will depend to some extent on our success in keeping the costs of our journals under control, and increasing their profiles.
By the term *mathematics* I mean all of the various branches of human thought based on the rules of formal (first order) logic and reasoning in an abstract setting. This includes major components of both philosophy and logic. Mathematics is informed by, and informs, disciplines based upon experimental sciences and other aspects of human endeavour. Mathematics is a science, a philosophy and an art, and it uses a very specialised language.

1 Mathematics – a discipline of declining importance

The following are held to be self-evident.

There is no shortage of mathematicians in Australia. The law of the market place (profit-oriented economics) and politics (maintaining and improving our social and economic well-being) provides a clear proof of the validity of this statement. It is a proof which mathematicians may not accept, but we are constrained by the limitations of formal logic and axiomatic systems, so the proof falls outside our restrictive set of parameters.

Far fewer mathematicians are needed in our society if we consider the socio-economic reality. The number of applicants for a mathematics lectureship (level B or C) at Charles Darwin University (CDU) attracted over 60 applicants in 2004. Application numbers in excess of 100 were reported for similar level positions at the University of Melbourne at the same time.

There is no shortage of teachers of mathematics. This was reportedly stated by a state Minister for Education (identity purposely not included). The logic is simple but clearly appropriate. Every mathematics class has a teacher in charge of the class. As the elected member chosen to lead our state education system, we must rely upon the skills and knowledge of that Minister and his advisors.

There is no reason to improve the funding ratios for mathematics in universities. At a time when mathematics faculties are continuing to be trimmed, or staff re-allocated to serviced areas it is a clear economic matter that funding should not be artificially pumped into supporting a declining industry.

There is no need for Government or industry to increase funding levels for mathematical research. It is clear from advertisements in various media outlets that industry needs useful people, ones who can work on a team project to solve practical problems in reasonable time for near-term financial advantage. This is what graduates in more vocational disciplines are trained to do – engineers solve practical problems in practical ways.

Mathematicians are constrained by their lack of practical skills and their desire to find optimal solutions and to generalise a specific problem. Industry needs practical outputs not theoretical niceties. Most of us will have published esoteric papers which take longer to write than it takes to build a house. Which is more valuable? Would we rather have a house or ten pages of abstraction read by two referees and maybe some current research students?

All mathematics conference papers should be refereed. In this way mathematics can partially respond to our modern
economic system even if it is totally unsuitable for our discipline. We can attract a greater share of resources through the rewards of having refereed papers. Our publication rates will increase, and then we can be seen to be more productive. We should follow the lead of our progressive disciplines like education and information technology, or pragmatic disciplines such as engineering.

Mathematicians should be subject to professional registration and regulation like the progressive and expanding disciplines of law, accounting, education, nursing, and engineering. Clearly our standards will improve and our economic value will increase.

Mathematics should adopt outcome-based learning principles and a postmodernist aesthetic. For too long we have poured mathematical knowledge into the brains of students. For too long we have ignored the student’s need for self-determination and self-expression. Each individual needs to find their own time, place and methods of absorbing the components of the discipline without the artificial methods and the induced stress of learning formulas and sitting examinations. Self-efficacy is much more important for success.

Because of improvements in technology, mathematics is less important than it was before computers. Mathematicians need to move into the modern world where calculators and computers can do all of the necessary calculations. So much time is wasted teaching problem solving, formulas and equations when methods can be found on the web and there is software to work out the answers.

Mathematics has been made much too complex and takes up too much time in the school curriculum. Modern education is based upon good classroom management, which includes individual learning programs and extensive report writing. Text books are insensitive to the needs of individuals, so it is imperative that every teacher develops an individual program for each student, that reports emphasise the positive aspects of a student’s learning outcomes, and feedback should only be expressed in terms of outcomes relative to the student’s own educational aspirations.

The training of mathematically competent people is archaic. It takes too much time and individual effort. Our ever-changing society would benefit from much shorter training times so that mathematicians become productive sooner, more flexible and are able to be produced on demand.

Mathematics should be optional at school. A lot of kids don’t like it and many teachers don’t like teaching it. Besides it can be learnt as needed when studying other subjects or in bridging courses for TAFE or university. Just-in-time learning is much more efficient, and students are more motivated because it is more relevant.

2 The Australian reality

Forty percent of next year’s first-year engineering students will need to do a six months remedial maths course ([1])

How is it that we have gone from teaching Latin in year 12 to teaching remedial English in first year of university? ([2])

The issues that we face in mathematics are common to all core disciplines. Processes and training have replaced education. Long-standing government policies and funding models are continuing to undermine education in Australia.

Recent debate concerning the control of education in Australia is pertinent but still seems to miss the point, as the debate appears to be more about control than about fixing a major problem. An indication of real desire to address the problem could begin with appropriate funding to recognise and support the rebuilding of core disciplines at universities, which
the Federal Government effectively controls through funding and legislation.

The current EFTSL funding model is destroying staff and destroying core disciplines, and every time we lose a core discipline from one university it affects other core disciplines and other universities.

It also follows that there has been a lowering of demand for fundamental education, as the mass of the population has not been exposed to its possibilities. This seems to be the situation for a whole generation and its harmful effects are showing. This includes the issue of many students in regional or “unlucky” areas often subject to significant disadvantage in educational opportunities.

A common reply to this sentiment is that we have a market economy and it is driven by demand. I recall a federal politician extolling the positive contribution of a fast-food chain in Australia. They do provide clean efficient service and employ lots of young people at low rates of pay and the demand for fast food is enormous. Now we have an epidemic of obese people. What is the educational equivalent of being obese, through being fed on a diet of poor fundamental education?

At the moment we sit comfortably, importing products to satisfy our insatiable demand for technical gadgets, all paid for by the resource boom. We import manual workers, qualified tradesmen and professionals whilst many good Australian brains have been hulled into a stupor of mediocrity.

We see so many students who are cognitively still at lower secondary level, but they are now at a university completing a degree with meaningless entry and exit standards and an expectation, encouraged by the universities, to think that they should be able to pass in any subject in which they choose to enrol.

Compounding the issue is that the contemporary course of society in Australia has been to move the balance away from education towards legislation. Legislation appeals to politicians and bureaucrats. They understand it, they can partially control it, and they can build their short-term empires, but they also have the power to address the issues, if there is the will.

The will does not seem strong, and it is an inherent weakness in our Australian system. The perceived reasons for the economic success of Australia has changed over time, but there is one common thread – export our large amounts of natural resources, or more recently attract visitors, namely tourists and students. Australia will continue to thrive on a temporary basis with diggers and waiters – the diggers to mine the resources, and the waiters to serve the tourists. What do we do when the resource bubble bursts or in the next major economic downturn?

One has to hang on the slim hope that the current review of Mathematical Sciences in Australia will be more than a political process, as it cannot deal with the broader underlying issues.

3 The University reality

Mathematics cannot justify itself on immediate economic return. It justifies itself in its influence on the long-term cognitive skills of its citizens. The long-term benefits are enormous, but this has often become lost in the immediacy of short-term goals.

Mathematics and all of the core cognitive disciplines have been in decline for some time. Society needs to make major changes in its long-term expectations for the negative aspects of our education system to be reversed.

Many mathematicians in Australia work in universities. Universities are formal organisations subject to the usual influences in any society. Unfortunately the contemporary influences work strongly to undermine the role of universities, and there is no one influence that dominates. However, influential social and financial decision making involve some major players including Governments, and these can act to modify
the situation which they have helped to create.

Consider the following:

- Current EFTSL funding models have caused artificial structural inefficiencies in university education – with each discipline being forced to claim as many students as it can within its own discipline codes.
- The demand for core educational disciplines is subsequently lowered.
- School counsellors partially respond to universities and inform students that the core subjects are now less important, so why not do the easier or “sexier” subjects.
- Hence core disciplines at school become weaker, and so fewer well-educated students complete school.
- Hence universities drop entry standards.
- Hence the demand for the core disciplines becomes less, and subjects become more vocational, meaning based more on process and less on considered thought and deeper understanding.
- The next generation of teachers further the decline in the core disciplines.
- The workforce employs lots of people with “people and communication skills”, but many of them do not have deeper cognitive skills, and we are less competitive as a workforce.

We are at the stage in many universities that we have neither the time nor energy to interact with the diverse disciplines or industries which would benefit from some of our skills and knowledge. Because of the decrease in mathematics education in many disciplines, there are fewer members of disciplines or industries who can sense that some mathematical input might be relevant to their considerations. Hence our opportunities for collaboration (direct usefulness?) are decreased. Although it is not always the case, it seems to me that applied mathematics will tend to be driven by the needs of other disciplines or industry, and it is hard for mathematicians to anticipate those needs without a strong desire or guidance from those parties.

I began my tertiary teaching career in TAFE – matriculation maths entry for technical certificates, and compulsory maths for secretaries. It now appears that I will finish my career teaching bridging mathematics or year 9 mathematics to trainee schoolteachers. Luckily there have been periods of joy within the system, provided by many reasonable students whom I have had the pleasure of teaching.

I’ve never had trouble dealing with students who have difficulties – restating the same basic ideas in many different ways. However, when every basic idea needs restating then the system is moribund, and that’s close to where we are now in mathematics education. Our one sad compensation is that we are not as badly off as physics or classics, for example.

The reality of the current system:

- Academically low-level people beginning teacher training (3 × 10 is unknown to some). Year 8 basic calculations (or less) are being seen as sufficient for teacher training. Trainee secondary mathematics teachers who “don’t like surds” – their basic mathematics skills are poor. A large number of functionally innumerate people have graduated as teachers.
- Matriculation mathematics entry were in place for technical certificates 25 years ago, and now there are engineering degrees with no physics or chemistry expected for entry and lower level mathematics entry.
- Year 9 teachers of mathematics simultaneously undertaking bridging mathematics units so that they have some of the basic manipulative skills that they are teaching the next day.
- There are also the teachers with the same skill levels who don’t undertake bridging.
• Apprentices unable to do the most basic trade calculations and employers unable to rely upon school reports for an accurate indication of the achieved skills of their potential employees.
• Contractors preparing tenders who cannot calculate basic quantities (such as the amount of soil needed to fill \( x \) cylindrical holes).
• Graduate scientists with no mathematics – ecologists who have never heard of exponential population growth models.
• IT graduates who can’t program and who know none of the basic mathematical models/tools essential for intelligent computing professionals.

4 Education not Training

Education should inspire and open the minds of students beyond the simplicity of childhood and pre-abstract understanding. This is necessary to lead to an informed consideration of aspirations. Current education practice often reinforces aspirations – expect to be mediocre and we’ll make sure that you are mediocre. Education does not provide all of the answers, rather it provides some answers and poses many questions.

We have removed fundamental principles from education in language, in mathematics, in science, in art (post-modernism) and have replaced it by “self-expression”. There are few formal skills now being imparted to facilitate genuine thoughtful expression, and there is a loss of lifetime skills development to deal with the complex, rather than the procedural aspects of modern life and the modern workforce. Moreover, emotional responses are left as the only tools of analysis without the benefit of logic to solve logical dilemmas.

By its very nature, education as opposed to training, can lead to deep or unusual intellectual thoughts, both of which are difficult to measure or control. There is an increasing abundance of intrusive and unproductive measures of control, with reporting and auditing processes being imposed by a bureaucratic system onto an education system already pushed into becoming process-oriented training factories.

There is an alternative, but it is not a simple path, given our national decline in core education. Throw out competency-based learning and outcome-based learning and all of the related processes that not only trivialise but also work to destroy true education. Replace these by education based upon deep and broad understanding of fundamental principles of the diverse core disciplines needed for creative or skilled problem solving. Of course, there is the increasing problem of who is competent to teach this as people significant discipline knowledge is essential.

Unless the notion of core structural principles are reintroduced as central components across a range of core curriculum subjects, it is almost impossible for students to be motivated to learn, or able to grasp the fundamental nature of mathematics as a complex formal discipline – a discipline in which there are few shortcuts in the process of developing the mind’s ability to think and reason logically, inductively and deductively.

A comment on teaching

If foundations are not laid then it is almost impossible to build a useful structure in the future – the influence of a good or bad teacher echoes down the generations. I would not like to be a student in the current system. The chances of having a set of suitably educated teachers across the curriculum are too low, at least in the mass public education systems.

We cannot isolate teachers to blame for this. Teachers are a product of a system over which they often have little or no influence, and many of those who care leave the system in despair.

A frequent criticism is that too many people who understand mathematics are poor teachers. I can accept this. However, it is clear that a good teacher who knows
the subject will be a much better teacher of mathematics than one who has little subject knowledge. A good teacher can deconstruct mathematics into its constituent parts and reassemble it into a rich and meaningful experience, building on the natural inquisitiveness of children.

It is easy to enjoy aspects of pop culture, but there can never be a satisfactory pop mathematics. If one doesn’t understand mathematics then what is one teaching? It’s not mathematics.

It is depressing to see that natural spirit of children being diluted by trivia, and idle minds certainly contribute to inappropriate child behaviour.

It is unquestionably a difficult task to spend year after year with each new group of 15-year-olds trying to extend them beyond their hormones, so it can be partially understood how the education system has taken the easy way out. School teaching is no longer a profession, and soon university lecturing will be the same (if it isn’t already).

Current expectations suggest that teaching does not require high-level cognitive skills (and exams of course are not appropriate). Nor is an ongoing professional development based upon discipline-oriented education. Teachers are treated as process workers, managing day-to-day, often meaningless tasks, and distracted by writing copious volumes of lesson documents and reports.

I have fond memories of most of my high school teachers. Their ages ranged from 25 to 65, and each offered unique and interesting experiences when they seemed to be well informed about their disciplines. It was a natural joy and inspiration to learn, and actual learning was a common part of the process – appreciated even more with hindsight. The occasional teachers who did not have mastery of their discipline, stood out as sad and uninspiring.

I find it quite odd that at least some “Education staff” seem to think that current Education graduates are better teachers, despite the fact that current training has significant components of indoctrination into fashionable, narrow and harmful educational dogma and little content. They are trained in process rather than content.

Mathematics is not tactile, it doesn’t feel nice, and you can’t touch it. Mathematics is an artificial construct based upon abstraction. It is an edifice built on 2500 years of human endeavour, and few appreciate its secrets and beauty. By its very nature each small part may be seen as not being very practical, but as a whole it is essential in our society.

Abstraction is based solely within the brain and it takes years of concerted effort to develop a skilful appreciation of it. Many undereducated teachers have not developed the basis of abstract reasoning and thinking and so they cannot develop the basis of abstract thinking in their students.

On the other hand teachers can be overeducated. I recall a debate in the 1980s at an AustMS conference; I was younger and did not speak at the time, but I knew the proposed resolution was stupid – every mathematics teacher should have an honours degree in mathematics. We need to be more realistic.

How could one spend 30 years teaching 15 year olds the same algebra? There is a need for a balance in teacher training and regular in-service to help teachers refresh, to remain interested and to share ideas and enthusiasm. The AustMS needs to provide leadership.

A comment on university teaching

School education has seen the implementation of structure and processes which are almost meaningless in terms of core education. The same problem is now permeating universities. Many academics have lost the freshness and freedom essential for inspired teaching or research. The best teaching is
Mathematics is an art form. It is being reduced to a poor craft.

University staff are finding themselves in an increasingly frustrating battle. The pragmatic way to solve this social problem is by a significant number of old and irrelevant people retiring, allowing in a new generation who have been indoctrinated into the training industry, rather than into education. The best way to solve this problem is quite different.

Teaching mathematics
Mathematics is a way of thinking and it has precise rules of syntax and semantics. The language aspects of mathematics need to be recognised as a major separate component of learning mathematics. It is not easy and it takes time – lots of time.

Almost all aspects of mathematics are of limited importance in parts but the whole is much stronger than the parts, and the lack of recognition of this is a major flaw in our Australian education system.

Many people see the beauty of pattern and design in the world but few see it with the insight of mathematics. There are so many practical and day-to-day experiences where the mind is needed to solve problems of various complexity, many of the population do not have the reasoning skills necessary to partition problems into their constituent parts and solve them with abstract models.

In this sense each individual part of mathematics is not practical. How many people actually use a quadratic expression at any time in their adult life? On the other hand, how can people deal meaningfully with the notion of ecological models or personal finances (loans, compound interest, ...) when they do not understand the difference between polynomial and exponential growth patterns?

General comments
The language of mathematics is unique and like any highly expressive language it takes significant time to learn. This requires regular appropriate exposure over many years, slowly increasing the complexity and hence the power of the language. The language cannot be absorbed by osmosis or informal exposure. Mathematics suffers from the fact that many undereducated people treat it like other disciplines and therefore understand that it requires proportionately more time than most other disciplines. It cannot be taught in the same way as other disciplines.

There are many people in our society who are naturally gifted teachers, and I am certain that we have lost a large number of them to other more fulfilling jobs. A gifted teacher will have significant subject knowledge, but will also have a strong emotional commitment to their teaching. Entering a classroom is like going on stage, and when we leave the stage we are exhausted. The system does not support this necessary emotional response.

How many of us have been on academic boards, receiving hundreds of pages of documentation with little lead-time, and as a result no serious academic debate occurs? Instead there is a trail of paper and frustration. When do we, as mathematicians, get the opportunity to be involved in a balanced discussion on the importance of mathematics as a provider of core skills for other disciplines? Such interactions have almost disappeared in the EFTSL driven funding model of a mass “miss-education” system.

5 What we need

- Concerted in-service effort for existing teachers.
- Rewards for genuine subject skills upgrading rather than educational theory.
- Replacing many BEd programs so that they are based on the core knowledge of disciplines and their ethos. Education is based upon knowledge. Let teachers gain sufficient discipline so that they can consider how to impart it. At the moment, the “how to” is often being
taught without the discipline knowledge in place. Also don’t forget the pragmatics – teachers are often asked to teach out of their discipline or at different levels to which they are trained – hence the discipline training must be comprehensive.

• For most of us the ARC is irrelevant. Instead provide $7000 for each active mathematician to support collaborations (travel/conferences).

• Recent comments from students at an AustMS supported workshop, Dry and Discrete, have provided an interesting insight into possible future activities by mathematicians. The workshop featured some world leaders and open problems in various aspects of discrete mathematics and postgraduate students. Student comment was made that it was very beneficial to see the “gurus” struggling with these problems and they felt the benefit of the interaction with guru or student each making valuable comment as equals. I suggest this as a positive format to be adopted more often, rather than the less interactive but useful “Look what I’ve been doing” conference paper format.

References


About this article

This article expresses some ponderings of the author in response to a request for

...the regular column “Math Matters” in which a prominent Australian mathematician voices his/her opinion. The subject of this column is entirely up to you, but we would prefer it to be somewhat provocative, stimulating a debate.

Opinions/provocations expressed are based upon 30 years involvement in (Pure) Mathematics, Mathematics Education, Mathematics Research and the perceived views of others. The author has been a student in NSW and WA and has taught/supervised at high school, TAFE, bridging, undergraduate and postgraduate level in the NT, and has been awarded various undergraduate teaching and postgraduate supervision awards. Relevant studies include a research masters in Functional Analysis, a PhD in Combinatorics and a Graduate Diploma in Education. Research/scholarship has been in pure mathematics, mathematics education, or theoretical computer science with collaborators in a range of states and countries.

Working in Darwin has provided a better than average experience of the students from various state systems for the very simple reason that Darwin is a transient city; we are constantly dealing with students from different states and having to assess and respond to their differing educational experiences. Certainly some states rank poorly when it comes to their school mathematics curriculum, and it is getting worse.

The word ‘prominent’ above deserves a bit of a chuckle, but at least I’m interested and experienced. One of the strategies to succeed in isolation is to be cheeky enough to forge relationships with prominent academics or societies, and so to experience a breadth and depth of experiences that are otherwise impossible in an isolated environment. The advantages of isolation include the ability to deal with educational issues of a much broader nature, to
write and teach a complete curriculum which need not be constrained by the curriculum – inertia obvious in some longer standing mathematics departments, and to enjoy regular relationships with people from other disciplines.

For most of my career I have been a mathematics educator who has also undertaken research, although that balance is changing by personal choice, and partly motivated by the declining circumstances in mathematics in universities.
1 Problems Pertaining to People at Parties

Suppose that you are at a party and you notice that there are three people, all of whom know each other — hardly a surprising observation, one must admit. But at another party the following night, you happen to notice that there are three people, all of whom do not know each other. Following that, you wonder whether it might always be the case that a party must include either three mutual friends or three mutual strangers. Of course, this statement would not apply to a party of one or two people, but perhaps there is a certain critical mass, so that parties with enough people do possess this property.

How many people do you need at a party to guarantee that there are three people all of whom know each other or three people all of whom do not know each other?

Of course, we can pictorially represent our party by replacing each person with a point in the plane and using a red line segment to join people who are acquainted with each other and a blue line segment to join people who are not. Thus, we find ourselves within the realm of graph theory, and our party problem can be rephrased in the following less social, though more colourful, terminology.

What is the smallest value of $N$ such that if the edges of $K_N$ are coloured red or blue, then the resulting graph must contain a red $K_3$ or a blue $K_3$?

Here, we have used the notation $K_N$ to represent the complete graph on $N$ vertices — that is, the graph with $N$ vertices and an edge between every pair of them. For example, $K_1$ represents a vertex, $K_2$ represents an edge between two vertices and $K_3$ represents a triangle. The answer to our problem, can now be stated as follows.

If the edges of $K_6$ are coloured red or blue, then the resulting graph must contain a red $K_3$ or a blue $K_3$. Furthermore, it is possible to colour the edges of $K_5$ red or blue so that the resulting graph does not contain a red $K_3$ or a blue $K_3$.

The proof of this statement is delightfully simple and elegant. Consider any vertex $V$ in the coloured $K_6$ and the five adjacent edges. By the pigeonhole principle, at least three of these edges, $VA$, $VB$, $VC$ are of the same colour and, without loss of generality, we may assume that they are red. If our graph is to avoid red triangles, then the edge $AB$ is forced to be blue. Similarly, the edges $BC$ and $CA$ are forced to be blue, thereby creating the blue triangle $ABC$. So any $K_6$ whose edges have been coloured red or blue must contain a red triangle or a blue triangle.

To complete the proof, it suffices to demonstrate a colouring of the edges of $K_5$ which is devoid of red or blue triangles — an example is pictured below\(^1\).

---

\(^1\)Due to the monochromatic nature of the hardcopy Gazette, the blue lines are dashed.
Problem: Prove that if the edges of $K_6$ are coloured red or blue, then the resulting graph must actually contain two distinct, though not necessarily disjoint, monochromatic copies of $K_3$.

2 Ramsey’s Theorem and Ramsey Theory

Of course, there is no need for us to restrict our attention to trios of friends or strangers. More generally, we can ask the following question.

How many people do you need at a party to guarantee that there are $m$ people all of whom know each other or $n$ people all of whom do not know each other?

Once again, we can state the problem in graph theoretic terms.

What is the smallest value of $N$ such that if the edges of $K_N$ are coloured red or blue, then the resulting graph must contain a red $K_m$ or a blue $K_n$?

Such problems are central to the domain of mathematics known as Ramsey theory. In keeping with the accepted notation, let us denote the answer to this problem by $R(m,n)$. Note that in asking for the smallest such value of $N$, we are presuming that there is indeed a value of $N$ in the first place. A priori, it is not at all clear that this is the case. For example, are all parties with a sufficiently large attendance guaranteed to have either a million people all of whom know each other or a million people all of whom do not know each other? That this is indeed the case is the conclusion of Ramsey’s theorem.

Ramsey’s theorem:
For every pair of positive integers $m$ and $n$, the value of $R(m,n)$ is finite. In other words, there is a positive integer $N$ such that if the edges of $K_N$ are coloured red or blue, then there exists a red $K_m$ or a blue $K_n$.

Proof. It requires only a moment’s thought to conclude that $R(2,n) = R(n,2) = n$. We will prove by induction on $m + n$ that

$$R(m,n) \leq R(m-1,n) + R(m,n-1)$$

2 Ramsey theory is named after Frank Plumpton Ramsey (1903–1930), a most remarkable man who made significant contributions not only to mathematics, but also to economics and philosophy. After learning to read German in little over a week, he was handed the task of translating the text of Wittgenstein’s Tractatus Logico Philosophicus while still at the tender age of 19. John Maynard Keynes himself encouraged Ramsey to try his hand at economics, resulting in him contributing three important papers. In a biographical article on Ramsey [3], Mellor indicates that “Ramsey’s enduring fame in mathematics…rests on a theorem he didn’t need, proved in the course of trying to do something we now know can’t be done!” Despite this, the evidence clearly indicates that Ramsey was a first-rate mathematician. However, as is so often the case in mathematics, Ramsey’s career shone brightly yet all too briefly. He died at the age of 26 after complications from an abdominal operation.
which will provide us not only with a proof that \( R(m, n) \) exists, but also with a computable upper bound for its value.

Consider a complete graph on \( R(m - 1, n) + R(m, n - 1) \) vertices and pick any vertex \( v \) from the graph. Let \( V_R \) denote the set of vertices which are connected to \( v \) by a red edge and let \( V_B \) denote the set of vertices which are connected to \( v \) by a blue edge. Then by a simple application of the pigeonhole principle, either \( |V_R| \geq R(m - 1, n) \) or \( |V_B| \geq R(m, n - 1) \) and, without loss of generality, we may assume that the former of these two inequalities is true.

Now by the very definition of \( R(m - 1, n) \), either \( V_R \) contains a blue \( K_n \) or a red \( K_{m-1} \). If the former is true, then we are done. And if the latter is true, then the complete graph formed by the red \( K_{m-1} \) and the edges connecting it to \( v \) will give a red \( K_m \). □

There are various directions in which we can hope to generalize Ramsey’s theorem. Rather than restricting ourselves to red and blue edges, we might extend our palette to include an arbitrary, though finite, number of colours. Furthermore, rather than looking for monochromatic complete graphs, we might wish to look for other particular graphs of a given colour. For example, one can ask whether there exists a number \( N \) such that if the edges of \( K_N \) are coloured red, orange, yellow, green, blue, indigo or violet, then there must exist one of the following graphs:

- a red cycle of 2006 edges;
- an orange path of 11 edges;
- a yellow complete graph on 163 vertices;
- a green graph consisting of 42 edges sharing a common vertex;
- a blue graph consisting of 239 triangles sharing a common vertex;
- an indigo complete graph on 1729 vertices; or
- a violet subgraph whose vertices are in correspondence with all of the words in the English language and whose edges join two vertices if and only if they represent two distinct words with at least one letter in common.

To simplify the situation, note that when looking for a red cycle of 2006 edges, it is entirely sufficient, though far from necessary, to guarantee the existence of a red complete graph on 2006 vertices. Similarly, when looking for a graph on \( V \) vertices of a particular colour, it suffices to show the existence of a complete graph on \( V \) vertices of that colour. Therefore, we can restrict our attention to complete graphs. So we are now left with the problem of whether Ramsey numbers exist when more than two colours are allowed. This problem is answered by the following extension of Ramsey’s theorem.

**Ramsey’s theorem (colourful version):**

For every tuple of positive integers \((m_1, m_2, \ldots, m_C)\), there is a positive integer \( R = R(m_1, m_2, \ldots, m_C) \) such that if the edges of \( K_R \) are coloured in one of the “colours” 1, 2, \ldots, \( C \), then there exists a complete subgraph on \( m_k \) vertices, all of whose edges have the colour \( k \), for some value of \( k \).

**Proof.** The \( C = 2 \) case is precisely the statement of Ramsey’s theorem given earlier. We will now prove that

\[
R(m_1, m_2, \ldots, m_C) \leq R(R(m_1, m_2), m_3, \ldots, m_C),
\]

which shows by induction that the Ramsey numbers exist for any number of colours.

Consider a complete graph on \( R(R(m_1, m_2), m_3, \ldots, m_C) \) vertices whose edges have been coloured in one of the colours 1, 2, \ldots, \( C \). Now suppose that the colours 1 and 2 correspond to red and green, respectively. Then a person who is red-green colour blind would only see \( C - 1 \) colours. By definition, the graph must have a \( K_{m_k} \) whose edges are coloured \( k \) for
Problem: Prove that at any party with nine people, there are either three people, all of whom know each other, or four people, all of whom do not know each other.

3 Calculating the Ramsey Numbers

The two colour Ramsey numbers are of central importance in Ramsey theory and we will concentrate on them for the remainder of the article. Given that the numbers \( R(m, n) \) are guaranteed to be finite by Ramsey’s theorem, the problem of calculating what the numbers actually are is entirely natural. As mentioned earlier, it is a trivial matter to prove that \( R(2, n) = R(n, 2) = n \) for all \( n \). The next case to consider is \( R(3, 3) = 6 \), which corresponds with the first party problem discussed in the article. And the problem posed at the end of the previous section asks to verify that \( R(3, 4) = R(4, 3) = 9 \). Unfortunately, despite the best efforts of mathematicians, there is no known formula for the Ramsey numbers in general. In fact, only seven other Ramsey numbers \( R(m, n) \) are known for \( m \leq n \). The following table shows these numbers, as well as the known upper and lower bounds for many of the other Ramsey numbers [4].

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So why are the Ramsey numbers so difficult to calculate? Well, suppose we decide to use a brute force approach to calculate the value of \( R(5, 5) \). As witnessed from the above table, we have the quite reasonable bounds \( 43 \leq R(5, 5) \leq 49 \). If we actually believed the answer to be 43, we might consider simply drawing all of the possible two-colourings of the complete graph on 43 vertices. Then, it would be a simple matter to examine each one to determine whether or not it contained a monochromatic copy of \( K_5 \). However, the number of edges in a graph on 43 vertices is precisely \( \binom{43}{2} = 903 \). Therefore, the number of distinct
ways to colour the edges of the graph red or blue is $2^{903} \approx 6.76 \times 10^{271}$. So even if it were possible to analyze $10^{39}$ cases per second, the time required would still be of the order of $2 \times 10^{244}$ years! Of course, it is possible to narrow down the number of cases by many orders of magnitude, but the computation is still far beyond our current technological capabilities.

To indicate the difficulty in calculating the Ramsey numbers, Paul Erdős, one of the most prolific mathematicians ever and a Ramsey theory enthusiast, used to tell the following story. He would claim that if a technologically superior race of aliens landed upon Earth and demanded the calculation of $R(5, 5)$ within a year, then our best chance for survival would be to gather together all of the mathematicians and computing power in the world to work on the problem. On the other hand, if they demanded the calculation of $R(6, 6)$, then Erdős claimed that our best chance for survival would be to gather together the world’s military power in an attempt to destroy the aliens!

Given that the Ramsey numbers are considered to be so difficult to calculate, it seems reasonable to ask whether we can at least find bounds for them. Of most interest to mathematicians is the computation of $R(n, n)$, in which case we have the following bounds.

$$2^n \leq R(n, n) \leq 2^{2n-3}$$

The upper bound can be obtained quite easily from the inequality which was proved earlier,

$$R(m, n) \leq R(m-1, n) + R(m, n-1).$$

In conjunction with the initial values $R(2, 2) = 2$ and $R(3, 3) = 6$, and the well-known recursion

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k},$$

we obtain

$$R(m, n) \leq \binom{m+n-2}{m-1}.$$ 

Now it suffices to observe that

$$R(n, n) \leq \binom{2n-2}{n-1} = \binom{2n-3}{n-2} + \binom{2n-3}{n-1} \leq 2^{2n-3}.$$ 

The lower bound is more difficult to obtain, but its proof will allow us to showcase one of the mathematical legacies of Paul Erdős — namely, the probabilistic method. It is an extremely general principle which can be stated as follows.

Suppose that the probability of an element chosen from a particular set having a certain property is less than 1. Then there must exist an element of the set with the desired property.

Simple as it seems, the addition of the probabilistic method to the combinatorialist’s arsenal has yielded many new results as well as beautiful proofs of old results. The following was included in *Proofs from the Book* [1], an approximation to The Book, where Erdős believed that God stored the perfect proofs for mathematical theorems.

**Theorem:** $R(n, n) \geq 2^n$

**Proof.** Since $R(2, 2) = 2$ and $R(3, 3) = 6$, the theorem certainly holds true for $n = 2$ and $n = 3$. We will now prove that for $n \geq 4$ and $N < 2^n$, there exists a colouring of the edges of $K_N$ red or blue which does not contain a monochromatic $K_n$. Observe that there are $2^{n^2}$ colourings in total. If we consider declaring each edge red or blue independently with probability $\frac{1}{2}$, then it is clear that any particular colouring occurs with probability $2^{-\binom{n}{2}}$.

Now given a set of vertices $V$, let $V_R$ denote the event that the edges between the vertices of $V$ are all red. Then the probability of the event $V_R$ is simply $2^{-\binom{|V|}{2}}$. Let $X_R$ denote the event that there is some complete graph on $n$ vertices which is coloured red.
Mathellaneous 311

\[ \Pr(X_R) = \Pr(\bigcup_{|V|=n} V) \leq \sum_{|V|=n} \Pr(V_R) = \binom{N}{n} 2^{-\binom{n}{2}} \]

\[ = \frac{N(N-1)(N-2)\cdots(N-n+1)}{n(n-1)(n-2)\cdots1} 2^{-\binom{n}{2}} \]

\[ \leq \frac{N^n}{n(n-1)(n-2)\cdots1} 2^{-\binom{n}{2}} \leq \frac{N^n}{2^{n-1}} 2^{-\binom{n}{2}} \]

\[ < 2^{\frac{n^2}{2} - \binom{n}{2} + 1} = 2^{1 - \frac{\binom{n}{2}}{2}} \]

\[ \leq \frac{1}{2} \]

So we have shown that for \( n \geq 4 \) and \( N < 2^{\frac{n^2}{2}} \), the probability of a red \( K_n \) is less than \( \frac{1}{2} \). By the symmetry of the problem, the probability of a blue \( K_n \) is also less than \( \frac{1}{2} \). Therefore, the probability of a monochromatic \( K_n \) is less than 1, so there must exist a colouring which contains neither a red \( K_n \) nor a blue \( K_n \).

\[ \square \]

4 Complete Disorder is Impossible

Ramsey theory is concerned with more than simply determining the Ramsey numbers. In fact, there is a myriad of interesting results which possess the same flavour as the party problems considered above. The overarching theme behind Ramsey theory is the fact that within sufficiently large mathematical systems, there must exist subsystems containing a certain degree of order. This is often succinctly described by the Ramsey theorists’ catch phrase, “Complete disorder is impossible!” We will conclude the article with a brief look at three Ramsey-type results.

**Ramsey’s Theorem — Infinite Version.** The first result is Ramsey’s theorem, as stated by Ramsey himself in his 1930 paper entitled, *On a problem of formal logic*. As witnessed by the title, he did not consider the result to be of great combinatorial importance, and it appeared only as a lemma towards what he considered a more substantial problem of formal logic. This presumably was a result of the fact that the foundations of mathematics were Ramsey’s great passion, combined with the fact that combinatorics was a far less fashionable subject at that time than it is today. Ramsey’s original result can be considered an infinite version of the theorem which is now attributed to him.

For every pair of positive integers \( C \) and \( N \), if the subsets of \( \mathbb{N} \) with \( N \) elements are coloured in \( C \) colours, then there exists an infinite subset \( X \) of \( \mathbb{N} \) such that all subsets of \( X \) with \( N \) elements are of the same colour.

**Problem:** Prove the infinite version of Ramsey’s theorem and show that the finite version of Ramsey’s theorem follows from the infinite version.

**Van der Waerden’s theorem.** In 2004, Ben Green and Terence Tao announced their celebrated result that the primes contain arbitrarily long arithmetic progressions. One of the main ingredients in their proof was Szemerédi’s theorem which states that if we take any positive fraction of the set of positive integers, a notion which can be made mathematically precise, then the resulting set must contain arbitrarily long arithmetic progressions. A precursor to Szemerédi’s theorem is the following result, proved by Van Der Waerden in 1927.
For every pair of positive integers \(C\) and \(P\), there is a positive integer \(N\) such that if the numbers from 1 up to \(N\) are coloured in \(C\) colours, then there exists at least \(P\) numbers in arithmetic progression, all of the same colour.

The Hales-Jewett theorem. The final result we will consider is motivated by the game tic-tac-toe, in which players take turns to mark the squares of a \(3 \times 3\) grid with the aim of occupying three squares along a column, row or diagonal. It is a well-known fact that in this traditional form of tic-tac-toe, both players can force a draw with optimal play. In stark contrast is the game of three-dimensional tic-tac-toe, played in a similar manner on a \(3 \times 3 \times 3\) grid of cubes, where the first player has an easy win by occupying the central position on the first move. Actually, it is impossible to play out a draw in three-dimensional tic-tac-toe since any partition of the 27 cubes into two colours will always include a monochromatic column, row or diagonal. If we consider instead \(N\)-in-a-row tic-tac-toe played between \(C\) players, then the Hales-Jewett theorem guarantees similar behaviour.

For every pair of positive integers \(C\) and \(N\), there is a positive integer \(D\) such that if the unit hypercubes in a \(D\)-dimensional \(N \times N \times \cdots \times N\) hypercube are coloured in \(C\) colours, then there exists at least one row, column or diagonal of \(N\) squares, all of the same colour.

Problem: Show that Van Der Waerden’s theorem follows from the Hales-Jewett theorem.

Anyone wanting to find out more is strongly encouraged to consult the monograph entitled Ramsey Theory by Ronald Graham, Bruce Rothschild and Joel Spencer [2].

References


Department of Mathematics and Statistics, The University of Melbourne, VIC 3010
E-mail: N.Do@ms.unimelb.edu.au
Favour the present tense

I recommend the rule “if in doubt use the present tense”.

Higham (1998), [1], §4.29

The present tense works well for scientific writing. The eternal truths we present in an article should all be in the present tense: instead of “experiments have shown that”, prefer “experiments show that”. The present tense helps make more active writing.

Write derivations in the present tense: not the past tense of “where we have assumed \( R = \{k_1, \ldots, k_r\} \)” but instead “where we assume \( R = \{k_1, \ldots, k_r\} \)”;
or the future tense of “As we will scale later the Hamiltonian with the inverse temperature”, but instead “As we scale the Hamiltonian with the inverse temperature, Section 4.”. Generally avoid “will”: prefer “The potential in Example 2 is central” to “The potential in Example 2 will be central”.

In summarizing the action of a drama, the writer should always use the present tense. In summarizing a poem, story, or novel, he should preferably use the present,

Strunk Jr (1918) [2], §17

However, report actions undertaken in computational experiments using the past tense. For example, “All simulations used a fine lattice of size \( N = 512 \)”. Similarly, instead of “we solve the linear equations (4.9)-(4.12) with \( \Delta t = 0.0063 \) and 8192 FFT points”, prefer “we solved the linear equations . . . “.

Refer to previous work in an earlier article using either the past tense or the present tense. Choose depending upon whether your main emphasis is the historical development (use the past tense) or whether your emphasis is the eternal truths in the work (use the present tense). In this fragment the emphasis is on the result, “theoretical studies [10, 11] have shown that”, so prefer the present tense of “theoretical studies [10, 11] show that”. However, the past tense suitably fits the historical aspect in “Nolasco and Dahlen [15] and Guevara et al. [16] demonstrated that”.

Use future tense to refer to future work—that is, work forecast to be in a different article. Such discussion usually only occurs in the conclusion.

Other than in conclusions, future tense is rarely used in science writing.

Zobel (2004) [3], p.40

Summary. This quote say it all.

Facts are true: use the present tense to denote unchanging truths. When telling what the authors or other researchers did, use the past tense. For what is being done in the paper, use the past tense for referring back (“in Section 5 it was shown that. . . ”). For referring ahead, use . . . the present tense if the writer is thinking of how the paper is set out (“in Section 7 it is shown that. . . ”).\(^1\)


\[^1\]But remember to write actively, not passively. In particular, avoid “it was/is”. I recommend the two parenthetical examples in this quote be “Section 5 showed that. . . ” and “Section 7 shows that. . . ”.
References

Bilateral average intelligence increase through tourism
Andrew Conway

The old joke goes “tourist from New Zealand visited Australia, and raised the average I.Q.s of both countries.” This piece of anti-Australian propaganda must be quantified in these days of heightened border security concerns. This paper establishes the expected number of such tourists to arrive in Australia per year.

Suppose that the intelligence of both Australians and New Zealanders follows a normal distribution for each individual with mean $\mu$ and variance $\sigma^2$. Then the average intelligence for Australia ($I_A$) and New Zealand ($I_{NZ}$) will both follow a normal distribution with mean $\mu$ and variance $\sigma^2/p_A$ and $\sigma^2/p_{NZ}$ respectively, where $p_A$ is the population of Australia and $p_{NZ}$ is the population of New Zealand. The average mental superiority $S = I_{NZ} - I_A$ of New Zealanders over Australians will then be distributed normally with mean 0 and variance $\lambda^2$ where $\lambda^2 = 1/p_A + 1/p_{NZ}$.

Suppose that a random tourist $T$ with I.Q. $I_T$ from New Zealand to Australia also has the same I.Q. distribution. Assume further that this is independent of $S$, which is close to true assuming that the populations are large so that no individual has a significant impact. In order for $T$ to fulfil the conditions of the joke, we need $I_A < I_T < I_{NZ}$. If we assume again that $I_A$ and $I_{NZ}$ are both very close to $\mu$ compared to $\sigma$ (which is the same as saying $\lambda \ll 1$), then the probability density function for $I_T$ will be pretty close to constant $1/\sigma \sqrt{2\pi}$ in the region $I_A$ to $I_{NZ}$. Integrating this between $I_A$ and $I_{NZ}$ gives $S/\sigma \sqrt{2\pi}$, assuming $S > 0$. The probability $P_T$ that $T$ fulfils the conditions of the joke then becomes the expectation of this probability over the distribution of $S$:

$$P_T = \int_{-\infty}^{\infty} \frac{s}{\sigma \sqrt{2\pi}} e^{-s^2/2\lambda^2 \sigma^2} ds = \frac{\lambda}{2\pi} \int_{-\infty}^{\infty} xe^{-x^2/2} dx = \frac{\lambda}{2\pi}.$$

If the number of visitors to Australia from New Zealand each year is $N_V$, then the expected number making the joke true each year is $N_V P_T$, or

$$\frac{N_V}{2\pi} \sqrt{\frac{1}{p_A} + \frac{1}{p_{NZ}}}.$$

We now put concrete numbers behind this. For the year ending 31 Jan 2006, $N_V = 1,104,600$. At the end of 2005, $p_A = 20,352,013$, and $p_{NZ} = 4,100,600$. These give an expected number of New Zealand visitors to Australia each year causing an affront to national pride of 95.16. It is not clear what the government is doing about this.

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1. This of course may be negative.
2. This may not be true as tourists to Australia may not be unbiased with respect to I.Q., however it is likely that a reasonable portion of them occupy the critical I.Q. close to $\mu$ point.
The second ICE-EM Australian Graduate School in Mathematics was held at the University of Queensland, St Lucia Campus, Brisbane from 3 to 21 July 2006 and was another outstanding success. The three themes this year were Computation, Geometric Analysis and Mathematical Physics. Forty nine students attended from Australia, New Zealand, Vietnam, Indonesia and Thailand. We had seven internationally renowned presenters from Australia and overseas.

The lecturers were able to sustain a level of enthusiasm throughout the three weeks and many students reported having gained new perspectives and insights they could apply in their own areas of study. All students also gave presentations which were of a very high standard. Another valuable aspect of the School for the students was interaction with a large population of other math postgraduates. Social activities included pool and whiteboard notations in the College common room, volleyball at morning tea, a Conference Dinner and weekend trips to the Coast.

The Graduate School is funded by the Federal Government through the International Centre of Excellence in Education in Mathematics and the Australian Mathematical Sciences Institute. The University of Queensland provides a major contribution through the use of its facilities. Other contributors this year were The Queensland Parallel Supercomputing Foundation and CSIRO Mathematical and Information Sciences Division. ICE-EM gratefully acknowledges their support and sponsorship. Following are individual reports from four participants.

Agah Garnadi

I was a (foreign) student participant in the IAGSM 2006 Computation stream. I am a university lecturer at a maths department in Indonesia, lecturing for first year students in Calculus. I also teach scientific computing for maths and physics students. At the same time, I am preparing a dissertation for a doctorate as an external student. My work focuses on numerical simulation and reconstruction of Electrical Impedance Tomography for process instrumentation device. It is for this reason that I decided to apply for participation in the IAGSM 2006, as one of the topics is PDE constrained Optimization which is closely related to my work (actually, the problem can be considered as an example of the topics) and a bonus for my own studies as well. One of the problems I faced, were bureaucratic bungles in my country, as the Australian embassy is hundreds of kilometers away from my workplace. Also, during the process, my visa application went missing. It disappeared into the jungle of papers in the embassy. Thanks to technology (through e-mail exchanges), the embassy got hold of my applications and issued my visa, just three days before IAGSM started. And thank God, I got a flight at the last minute as I had to cancel my previously booked flight due to visa uncertainty. I arrived at Emmanuel College right before lunch on Monday and I noticed a couple of students wearing green T-shirts with IAGSM logos. I missed half a day of lectures and the opening session.

The lectures were not difficult for me to follow. With the help of the knowledge of lectures I got during my years at the ANU a long time ago, I was able to sail through. Some parts of the course filled in the gaps in my knowledge where I, due to the fact that books and journals are hard to obtain in Indonesia, only had a vague idea of a certain subject. But I enjoyed both of the lecturers and some of the participants’ contributions were also quite enlightening for me. Unfortunately, due to the wide interests of the
participants, working towards collaboration is difficult to achieve. But, at least I am aware of what people are doing Australia wide. Social lives between the participants were quite good, thanks to the common and TV rooms, as well as the dining hall in the college.

One thing that I think would be of benefit is some sort of modeling/problem solving session, similar to the one at MISG, integrated within the school.

Chris Goddard

In my opinion the 2006 ICE-EM Graduate School was a great success. As one of the students taking the course on geometric evolution equations I was treated to an excellent overview of this exciting new area. The presenters were all outstanding and we benefited greatly from our time with them. Our main speakers for the first couple of weeks were Gary Lieberman, from Iowa State University, and Ben Andrews from the ANU. They had quite different styles of presentation, but both were superb teachers. Gary spoke about existence theory for parabolic PDE, showing how one could prove regularity results for the solutions of quite general systems from an understanding of the boundary and initial conditions. Although quite pure and technical, this course had an underlying importance to the theory of geometric evolution equations, since they are all of a type of PDE known as weakly parabolic type – a type of PDE that can, after a special type of reparametrisation, be written as parabolic PDEs.

Ben’s course was the centrepiece, however. He spoke about different types of geometric evolution equations, starting from the curve shortening flow, then working to the mean curvature flow, and finally, the Ricci Flow. All the ideas were extremely well motivated, and from my impressions the core idea of the course was to develop analogues of key results for the full blown Ricci Flow in terms of much simpler examples. Even though he covered a huge amount of territory, his delivery was concise, efficient, and extremely understandable. In the words of another student, shortly after he summarised the Hamilton-Perelman program in the final lecture “That was probably the best account of the Ricci flow that you’ll currently get anywhere.” Our final speaker was Gerard Huisken of the Max Planck Institute in Germany. He talked quite a bit about a possible application of the idea of geometric evolution equations, demonstrating how one could make sense of the mass of a Black Hole by considering the inverse mean curvature flow, flowing backwards to infinity starting from the event horizon, and defining a mass in terms of a generalised isoperimetric inequality. To make this all precise he demonstrated that he was able to get control on the rate of expansion of the expanding surfaces, bounding them between spheres of particular radii. In addition, he was able to show that such a mass was bounded below by the AdM mass, the notion of mass usually used to resolve such issues, and bounded above by the Hawking mass.

However, there was more to the Graduate School than just gaining mathematical knowledge. We had a whole new city to explore and on several occasions groups of us went out to interact with the city’s people
and explore its environs (while hopefully not getting too lost along the way).

A great deal of time was also spent at the dinner table. Not eating per se, but talking and listening to one another. Unfortunately, I have a very poor memory for details, but I do remember the conversations as being occasionally insightful, sometimes outrageous, and always amusing.

Of course I won’t forget the soccer matches we had while at the university. UQ has an enormous amount of land and easily half of that is devoted to ovals and various other sporting amenities. Deciding one day that I really must go out to get a ball (heaven forbid that I actually work during my spare time!), a large number of us quickly discovered the joys of this particular diversion. All in all, I enjoyed myself immensely, and look forward to going again, if possible, in 2007.

Lucy Gow

I had never been to Brisbane before this year’s ICE-EM Graduate School, so when I arrived I looked forward to checking out the various tourist sights on offer. In the end though, the mathematics and fellow mathematics students kept me so occupied and entertained that I had to put tourism on hold for the next time I visit.

Each morning we had two lectures, one from Gustav Delius on Symmetry and another on The Importance of Being Integrable by Murray Batchelor. The Symmetry lectures included a seemingly whirlwind account of modern physics that gradually took on the shape of subjects more familiar to me, like Lie algebras and quantum groups. Meanwhile, Murray Batchelor’s lectures carefully developed the Bethe Ansatz for the spin-half Heisenberg Hamiltonian, before broadening their scope to considering how solutions of the Yang-Baxter relation could give new models using a variety of algebraic techniques. Both courses gave me a glimpse of how the tiny bit of mathematics that I know is used in mathematical physics, and how it relates to other areas of maths. Sometimes I felt like I could understand, but at other times I was left feeling a little overwhelmed and confused. Nevertheless I finished the School with a strong desire to learn more when I got home. I don’t think the lectures could have been better presented.

In the afternoons the students took the stage to give hour long seminars on their own research. It was lucky that giving a seminar was compulsory, since I’m not sure I would have chosen to do it otherwise, and it proved to be a good experience for me. Through the seminars it became clear just how diverse the other students’ particular interests were - as they presented on topics ranging from combinatorics, statistical mechanics, differential equations, star quasi-Hopf superalgebras and group theory, to modelling the forces on Thai buses as they make their way about obstacle courses and quantum entanglement. It was a lot of fun to meet students from other countries and also other cities in Australia. More importantly, I think the school did a good job of bridging the gap between students in different areas of mathematics. Before the conference I was chatting with my friend Yang Shi at the pub about which course we would do, and we realised for the first time that our areas of research were actually closely
related. We had never talked about mathematics before because we assumed we would have nothing in common. (She studies non-linear differential equations and I do algebra). As a result of the course I’ve found new interest in mathematical areas I had long ago given up on. My undergraduate-experience of differential equations left me never wanting to see them ever again, but I was won over by the enthusiasm of others and am now giving them another look.

We did, however, find some time for non-mathematical pursuits, and also to enjoy the peaceful UQ campus and surrounding restaurants. (The chicken sandwiches at Piccalilli come well recommended). Particularly memorable was the last night when, after enjoying a meal of pizza and chocolate cake, we filled a pub with dancing mathematics’ students and nearly everyone took their turn to sing karaoke to the locals of Indooroopilly. On the whole, it was a thoroughly enjoyable experience and I look forward to keeping in touch with the friends I made at the school.

Kaiser Lock

When I first heard about the IAGSM from my supervisor, I was quite nervous about leaving the security of my surroundings for this three-week winter course. A major part of this anxiousness was due to the compulsory seminar each of us had to present (it would only be my second time if the honours talk to my own department counts). In the end, I thought “what the heck” and ran head long straight past the previously rigid boundaries that governed my comfort zone and right into the swirling eye of uncertainty and anxiousness. I am so glad I went. It was a fantastic experience!

The university is as breathtakingly beautiful as it is huge. There is no better place than Brisbane to get away from the cold winter, and the IAGSM provided a great forum to warm up ones mind with mathematical exercise. I attended the computation session lectured by Dr. Boris Vexler on “Optimization with PDEs” and Prof. Robert McLachlan on “Geometric Numerical Integration”. Boris took us through the motivations, theory, methodologies and applications of optimization problems with PDEs and finished off with a wonderful dynamic example. The two-week course covered the material very thoroughly and was presented in such a way that it stimulated learning. Although I was new to the subject matter, it provided me with a quick glimpse into the way the real world problems are approached. For another part of the computation session, Robert gave us a fast paced and once again, thorough introduction to geometric integration. He also provided a set of challenging practical exercises which I’m sure were meant to consolidate our learnings but only served to up my frustration levels as I attempted to find the elusive answers.

Our accommodations also provided a pleasant backdrop to our scholastic pursuits. The food served at college was great. Definitely love the hot breakfasts and after dinner desserts, and of course, how can you forget the kitchen ladies, who with their old fashioned charm and infectious zest for life made college feel that much closer to home. Like most things in life, it is the people around you that can greatly dictate your enjoyment of a particular occasion and I must
say, I thoroughly enjoyed this event. Actually, no – enjoyed is not a strong enough word – I loved every minute of my time here! I loved meeting like-minded people who didn’t stare back at me with blank faces when I mentioned partial differential equations. I loved the trek to the shops for our nightly hit of greasy fast food and the card games that ensued after our heart-attack inducing meals, volleyball and frisbee that occupied the morning tea session and the afternoon breaks. Thank you, Helen, for providing the volleyball. Even though the first night out pub-crawling was not such a great experience, I think we put paid to the ever so incorrect myth that mathematicians don’t know how to have fun during our karaoke night out on the last day. What started as a very civil affair with wine and pizzas quickly turned into a free for all that ended up with wrestling competitions. Fortunately, the inconvenience of traveling to the city does successfully prevent us from enjoying the nightlife to excess.

I have taken away so much from this conference. I have had the most memorable time and the exposure to people from all over the world, who bring with them refreshingly different perspectives on both mathematics and life in general has enriched me greatly. The staff were tremendous, the whole event was well structured and organized and the courses were both interesting and thought provoking. What more can I say, I have come out of the eye of this whirlwind experience, having had the boundaries of my comfort zone blown away and here I am, ever so thankful that I took up this wonderful opportunity.

Next year’s AGSM

Obituary

Alex Rubinov
28 March 1940 – 9 September 2006

Alex Rubinov was born in Leningrad (now St Petersburg), Russia in 1940 and did his undergraduate studies at Leningrad State University. This was followed by a Ph.D. in Novosibirsk, Siberia and an advanced doctorate at Moscow University. Subsequently, he held positions in Leningrad and Karlini in Russia. Then he joined his mentor, Nobel Prize winning mathematician and economist, Leonid Kantorovich, in the scientific research centre near Novosibirsk. He enjoyed 7 years there until he needed to escape anti-Semitism by moving to Baku, Azerbaijan where he was head of the mathematics institute. With the fall of the Soviet Union he moved to Ben Gurion University in Beersheba, Israel. After a comparatively short sojourn there, in 1996 he migrated to Australia and joined the staff of the University of Ballarat. He finally thrived in the peace and tranquillity of Ballarat, and Australia in order to escape anti-Semitism and Soviet oppression, but always nurtured his international circle of research colleagues and attracted many to visit Ballarat and some to join its staff.

The University of Ballarat quickly recognised its luck in having an international researcher and scholar of the calibre of Alex Rubinov and appointed him Professor of Mathematics. Six years ago, the University established the research centre CIAO (Centre for Informatics and Applied Optimization) with Alex as its founding Director. Under his leadership, CIAO was successful in becoming a research centre of both national and international repute. It is recognized for both its theoretical and practical research in non-smooth optimization. Over that period, the research output of CIAO increased ninefold with an annual commercial income of $2 million and annual grant income of some $500,000. Alex’s academic and personal impact was such that, at his funeral, scores of tributes from international scholars as well as many Australian colleagues were read. Professor Phil Howlett of the University of South Australia said “Alex was a very fine mathematician who made a magnificent contribution to mathematics in Australia. His passing is a great loss.”
Over his career Alex published 17 monographs and textbooks, 12 edited books and journal special issues and more than 200 research papers. His most cited work is the book Abstract Convexity and Global Optimization, published in 2000 by Kluwer Academic Publishers. While at the University of Ballarat, Alex was the recipient of many Australian Research Council grants, as well as other national competitive grants and research contracts. He continued to work in optimization and convexity and founded the area of monotonic analysis as well as initiating new optimization approaches to data mining. Alex supervised over 40 PhD students, half of these at the University of Ballarat. Along with his scholarly achievements, Alex was also recognised for his wonderful human qualities and his ability to relate, not only to his colleagues and students, but also to industry partners and clients. Many mathematicians do their best work before they are 40 and their publication rate drops off dramatically as they age. This could not be further from the truth in the case of Alex. In recent years he was at the pinnacle of his research productivity and to the end had plans and initiatives for the future. He was honoured days before his death as the 2006 Fellow for the Continuous Optimization Working Group of EURO (EUROPT - The Association of European Operational Research Societies). To focus, however, on Alex’s academic achievements alone would be to ignore Alex the person. The School of Information Technology and Mathematical Sciences (ITMS) of the University of Ballarat has eighty staff, all of whom liked and respected him as a scholar and as a person. Despite his great achievements, Alex was a modest and humble man and always a gentleman. Even when close to the end, when the Vice-Chancellor visited him in hospital, Alex stood up for him. Professor Mirka Miller of ITMS used the following words in connection with Alex: “excellence, integrity, gentleness, and excellence again.” Professor Karl Hofmann of Darmstadt University of Technology and his wife Isolde visited Ballarat for extended periods. He wrote as follows: “Alex will be remembered as an outstanding scholar, as one of the most prominent colleagues and personalities of the University of Ballarat, and as a very gentle, modest, and wonderful person. Isolde and I feel all the better for having known him and having enjoyed his friendship and hospitality.” At the University of Ballarat, he is greatly missed both as a colleague and as a friend. At Alex’s funeral I described Alex in the following way: “not only a gentleman and a scholar, but also a truly lovely person”. Condolences from around the world can be read at http://www.ballarat.edu.au/ard/itms/CIAO/condolences.shtml. Alex was very devoted to, and proud of, his family. He is survived by his wife Dr Zari Dzalilov (also a mathematician in ITMS), and children Eldar and Mikail, and children from his first marriage, Svetlana and Vadim.

Sid Morris
Head of the School of Information Technology and Mathematical Sciences, University of Ballarat
PO Box 663, Ballarat VIC 3353.
50th Annual Meeting of the Australian Mathematical Society
Macquarie University 25 – 29 September 2006
William Chen

The landmark 50th Annual Meeting of the Australian Mathematical Society took place in the last week of September 2006 at Macquarie University, with roughly 260 participants. For this special event, the organizers decided quite early on to encourage participation by research students and drastically lowered their registration costs to a symbolic $50. It was therefore particularly pleasing that among the participants, there were roughly 50 research students, and they contributed over 35 short presentations.

The plenary talks and a number of other main activities took place at Macquarie Theatre, while special session talks took place in 9 classrooms on the eastern end of the campus, in buildings E5A and E6A. Accommodation for participants was available at the Dunmore Lang College, the Travelodge, the Medina Serviced Apartments and the Stamford Grand North Ryde, all within walking distance of the main parts of the Macquarie University campus.

The Program Committee started work in the middle of 2005 and eventually organized 14 exciting plenary talks, covering a wide area of pure and applied mathematics. The plenary speakers and the titles of their talks are as follows:

- Pascal Auscher (Université de Paris-Sud) On T(b) Theorems
- Robert Bartnik (Monash University) Static Metrics and the Recognition Problem
- Michael Batanin (Macquarie University) Configuration Spaces from Combinatorial, Topological and Categorical Perspectives
- Frank De Hoog (Commonwealth Scientific and Industrial Research Organisation) Industrial Mathematics – a CSIRO Perspective
- Steven Evans (University of California, Berkeley) Phylogeny, Real Trees, Metric Geometry, and Dirichlet Forms
- Peter Forrester (University of Melbourne) Random Matrices and Painlevé Transcendents
- Andrew Hassell (Australian National University) The Time-Dependent Schrödinger Equation
- Adrian Lewis (Cornell University) Eigenvalues and Optimization
- Ngaiming Mok (University of Hong Kong) Global Extension of Local Holomorphic Isometries with respect to the Bergman Metric
- Christopher Skinner (Princeton University) What Do We Know About the Birch–Swinnerton-Dyer (and Related) Conjectures?
- Terence Tao (University of California, Los Angeles) Long Arithmetic Progressions in the Primes
- Katrin Tent (Universität Bielefeld) Buildings and Algebraic Groups
- Claire Voisin (Centre National de la Recherche Scientifique) Hodge Theory, Kähler Geometry and Projective Geometry
- Xu-Jia Wang (Australian National University) The Monge–Ampère Equation and its Applications

The Program Committee also chose fifteen areas for special sessions. Together with a general session, there were 188 shorter talks in 16 special sessions.

The Meeting opened on Monday morning with a brief welcome by Professor John Loxton, Deputy Vice Chancellor (Academic) of Macquarie University and Conference Director of the 33rd Annual Meeting of the Society in 1989 when it was last held at Macqua-
Award of the AustMS Medal to Andrew Mathas

The Meeting was then officially declared open by Professor Neil Trudinger, the first recipient of the Australian Mathematical Society Medal 25 years ago. The 2006 Australian Mathematical Society Medal was presented by Professor Michael Cowling, the President of the Society, to Dr Andrew Mathas of the University of Sydney. The 2006 George Szekeres Medal was presented by Dr Peter Szekeres to Professor Anthony Guttmann of the University of Melbourne. Dr Mathas and Professor Guttmann then gave short presentations.

A birthday cake for the Society was cut on Tuesday afternoon by Professor Bruce Craven and Professor Max Kelly who were both present at the First Annual Meeting of the Society in Sydney in 1957. This was followed by the presentation of the 2005 Australian Mathematical Society Medal to Professor Terence Tao of the University of California, Los Angeles. On Tuesday evening, following a champagne reception, the book on the history of Australian mathematics, written by Graeme Cohen and entitled *Counting Australia In*, was officially launched by Professor David Elliott, for many years the Secretary of the Society. At the end of the proceedings, Graeme Cohen was made an Honorary Life Member of the Society.

An Education Afternoon, sponsored by the International Centre of Excellence for Education in Mathematics, took place on Wednesday afternoon, with short talks on mathematics given by Steven Evans, Ngaiming Mok, Mary Myerscough and Terence Tao. A talk on mathematics education was delivered by Chris Wardlaw.

On Wednesday afternoon, there was also a Forum on the recent Review of Australian Mathematical Sciences led by Barry Hughes.

The Annual General Meeting of the Society took place on Thursday afternoon. Professor Peter Hall succeeded Professor Michael Cowling as the President of the Society. It was also confirmed that the 51st Annual Meeting of the Society would take place at La Trobe University, with Geoff Prince as Conference Director and Grant Cairns as Conference Treasurer.

The Conference Dinner took place on Thursday evening at the Stamford Grand North Ryde, not far from Macquarie University. Benjamin Wilson of the University of Sydney was awarded the Neumann Prize for the best talk given by a research student. Jan Thomas was made Honorary Life Member of the Society at the conclusion of her tenure as Executive Officer of the Society.

I would like to express my gratitude to my fellow members on the Program Committee – Peter Bouwknegt, Xuan Duong, Nalini Joshi, Amnon Neeman, Cheryl Praeger, Hyam Rubinstein, Ross Street and Rob Womersley – for their wonderful judgment and informed discussions that enabled us to arrive at such an exciting program of plenary talks, to all the special session organizers who went out of their way to organize such a rich supporting program, to my
wonderful team on the Organizing Committee – Xuan Duong, Andrzej Kozek, Chris Meaney, Ross Moore, Tanya Schmah, Ross Street and Rod Yager – for their amazing effort in ensuring the smooth running of the Meeting, to Victoria Benning who handled the mountain of registration and accommodation paperwork, and to my great friend Margaret, my wife Lily, my daughter Abby and the many student helpers who organized the refreshments throughout the week as well as the champagne reception on the Tuesday evening.

Lastly, but definitely not least, I would like to thank all the participants, without whom the Meeting would not have taken place.
Exact one hundred nontrivial composites
Alf van der Poorten

Fine advice John Conway once gave me on instant factorisation of three-digit numbers should be better known. The difficulty is that doing it seems to require one to memorise the 168 primes less than a thousand. “Surely you’re not saying that’s a problem, Alf?” John said askance to me. However, not to worry. There are only, and indeed exactly, one hundred (100) nontrivial composites less than a thousand.

Nontrivial composites

Usually one thinks of positive integers as being one of 1, prime, or composite. John recommends the more refined partition: 1, prime, trivially composite, or nontrivially composite.

Here, a composite integer is trivially composite if it is divisible by 2, 3, or 5.

Exercises

(a) List the nontrivial composites less than a thousand. Learn them. Annoy your friends by factorising every three digit number that comes your way.

(b) Let \( S \) denote the set \{1, 2, 3, \ldots, 100\}. As usual \(|S|\) denotes \#S, the number of elements in the set (in this case \#S = 100, of course). Let \( S_n = \{ a \in S : n \mid a \} \).

Compute

\[ |S| - (|S_2| + |S_3| + |S_5|) + (|S_6| + |S_{10}| + |S_{15}|) - |S_{30}| , \]

and, noting that there are just three nontrivial composites, namely 49 = 7\(^2\), 77 = 7 \cdot 11, and 91 = 7 \cdot 13, less than one hundred, find the number of primes less than 100.

(c) Similarly, now take \( S = \{1, 2, 3, \ldots, 1000\} \). Given that there are exactly one hundred nontrivial composites less than a thousand, find the number of primes less than 1000.

John Conway was saddened to find when I last met him (Calgary, June 2006, at a meeting celebrating the 90th birthday of Richard K. Guy) that I had not recently redone exercise (a) and could not instantly report 871 = 13 \cdot 67. He points out that exceeding our grasp by aiming to factorise not just three digit but four digit integers is the best way to internalise the first hundred nontrivial composites; the three bonus factorisations below promote that cause.
The First Hundred Nontrivial Composites

| 49 = 7²   | 301 = 7 × 43 | 497 = 7 × 71 | 679 = 7 × 97 | 841 = 29²   |
| 77 = 7 × 11| 319 = 11 × 29| 511 = 7 × 73 | 689 = 13 × 53| 847 = 7 × 11²|
| 91 = 7 × 13| 323 = 17 × 19| 517 = 11 × 47| 697 = 17 × 41| 851 = 23 × 37|
| 119 = 7 × 17| 329 = 7 × 47 | 527 = 17 × 31| 703 = 19 × 37| 869 = 11 × 79|
| 121 = 11² | 341 = 11 × 31 | 529 = 23²      | 707 = 7 × 101 | 871 = 13 × 67 |
| 133 = 7 × 19 | 343 = 7³      | 533 = 13 × 41 | 713 = 23 × 31 | 889 = 7 × 127 |
| 143 = 11 × 13 | 361 = 19²    | 539 = 7² × 11 | 721 = 7 × 103 | 893 = 19 × 47 |
| 161 = 7 × 23 | 371 = 7 × 53 | 551 = 19 × 29 | 731 = 17 × 43 | 899 = 29 × 31 |
| 169 = 13² | 377 = 13 × 29 | 553 = 7 × 79  | 737 = 11 × 67 | 901 = 17 × 53 |
| 187 = 11 × 17 | 391 = 17 × 23 | 559 = 13 × 43 | 749 = 7 × 107 | 913 = 11 × 83 |
| 203 = 7 × 29 | 403 = 13 × 31 | 581 = 7 × 83  | 763 = 7 × 109 | 917 = 7 × 131 |
| 209 = 11 × 19 | 407 = 11 × 37 | 583 = 11 × 53 | 767 = 13 × 59 | 923 = 13 × 71 |
| 217 = 7 × 31 | 413 = 7 × 59  | 589 = 19 × 31 | 779 = 19 × 41 | 931 = 7² × 19 |
| 221 = 13 × 17 | 427 = 7 × 61  | 611 = 13 × 47 | 781 = 11 × 71 | 943 = 23 × 41 |
| 247 = 13 × 19 | 437 = 19 × 23 | 623 = 7 × 89  | 791 = 7 × 113 | 949 = 13 × 73 |
| 253 = 11 × 23 | 451 = 11 × 41 | 629 = 17 × 37 | 793 = 13 × 61 | 959 = 7 × 137 |
| 259 = 7 × 37 | 469 = 7 × 67  | 637 = 7² × 13 | 799 = 17 × 47 | 961 = 31²    |
| 287 = 7 × 41 | 473 = 11 × 43 | 649 = 11 × 59 | 803 = 11 × 73 | 973 = 7 × 139 |
| 289 = 17² | 481 = 13 × 37 | 667 = 23 × 29 | 817 = 19 × 43 | 979 = 11 × 89 |
| 299 = 13 × 23 | 493 = 17 × 29 | 671 = 11 × 61 | 833 = 7² × 17 | 989 = 23 × 43 |

Centre for Number Theory Research, 1 Bimbil Place, Killara NSW 2071
E-mail: alf@math.mq.edu.au
I write this report after returning from the 50th anniversary meeting of the Australian Mathematical Society, held at Macquarie University. In my opinion, this meeting and the previous annual meeting held at the University of Western Australia, were outstanding successes. I haven’t attended other AustMS annual meetings this century but compared to those of last century, there are some notable developments.

More speakers develop interesting new mathematics around current applications. People will always have biases and tastes but there is an emerging professional profile of the “mathematical scientist” who is happy to develop or learn new mathematical theory and to apply it to other fields. It is not unnatural for an abstract thinker to have thoughts on concrete applications; neither is it unnatural for an applied scientist to be interested in deep foundations.

Annual conferences usually have associated public lectures, delivered after business hours by invited speakers who communicate well with heterogeneous audiences. Our relations with other disciplines and with the public at large are important for building support for mathematics. The public lectures are refreshing for most professional mathematicians because there is a strong chance that every detail can be understood in situ.

For a few years now, the annual meeting has run an Education Afternoon, which has gone from strength to strength. It is well attended by enthusiastic secondary school teachers. Our discipline relies on our teachers to inspire the next generation. It was heartening to see that even the latest Fields medallist provided some useful enrichment material for teachers. All academic and industrial mathematicians have a role to play in outreach.

Over the last five years, there has been increased government expenditure on full-time senior research fellowships and on research centres that employ full-time researchers. This concentration of resources continues to lead to some good research outcomes but there are some interesting secondary effects of moving our strong researchers away from teaching and service roles.

Our best researchers are no longer expected to provide research-inspired teaching or scholarly-informed administration, although some provide these things voluntarily. At the same time, the number of academic teaching staff is decreasing and class-room loads are rising. It is not uncommon for some mathematics lecturers in recognized universities to have 14 contact hours per week, something that was unheard of 20 years ago.

The full-time teachers are then exhorted by their institutions to apply for research grants, in competition with full-time researchers. The inevitable consequence is that the research-rich will become richer and the research-poor will become poorer. The looming Research Quality Framework is reinforcing this trend by encouraging universities to poach promising researchers by offering research-only positions. Some regard this as a good policy. It certainly needs to be openly discussed.

As well as acknowledging increasing government expenditure on some research concentrations, it is fair to add that there’s a separate well-functioning support system for University Learning and Teaching through the Carrick Institute. The Carrick Institute has a number of interesting schemes and programs that have funded a number of projects in mathematics and
statistics education. A large AMSI-focussed group is committed to work on the disciplines-based initiative project, “Mathematics for 21st C Engineering Students”. Within a year, we will be running a national workshop on this important topic.

We are hoping also to attract some interest from mathematicians and education faculty to design a project on mathematics curriculum for primary education students. AMSI and ICE-EM have international contacts to support such an initiative.

In conclusion, I would like to congratulate the AMSI Scientific Committee Chair, Peter Hall on his election as President of AustMS, and the AMSI Executive Officer Jan Thomas on being awarded life membership of the society.


Australian Mathematical Sciences Institute, University of Melbourne, VIC 3010
E-mail: phil@amsi.org.au
Quadrance graphs

Le Anh Vinh

Abstract

In this note, we will see an application of universal geometry in combinatorics. Let $q$ be any odd prime power of the form $q = 4l + 3$ for some integer $l$, and $F_q$ be the finite field of order $q$. The following definitions follow the book “Divine porpotions: rational trigometry to universal geometry” by Wildberger (see [6]).

Definition 1 The quadrance $Q(A_1, A_2)$ between the points $A_1 = [x_1, y_1]$, and $A_2 = [x_2, y_2]$ is the number

$$Q(A_1, A_2) := (x_2 - x_1)^2 + (y_2 - y_1)^2.$$  

Definition 2 A circle $C_k(A)$ in a finite field $F_q$ with center $A = [x, y]$ and quadrance $k \in F_q$ is the set of all points $X$ in $F_q \times F_q$ such that

$$Q(A, X) = k.$$  

Let us begin the discussion with the following lemma about the number of intersection points between two circles in $F_q \times F_q$.

Lemma 1 For any $i, j \neq 0$ in $F_q$. Let $X, Y$ be two distinct points in $F_q^2$ such that $k = Q(X, Y) \neq 0$. Then the size of the intersections of two circles $C_i(X), C_j(Y)$ only depends on $i, j$ and $k$. Precisely, let $f(i, j, k) := ij - (k - i - j)^2/4$. Then the number of intersection points is $p_{ij}^k$, where

$$p_{ij}^k = \begin{cases} 0 & \text{if } f(i, j, k) \text{ is non-square}, \\ 1 & \text{if } f(i, j, k) = 0, \\ 2 & \text{if } f(i, j, k) \text{ is square}. \end{cases} \tag{1}$$

Proof. Suppose that $X = [m, n]$ and $Y = [m + x, n + y]$ for some $m, n, x, y \in F_q$ then $x^2 + y^2 = k$. Suppose that $Z \in C_i(X) \cap C_j(Y)$ where $Z = [m + x + u, n + y + v]$ for some $u, v \in F_q$. Then we have $u^2 + v^2 = j$ and $(x + u)^2 + (y + v)^2 = i$. This implies that $xu + yv = (i - j - k)/2$. But we have $(xu + yv)^2 + (xy - y)^2 = (x^2 + y^2)(u^2 + v^2)$ so

$$(xv - yu)^2 = kj - \frac{(i - j - k)^2}{4} = ij - \frac{(k - i - j)^2}{4} = f(i, j, k).$$

If $f(i, j, k)$ is a non-square number in $F_q$ then it is clear that there does not exist such $x, y, u, v$, or $p_{ij}^k = 0$. Otherwise, let $\alpha = (i - j - k)/2$ and $f(i, j, k) = \beta^2$ for some $0 \leq \beta \leq (p + 1)/2$ then

$$xv - yu = \pm \beta, \quad xu + yv = \alpha.$$  

Solving for $(u, v)$ with respect to $(x, y)$ we have

$$u = (\alpha x \pm \beta y)/k, \quad v = (\alpha y \pm \beta x)/k.$$  

If $\beta = 0$ then we have only one $(u, v)$ for each $(x, y)$, but if $\beta \neq 0$ then we have two pairs $(u, v)$. This implies (1) and concludes the proof of the theorem. $\square$
The quadrance graph $V_q$ is defined on the vertex set $F_q \times F_q$. The pair $(X, Y)$ with $X, Y \in F_q \times F_q$ is an edge of $V_q$ if and only if $Q(X, Y)$ is a (nonzero) square in $F_q$. We will first show that $V_q$ gives us a critical colouring for Ramsey number $R(K_4 - e, K_4 - e)$; recall that $K_4 - e$ is the complete graph with 4 vertices with one edge deleted, and the Ramsey number $R(K_4 - e, K_4 - e)$ is the minimal number $n$ such that for any 2-colouring of edges of $K_n$, we can find a monochromatic subgraph $K_4 - e$.

**Example** We will show that neither $V_3$ nor $\overline{V}_3$ contains a $K_4 - e$. Suppose that $V_3$ contains a $K_4 - e$ then there exists 4 points $X, Y, Z, T$ in $F_3 \times F_3$ such that

$$Q(X, Y) = Q(X, Z) = Q(X, T) = Q(Y, Z) = Q(Y, T) = 1.$$ 

It implies that the circle $C_1(X)$ intersects $C_1(Y)$ at $Z$ and $T$. We have $f(1, 1, 1) = 0$ in $F_3$. From Lemma 1, for any $X, Y$ with $Q(X, Y) = 1$, the circle $C_1(X)$ intersects $C_1(Y)$ at exactly one point. It implies that $Z \equiv T$, which is a contradiction. Thus, $V_3$ does not contains $K_4 - e$. Similarly, $f(2, 2, 2) = 0$ in $F_3$ so $\overline{V}_3$ does not have a subgraph $K_4 - e$.

Therefore, $R(K_4 - e, K_4 - e) \geq 10$ since our graph has 9 vertices. In [3], we know that $R(K_4 - e, K_4 - e) = 10$. It has been verified (by computer) that there is only one (up to graph isomorphism) 2-colouring of $K_9$ with no monochromatic subgraph $K_4 - e$. Perhaps the construction here is the most intuitive one.

Using Lemma 1, we can show that the graph $V_q$ is a strongly regular graph with parameters \( \{q^2, (q^2 - 1)/2, (q^2 - 5)/4, (q^2 - 1)/4 \} \): $V_q$ is $(q^2 - 1)/2$-regular, any two adjacent vertices have $(q^2 - 5)/4$ common neighbours and any two non-adjacent vertices have $(q^2 - 1)/4$ common neighbours. The basic idea is for a fixed $k \in F_q$ we need to count the number of pair $(i^2, j^2) \in F_q \times F_q$ such that $f(i^2, j^2, k)$ is a nonzero square in $F_q$. The direct proof, however, is lengthy and technical and will appear elsewhere.

Note that the well-known Paley graph is defined in a similar way. The Paley graph is defined on a vertex set $F_q$ for some odd prime power $q$ of the form $q = 4l + 1$. The pair $(i, j)$ with $i, j \in F_q$ is an edge of $P_q$ if and only if $i - j$ is a nonzero square in $F_q$. When $q \equiv 1 \pmod{4}$ then $-1$ is square in $F_q$ so the Paley graph is well-defined. We know that (see [1], page 315-323) the Paley graph $P_q$ is also a strongly regular graph with parameters \( \{g, (g - 1)/2, (g - 5)/4, (g - 1)/4 \} \). Thus, the quadrance graph $V_q$ has the same parameters with the Paley graph $P_{q^2}$ for all prime power $q$ of the form $q = 4l + 3$. A natural question is whether the two graphs are isomorphic for all prime power $q \equiv 3 \pmod{4}$. Interestingly, the answer is affirmative.

**Theorem 1** Let $q \equiv 3 \pmod{4}$ be a prime power. Then the quadrance graph $V_q$ is isomorphic to the Paley graph $P_{q^2}$.

**Proof.** Let $f(x) = x^2 + 1 \in F_q[x]$ and $\alpha$ be a root of $f(x)$. Since $q \equiv 3 \pmod{4}$, we have $f(x)$ is irreducible over $F_q$. Thus, each element $\beta \in F_{q^2}$ can be uniquely represented in the form

$$\beta = x_\beta + y_\beta \alpha \quad \text{with} \quad x_\beta, y_\beta \in F_q. \quad (2)$$

Let $X_\beta = [x_\beta, y_\beta] \in F_q \times F_q$ for $\beta \in F_{q^2}$. We show that the isomorphism which maps $\beta = x_\beta + y_\beta \alpha \in F_{q^2}$ to $[x_\beta, y_\beta] \in F_q \times F_q$ induces a graph isomorphism between the Paley graph $P_{q^2}$ and the quadrance graph $V_q$. Both $P_{q^2}$ and $V_q$ are $(q^2 - 1)/2$-regular so it suffices to show that $(\beta, \gamma)$ is an edge of $P_{q^2}$ then $X_\beta, X_\gamma$ is an edge of $V_q$.
Suppose that \((\beta, \gamma)\) is an edge of \(P_{q^2}\) then \(\beta - \gamma = (x_\beta - x_\gamma) + (y_\beta - y_\gamma)\alpha\) is square in \(F_{q^2}\). Then there exists \(\rho, \tau \in F_q\) such that \(\beta - \gamma = (\rho + \tau\alpha)^2\). From (2), we have
\[
\begin{align*}
    x_\beta - x_\gamma &= \rho^2 + \tau^2 \alpha^2 = \rho^2 - \tau^2, \\
    y_\beta - y_\gamma &= 2\rho\tau.
\end{align*}
\]
This implies that
\[
Q(X_\beta, X_\gamma) = (\rho^2 - \tau^2)^2 + 4\rho^2\tau^2 = (\rho^2 + \tau^2)^2
\]
is square in \(F_q\).
Hence \((X_\beta, X_\gamma)\) is an edge of \(V_q\). This concludes the proof. \(\square\)

Thus, we can deduce that the graph \(V_q\) is a strongly regular graph with parameters \(\{q^2, (q^2 - 1)/2, (q^2 - 5)/4, (q^2 - 1)/4\}\) from Theorem 1 and the standard fact of the Paley graph. Furthermore, we can also show that the quadrance graph \(V_q\) is a special case of the graph defined in Example (2.21) in [4].

References

Trisecting the equilateral triangle with rational trigonometry

N.J. Wildberger

What is rational trigonometry?

Rational trigonometry is a dramatically simpler way of studying the measurement of triangles. It replaces the quasi-linear concepts of distance and angle with the quadratic algebraic concepts of quadrance and spread, and replaces the usual swag of trigonometric laws with simpler polynomial ones, involving no transcendental functions. It is much easier to learn, more powerful, and more accurate too. This new theory was introduced last year in [2], and has seen a fair amount of internet discussion.

Here we give a simple but instructive example, which showcases the basic laws of the subject and demonstrates the power of the method. Suppose that \(ABC\) is an equilateral triangle in which we have trisected each of the sides, and joined all these new points to the opposite vertices as in Figure 3. What are all the angles in this diagram?

Although this appears a reasonable question, it turns out that the answers are not particularly interesting, with approximate values such as 19.106605°, 81.786789° and 38.213210°. This is why you are probably not familiar with the situation. Let’s turn to rational trigonometry to find the correct question, so that the answers are interesting after all.

The quadrance \(Q(A_1, A_2)\) between two points in the plane is the square of the distance, so that in terms of coordinates

\[
Q(A_1, A_2) = (x_2 - x_1)^2 + (y_2 - y_1)^2.
\]

The spread \(s(l_1, l_2)\) between two lines in the plane is the square of the sine of the angle between them, but we use a better definition. Suppose the lines meet at a point \(A\), and \(B\) is any other point on either of the two lines, with \(C\) the foot of the perpendicular from \(B\) to the other line as in Figure 1. Then

\[
s(l_1, l_2) = \frac{Q(B, C)}{Q(A, B)} = \frac{PR}{AB}.
\]

Figure 1. Spread between two lines
If the lines have equations \(a_1 x + b_1 y + c_1 = 0\) and \(a_2 x + b_2 y + c_2 = 0\) then the spread is the rational expression
\[
s(l_1, l_2) = \frac{(a_1 b_2 - a_2 b_1)^2}{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}.
\]
In terms of coordinates of points,
\[
s(A_3A_1, A_3A_2) = \frac{((y_1 - y_3)(x_3 - x_2) - (y_2 - y_3)(x_3 - x_1))^2}{(x_1 - x_3)^2 + (y_1 - y_3)^2}(x_2 - x_3)^2 + (y_2 - y_3)^2).
\] (1)

The spread \(s\) is always a number between 0 and 1, being 0 when the lines are parallel and 1 when the lines are perpendicular. An angle of \(45^\circ\) or \(135^\circ\) is a spread of \(1/2\), an angle of \(30^\circ\) or \(150^\circ\) is a spread of \(1/4\), and an angle of \(60^\circ\) or \(120^\circ\) is a spread of \(3/4\). You can make a spread protractor that measures spreads in the same way that an ordinary protractor measures angles. Here is one you can download from the internet [1].

![Figure 2. A spread protractor](image)

Of course it takes a bit of getting used to the fact that the spread is not ‘linear’. However the advantages quickly become apparent when we look at the main laws of trigonometry expressed with these concepts.

If a triangle has quadrances \(Q_1, Q_2\) and \(Q_3\), and corresponding spreads \(s_1, s_2\) and \(s_3\) then the **Spread law** states that
\[
\frac{s_1}{Q_1} = \frac{s_2}{Q_2} = \frac{s_3}{Q_3}
\]
while the **Cross law** states that
\[
(Q_1 + Q_2 - Q_3)^2 = 4Q_1Q_2(1 - s_3).
\]
These are direct analogs of the Sine law and Cosine law. The usual ‘Sum of angles is 180 degrees law’ is replaced by the **Triple spread formula**, which states that
\[
(s_1 + s_2 + s_3)^2 = 2(s_1^2 + s_2^2 + s_3^2) + 4s_1s_2s_3.
\]
The other two main laws are just special cases of the Cross law. When \(s_3 = 0\) the three points are collinear and the three quadrances satisfy \((Q_1 + Q_2 - Q_3)^2 = 4Q_1Q_2\) or equivalently
\[
(Q_1 + Q_2 + Q_3)^2 = 2(Q_1^2 + Q_2^2 + Q_3^2).
\]
That’s the **Triple quad formula**. Note that the Triple quad formula and the Triple spread formula differ only by a single cubic term. When \(s_3 = 1\) in the Cross law, you get **Pythagoras’ theorem** in the more basic form \(Q_1 + Q_2 = Q_3\). A complete derivation of these laws
from first principles takes a few days to explain to an average high school maths student (as opposed to a few months for the current theory of trigonometry).

**Trisecting the equilateral triangle**

Label the equilateral triangle as shown, and suppose that the triangle has been scaled so that each of the boundary segments such as $AD$, $DG$ and so on have quadrance $Q = 1$. This figure also appeared in [3], and has a projective analog that encodes the Platonic solids very efficiently, as I will show elsewhere. We are now going to derive the spreads in the diagram. Symmetry will of course cut down on the possibilities.

The triangle $ACD$ has quadrances $Q(A, D) = 1$ and $Q(A, C) = 9$ since quadrance scales quadratically, and spread $s(AD, AC) = 3/4$.

The Cross law then asserts that $Q = Q(C, D)$ satisfies

$$(9 + 1 - Q)^2 = 4 \times 9 \times 1 \times (1 - 3/4)$$

or

$$(Q - 10)^2 = 9.$$ 

There are two solutions and the relevant one is $Q = 10 - 3 = 7$ since the other one corresponds to the situation where $D$ is on the other side of $A$. The Spread law asserts that

$$\frac{3/4}{7} = \frac{s(CA, CD)}{1} = \frac{s(DA, DC)}{9}$$

so that $s(CA, CD) = 3/28$ and $s(DA, DC) = 27/28$.

The Spread law applied now to the triangle $ACG$ gives

$$\frac{3/4}{7} = \frac{27/28}{9} = \frac{s(CA, CG)}{4}$$

so that $s(CA, CG) = 3/7$.

Both triangles $ADJ$ and $AGQ$ have spreads of $3/28$ and $27/28$, so the third spread $s$ must satisfy the Triple spread formula

$$\left(\frac{3}{28} + \frac{27}{28} + s\right)^2 = 2 \left(\left(\frac{3}{28}\right)^2 + \left(\frac{27}{28}\right)^2 + s^2\right) + 4 \times \frac{3}{28} \times \frac{27}{28} \times s.$$
This is a quadratic equation which turns out to factor as
\[(4s - 3)(49s - 48) = 0.\]
Thus \(s(JA, JD) = 3/4\) (as we could have also seen by symmetry since \(JKLM\) is an equilateral triangle) and \(s(QA, QG) = 48/49\).

The Triple spread formula simplifies in a pleasant way when two of the spreads \(s\) are equal—in this case the third spread is either 0 or 4\(s(1 - s)\). Thus in the isosceles triangle \(ABM\)
\[s(MA, MB) = 4 \times \frac{3}{28} \times \left(1 - \frac{3}{28}\right) = \frac{75}{196}.\]
Similarly in the isosceles triangle \(AEH\)
\[s(AE, AH) = 4 \times \frac{27}{28} \times \left(1 - \frac{27}{28}\right) = \frac{27}{196}.\]
So we get the following picture of spreads.

Figure 4. Spreads in the trisected equilateral triangle

It is now possible to find all the quadrances in the diagram too—just use the Spread law repeatedly. Of particular interest is the fact that
\[Q(A, J) = Q(J, K) = Q(B, K) = Q(K, L) = Q(C, L) = Q(L, J).\]
This property characterizes the spherical analog of this situation which is involved in the construction of the Platonic solids.

Classical trigonometry relies heavily on 90° − 45° − 45° and 90° − 60° − 30° triangles for examples and test questions, as these are largely the only ones that students can calculate with directly by hand. Once you make the switch to rational trigonometry, the opportunities widen immensely. For example, the triangle \(AGU\) in the previous diagram has spreads \(3/7, 27/28\) and \(3/4\). With an appropriate scaling, the quadrances may be taken to be 4, 9 and 7. This is now a triangle that you can use for demonstrations and further analysis.

As an aside, there are somewhat interesting relationships between the areas of the various polygons in this figure. You might enjoy proving for example that the area of the interior hexagon \(OPMQNR\) is one-tenth the area of the original triangle \(ABC\). This is an affine property, meaning that it is preserved under affine transformations, and so holds rather generally.
Geometry over other fields

This example has shown that rational trigonometry often involves solving quadratic equations, and that two possible solutions may arise from the same initial data. In [2] there are some additional laws, called the Triangle spread rules, that exploit convexity over the ‘real numbers’ to specify exactly which solution to take. They are a bit too involved to state here.

However it should be noted that rational trigonometry allows one to study metrical geometry over arbitrary fields, and in general different solutions to a quadratic equation correspond to alternative configurations.

In particular the equilateral trisection problem can also be studied in other fields if equilateral triangles exist, for which it is necessary that 3 be a square. Now in $\mathbb{F}_{11}$ the number $3 = 5^2$ is a square, and setting

$$A = [0,0] \quad B = [3,0] \quad C = [7,2]$$

you get

$$Q(A,B) = Q(B,C) = Q(A,C) = 9,$$

so that $\overline{ABC}$ is equilateral. Furthermore we may trisect the sides, taking

$$D = [1,0] \quad F = [1,5] \quad H = [2,5]$$

$$E = [8,8] \quad G = [2,0] \quad I = [6,8].$$

Then using the formula (1) and working in $\mathbb{F}_{11}$, you can check that

$$s(\overline{CA}, \overline{CD}) = 6 = 3 \frac{28}{3} \quad s(\overline{DA}, \overline{DC}) = 10 = 27 \frac{28}{1} \quad s(\overline{CA}, \overline{CG}) = 2 = 3 \frac{7}{1}$$

and so on.

To give a further example, the point $U$ is $[8,9]$, and the triangle $\overline{ACU}$ has quadrances 7, 2 and 4, and spreads 2, 10 and 9. Its circumcenter is $C = [1,9]$, its centroid is $[7,3]$ and its orthocenter is $O = [8,2]$. These points are collinear, since

$$\frac{2}{3} [1,9] + \frac{1}{3} [8,2] = [7,3].$$

Furthermore the center of the nine point circle of $\overline{ACU}$ is $N = [10,0]$, which is the midpoint of $O$ and $C$. Thus the usual relations in the Euler line of a triangle are alive and well.

Don’t be fooled by the simplicity of the above calculations. Rational trigonometry and its extension to Universal Geometry transform many aspects of metrical geometry and introduce new directions for algebraic geometry. The theory extends not only to general fields, but also to arbitrary quadratic forms, with a projective version that redefines both spherical and hyperbolic geometries, as described in [4].

References


School of Mathematics, University of New South Wales, Sydney NSW 2052
E-mail: norman@maths.unsw.edu.au

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A counting function for the sequence of perfect powers

M.A. Nyblom

1 Introduction

A natural number of the form $m^n$ where $m$ is a positive integer and $n \geq 2$ is called a perfect power. Unsolved problems concerning the set of perfect powers abound throughout much of number theory. The most famous of these is known as the Catalan conjecture, which states that the only perfect powers which differ by unity are the integers 8 and 9. It is of interest to note that this particular problem has only recently been solved using rather deep results from the theory of cyclotomic fields (see [4]). The set of perfect powers can naturally be arranged into an increasing sequence of distinct integers, in which those perfect powers expressible with different exponents are treated as a single element of the sequence. The first few terms of this sequence of perfect powers without duplication are

$$1, 4, 8, 9, 16, 25, 27, 32, 36, 49, 64, 81, 100, 125, 128, \ldots$$

and is listed in the On-Line Encyclopedia of Integer Sequences under A001597 (see [5]). The sequence in (1) has many properties, one being that the infinite sum of its reciprocals is convergent (see [3])—a clear indication of the scarcity of the perfect powers amongst the set of natural numbers. This latter fact is naturally reflected in the well known result that the sequence of perfect powers has zero asymptotic density that is, if $N(x)$ denotes the number of elements of (1) less than a positive real $x$, then $\lim_{x \to \infty} N(x)/x = 0$. In view of this result, one may question what is the precise nature of the growth rate of the counting function $N(x)$, in particular can an asymptotic estimate for $N(x)$ be found? We shall establish such a distributional result for the sequence of perfect powers by proving that

$$N(x) \sim \sqrt{x}$$

as $x \to \infty$. As will be seen, this asymptotic formula can be interpreted as stating that the perfect squares dominate the count of the sequence elements in (1) as $x \to \infty$. To contrast the main result, we shall in addition develop a closed-form expression for $N(x)$ using elementary sieve methods. As will be seen, this formula is some what reminiscent to Legendre’s counting function for the number of primes in the interval $(\sqrt{x}, x]$. In what follows every use of the character $p$ refers to a prime number and we denote the integer part of $x$ by $[x]$.

2 An asymptotic formula

To help establish the main results of this paper we shall first need to formally introduce the following family of sets.

Definition 3 Suppose $x \geq 1$ and $n \in \mathbb{N}\{1\}$, then let $A_n(x)$ denote the set of perfect powers having exponent $n$ and which are less than or equal to $x$ that is, $A_n(x) = \{k^n : k \in \mathbb{N}, k^n \leq x\}$.

We now establish the asymptotic formula for the counting function $N(x)$.

Theorem 1 If $N(x)$ denotes the number of sequence elements of (1) that are less than or equal to $x$, then $N(x) \sim \sqrt{x}$ as $x \to \infty$. 
A counting function for the sequence of perfect powers

Proof. The first step of the argument will be to obtain upper and lower functional bounds for $N(x)$. Assuming without loss of generality that $x \geq 4$, observe that $1 \in A_n(x)$ for each $n \in \mathbb{N}\{1\}$, but for $n$ sufficiently large $A_n(x)\{1\} = \emptyset$. Defining the auxiliary function $M(x) = \max\{n \in \mathbb{N}\{1\} : A_n(x)\{1\} \neq \emptyset\}$, we clearly see $M(x) \geq 2$, as $A_2(x)\{1\} \neq \emptyset$, and that $N(x)$ is equal to the number of elements of the set $A = \bigcup_{n=2}^{M(x)} A_n(x)$. Furthermore from the inequality $2^{\lceil \log_2 x \rceil} \leq x < 2^{\lceil \log_2 x \rceil + 1}$, it is immediately deduced that $M(x) = \lceil \log_2 x \rceil$. Since for large $x$ the family of sets $\{A_n(x)\}_{n=\lceil \log_2 x \rceil}^M$ are not mutually disjoint it follows that

$$N(x) = |A| \leq \sum_{n=2}^{\lceil \log_2 x \rceil} |A_n(x)| ,$$

and since $A_2(x) \subseteq A$, one also has

$$|A_2(x)| \leq |A| = N(x) .$$

Now as $A_n(x) \neq \emptyset$ there must exist a largest integer $m \geq 1$ such that $m^n \leq x < (m+1)^n$. By taking the $n$-th root through the previous inequality we deduce $m \leq \sqrt[n]{x} < m + 1$, that is $m = \lceil \sqrt[n]{x} \rceil$ and so $A_n(x)$ must contain $\lfloor \sqrt[n]{x} \rfloor$ elements. Consequently (2) and (3) together yield that

$$\lfloor \sqrt[n]{x} \rfloor \leq N(x) \leq \sum_{n=2}^{\lceil \log_2 x \rceil} \lceil \sqrt[n]{x} \rceil .$$

Using the upper and lower bounds in (4) we can establish the required asymptotic estimate for $N(x)$ as follows. Dividing (4) by $\sqrt[n]{x}$, observe for large $x$ the following chain of inequalities

$$\frac{\lfloor \sqrt[n]{x} \rfloor}{\sqrt[n]{x}} \leq \frac{N(x)}{\sqrt[n]{x}} \leq \frac{\lceil \sqrt[n]{x} \rceil}{\sqrt[n]{x}} + \sum_{n=3}^{\lceil \log_2 x \rceil} \frac{\lfloor \sqrt[n]{x} \rfloor}{\sqrt[n]{x}} \leq \frac{\lfloor \sqrt[n]{x} \rfloor}{\sqrt[n]{x}} + \sum_{n=3}^{\lceil \log_2 x \rceil} \frac{\sqrt[n]{x}}{\sqrt[n]{x}}$$

$$\leq \frac{\lfloor \sqrt[n]{x} \rfloor}{\sqrt[n]{x}} + \sum_{n=3}^{\lceil \log_2 x \rceil} \frac{\sqrt[n]{x}}{\sqrt[n]{x}} = \frac{\lfloor \sqrt[n]{x} \rfloor}{\sqrt[n]{x}} + \frac{\lceil \log_2 x \rceil - 2}{\sqrt[n]{x}} .$$

Via an application of L’Hopital’s rule, it is easily seen that

$$0 \leq \frac{\lfloor \log_2 x \rfloor - 2}{\sqrt[n]{x}} < \frac{\log_2 x}{\sqrt[n]{x}} \to 0$$
as $x \to \infty$, moreover by recalling $\lim_{x \to \infty} \lfloor x \rfloor / x = 1$, we finally deduce from (5) that $N(x) / \sqrt[n]{x} \to 1$ as $x \to \infty$. □

Remark Since the number of perfect squares less than or equal to $x$ is given by $\lfloor \sqrt[n]{x} \rfloor$ and as $\lfloor \sqrt[n]{x} \rfloor \sim \sqrt[n]{x}$ we can interpret Theorem 2.1 as stating that the perfect squares dominate the count of the sequence elements of (1) as $x \to \infty$.

3 An exact formula

One of the earliest known sieve methods was a simple effective procedure for finding all prime numbers up to a certain bound $x$. This procedure which involves the systematic deletion of all multiples of primes less than or equal to $\sqrt[n]{x}$ was captured succinctly by Legendre using a theoretical analog of the sifting process, known today as the Inclusion-Exclusion Principal, to study the prime counting function $\pi(x) = |\{p \leq x : p \text{ a prime }\}|$. His method led to an
exact formula for the number of primes in the interval \((\sqrt{x}, x]\) in particular, if \(\mu(\cdot)\) denotes the Möbius function then

\[
\pi(x) - \pi(\sqrt{x}) = -1 + \sum_{d \mid P_x} \mu(d) \left\lfloor \frac{x}{d} \right\rfloor ,
\]

where the sum is taken over all divisors of \(P_x = \prod_{p \leq \sqrt{x}} p\) (see [2, pg.15]). In this section we shall employ the same elementary sieve method of Legendre to establish an exact formula for the counting function \(N(x)\) which is similar in form to (6). We begin with a technical lemma for the sets of Definition 2.1.

**Lemma 2** For any set of \(m\) positive integers \(\{n_1, \ldots, n_m\}\) all greater than unity

\[
\bigcap_{i=1}^{m} A_{n_i}(x) = A_{[n_1,\ldots,n_m]}(x) ,
\]

where \([n_1, \ldots, n_m]\) denotes the least common multiple of the \(m\) integers \(n_1, \ldots, n_m\).

**Proof.** We begin by demonstrating that \(A_n(x) \cap A_m(x) = A_{[n,m]}(x)\) for any \(n, m \in \mathbb{N}\setminus\{1\}\), which is the base step of our inductive argument. Now since \(n\) divides \([n,m]\) and \(m\) divides \([n,m]\), any number of the form \(k^{[n,m]}\) where \(k \in \mathbb{N}\), can be rewritten as a perfect power having an exponent \(n\) and \(m\), thus \(A_{[n,m]}(x) \subseteq A_n(x) \cap A_m(x)\). Let \(s \in A_n(x) \cap A_m(x)\) with \(s \neq 1\), then \(s = k_1^n = k_2^m\) for some \(k_1, k_2 \in \mathbb{N}\setminus\{1\}\). We have to produce a \(k \in \mathbb{N}\setminus\{1\}\) such that \(s = k^{[n,m]}\). As \(k_1^n = k_2^m\) both \(k_1\) and \(k_2\) must have the same prime divisors.

Writing \(k_1 = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}\) and \(k_2 = p_1^{\beta_1} p_2^{\beta_2} \cdots p_r^{\beta_r}\) we deduce from the equality \(k_1^n = k_2^m\) that \(n\alpha_i = m\beta_i\) for each \(i = 1, 2, \ldots, r\). Consequently \(n\) divides \(n\alpha_i\) and \(m\) divides \(m\beta_i\) and so \(n\alpha_i = [n,m]\gamma_i\) for some \(\gamma_i \in \mathbb{N}\). Thus \(s = k^{[n,m]}\) where \(k = p_1^{\gamma_1} p_2^{\gamma_2} \cdots p_r^{\gamma_r}\) which establishes that \(A_n(x) \cap A_m(x) \subseteq A_{[n,m]}(x)\).

Now suppose for \(m > 1\) the set identity in (7) holds for an arbitrary set of \(m\) positive integers \(\{n_1, \ldots, n_m\}\) all greater than unity. Then as \([n_1, n_2, \ldots, n_m, n_{m+1}] = [n_1, \ldots, n_m, 1] = [n_1, \ldots, n_m, 1]\) observe from the inductive assumption and the base step that

\[
\bigcap_{i=1}^{m+1} A_{n_i}(x) = (\bigcap_{i=1}^{m} A_{n_i}(x)) \cap A_{n_{m+1}}(x) = A_{[n_1,\ldots,n_m]}(x) \cap A_{n_{m+1}}(x) = A_{[n_1,\ldots,n_{m+1}]}(x) .
\]

Hence (7) holds for \(m + 1\) arbitrary positive integers greater than unity and so the result is established by the principal of mathematical induction. \(\Box\)

**Theorem 2** If \(x \geq 4\) then the counting function for the sequence in (1) is given by the explicit expression

\[
N(x) = \lfloor x \rfloor - \sum_{d \mid P_x} \mu(d) \lfloor \frac{x}{d} \rfloor ,
\]

where the sum is taken over all divisors of \(P_x = \prod_{p \leq \log_2 x} p\).

**Proof.** We begin by establishing a slight reformulation for the set \(A\) of Theorem 2.1. Recalling that \(A = \bigcup_{n=2}^{\log_2 x} A_n(x)\), we claim that if \(p_1, \ldots, p_m\) are the first \(m\) primes less than or equal to \(\lfloor \log_2 x \rfloor\), then in fact \(A = B\) where

\[
B = \bigcup_{r=1}^{m} A_{p_r}(x) .
\]
The inclusion $B \subseteq A$ follows automatically by definition as each set $A_{p_i}(x)$ is included in the union of sets which form $A$. To establish the reverse inclusion $A \subseteq B$, first observe that as $p_1, \ldots, p_m$ represent the complete list of primes less than or equal to $\lfloor \log_2 x \rfloor$, every integer $n \in \{2, 3, \ldots, \lfloor \log_2 x \rfloor\}$ must be divisible by at least one of these primes since otherwise, by the fundamental theorem of arithmetic, $n$ would be divisible by a prime $p' > \lfloor \log_2 x \rfloor$ and so $n > \lfloor \log_2 x \rfloor$, a contradiction. Consequently if given any $s \in A_n(x)$, then $s = k^\gamma$ and one may write $n = p_i \gamma$ for some $r \in \{1, 2, \ldots, m\}$ and $\gamma \in \mathbb{N}$. Thus $s = (k^\gamma) p_i \in A_{p_i}(x)$ and so every element of $A$ is contained in the set $B$.

Now $N(x) = |A| = |B|$ and since for large values of $x$ the family of sets $\{A_{p_i}(x)\}_{i=1}^m$ are not mutually disjoint we deduce from an application of the Inclusion-Exclusion Principle applied to the set $B$ that

$$N(x) = \sum_{k=1}^{m} (-1)^{k+1} \sum_{1 \leq i_1 < \cdots < i_k \leq m} |A_{p_{i_1}}(x) \cap \cdots \cap A_{p_{i_k}}(x)|,$$

(9)

where the expression $1 \leq i_1 < \cdots < i_k \leq m$ indicates that the sum is taken over all ordered $k$-element subsets $\{i_1, \ldots, i_k\}$ of the set $\{1, 2, \ldots, m\}$. As the least common multiple of the $k$ prime numbers $p_{i_1}, \ldots, p_{i_k}$ is clearly the product $d = p_{i_1} p_{i_2} \cdots p_{i_k}$, observe from Lemma 3.1 that $|A_{p_{i_1}}(x) \cap \cdots \cap A_{p_{i_k}}(x)| = |A_d(x)| = \lfloor x^{\frac{1}{d}} \rfloor$, noting here we have again used the fact that the number of elements in the set $A_n(x)$ is $\lfloor x^\frac{1}{\gamma} \rfloor$. Defining $P_x = \prod_{p \leq \lfloor \log_2 x \rfloor} p$ we see that for each $k \in \{1, 2, \ldots, m\}$ the inner summation in (9) consists of adding $\binom{m}{k}$ terms of the form $\lfloor x^{\frac{1}{d}} \rfloor$, where $d = p_{i_1} p_{i_2} \cdots p_{i_k}$ is a divisor of $P_x$ having $k$ distinct prime factors. Consequently as $\mu(p_{i_1} p_{i_2} \cdots p_{i_k}) = (-1)^k$ the double summation in (9) must sum terms of the form $-\mu(d) \lfloor x^{\frac{1}{d}} \rfloor$ over all divisors $d$ of $P_x$ excluding $d = 1$. Finally by recalling that $\mu(1) = 1$ we deduce that the right hand side of (9) reduces to the right hand side of (8). □

Remark An immediate consequence of Theorem 3.1 is that the number of non-perfect powers less than or equal to $x$ is equal to $\sum_{d|P_x} \mu(d) \lfloor x^{\frac{1}{d}} \rfloor$.

4 Numerical example

We examine now how the explicit expression for $N(x)$ in (8) can be practically implemented to compute the number of perfect powers less than or equal to a given large positive real $x$.

For notational convenience let the inner summation of (9) be denoted by

$$S_k(x) = \sum_{1 \leq i_1 < \cdots < i_k \leq m} \lfloor x^{(p_{i_1} \cdots p_{i_k})^{-1}} \rfloor.$$

Observe that in order to evaluate each $S_k(x)$, one must sum the terms $\lfloor x^{(p_{i_1} \cdots p_{i_k})^{-1}} \rfloor$ over those subscripts $i_1 < \cdots < i_k$ whose values are chosen from the ordered $k$-element subsets of $\{1, 2, \ldots, m\}$, consequently the number of summands is $\binom{m}{k}$. Thus on first acquaintance, it would appear that the calculation of $S_k(x)$ would involve having to determine for each $1 \leq k \leq m$, all $\binom{m}{k}$ combinations of prime numbers from the set $\{p_1, \ldots, p_m\}$. However, for sufficiently large $x$ this may not be necessary since for certain values of $k$ one can show that $S_k(x) = \binom{m}{k}$ as follows.

To begin, consider for any $x > 2$ the arithmetic function $k(x) = \min\{k \in \mathbb{N} : p_{i+1} p_{i+2} \cdots p_k > x\}$, where again $p_i$ denotes the $i$-th prime number. We wish to first show that if there are $m$ primes less than or equal to $\lfloor \log_2 x \rfloor$, then $k(\lfloor \log_2 x \rfloor)$ will be at most $m - 2$ when $m > 5$. Recalling for any $n \geq 2$, there exists a prime strictly between $n$ and $2n$ (Bertrand’s
Postulate), observe as each \( p_i \geq 2 \), that
\[
p_{m-5} p_{m-4} (p_{m-3} p_{m-2}) > p_{m-5} (p_{m-4} p_{m-1}) > p_{m-5} p_m > p_{m+1} > \lfloor \log_2 x \rfloor.
\]
Thus when \( m > 5 \) we have \( p_1 \cdots p_{m-2} > \lfloor \log_2 x \rfloor \) and so \( k(\lfloor \log_2 x \rfloor) \leq m - 2 \). Now for \( \lfloor \log_2 x \rfloor \geq p_5 = 11 \) and \( k \geq k(\lfloor \log_2 x \rfloor) \) we note that in the summation \( S_k(x) \) all \( \binom{m}{k} \) combinations of products \( p_1 \cdots p_k \geq p_1 \cdots p_k(\lfloor \log_2 x \rfloor) > \lfloor \log_2 x \rfloor \). Consequently from the inequality \( 2^{\lfloor \log_2 x \rfloor} \leq x < 2^{\lfloor \log_2 x \rfloor + 1} \) it is immediate that
\[
1 < 2^{\lfloor \log_2 x \rfloor (p_1 \cdots p_k)^{-1}} \leq x(p_1 \cdots p_k)^{-1} < 2^{\lfloor \log_2 x \rfloor + 1 (p_1 \cdots p_k)^{-1}} \leq 2.
\]
Thus \( [x(p_1 \cdots p_k)^{-1}] = 1 \) and so the summation \( S_k(x) \) must consist of adding \( \binom{m}{k} \) terms all of which are identically 1, that is \( S_k(x) = \binom{m}{k} \). Hence for \( x > 2^{10} = 1024 \) the number of perfect powers less than or equal to \( x \) can be calculated by the alternate expression
\[
N(x) = \sum_{k=1}^{k(\lfloor \log_2 x \rfloor)-1} (-1)^{k+1} S_k(x) + \sum_{k=k(\lfloor \log_2 x \rfloor)}^{m} (-1)^{k+1} \binom{m}{k} \quad (10).
\]
For \( x > 2^{11} \) the value of the arithmetic function \( k(\lfloor \log_2 x \rfloor) \) will in practice be much smaller than the number of primes less than or equal to \( \lfloor \log_2 x \rfloor \), consequently in calculating \( N(x) \), we shall only have to evaluate \( S_k(x) \) for the few values of \( 1 \leq k < k(\lfloor \log_2 x \rfloor) \). In what follows the reader may wish to consult the table of perfect powers less than or equal to \( 10^9 \) by Serhat Sevki Dincer in [1].

**Example** Consider \( x = 2^{18} = 262144 \). From the table of perfect powers one can by inspection deduce that \( N(x) = 583 \). To demonstrate the use of (8) we shall apply the alternate expression in (10) to verify the number of perfect powers less than or equal to \( x \) is 583. Now \( \lfloor \log_2 x \rfloor = 18 \) and so there are \( m = 7 \) primes, namely 2, 3, 5, 7, 11, 13, 17 less than \( \lfloor \log_2 x \rfloor \). As \( 2 \cdot 3 \cdot 5 > 18 > 2 \cdot 3 \) we have that \( k(\lfloor \log_2 x \rfloor) = 3 \) and so from (10)
\[
N(x) = S_1(x) - S_2(x) + \sum_{k=3}^{7} (-1)^{k+1} \binom{7}{k} \quad (11).
\]
Using a calculator one finds in this instance that
\[
S_1(x) = \left[ \sqrt{2^{18}} \right] + \left[ \sqrt[3]{2^{18}} \right] + \left[ \sqrt[4]{2^{18}} \right] + \left[ \sqrt[5]{2^{18}} \right] + \left[ \sqrt[6]{2^{18}} \right] + \left[ \sqrt[7]{2^{18}} \right] = 512 + 64 + 12 + 5 + 3 + 2 + 2 = 600.
\]
To evaluate \( S_2(x) \) first recall from definition
\[
S_2(x) = \sum_{1 \leq i_1 < i_2 \leq 7} \left[ 2^{18(p_{i_1} p_{i_2})^{-1}} \right].
\]
Now if \( p_{i_1} p_{i_2} > 18 \) then \( \left[ 2^{18(p_{i_1} p_{i_2})^{-1}} \right] = 1 \). However, of the \( \binom{7}{2} = 21 \) combinations of products \( p_{i_1} p_{i_2} \) with \( 1 \leq i_1 < i_2 \leq 7 \), the only products less than 18 are \( 2 \cdot 3, 2 \cdot 5, 2 \cdot 7 \) and \( 3 \cdot 5 \). Thus the summation \( S_2(x) \) will consist of adding 21 \(- 4 = 17 \) terms all of which are identically 1, together with the sum of the terms \( \left[ \sqrt[2]{2^{18}} \right], \left[ \sqrt[3]{2^{18}} \right], \left[ \sqrt[4]{2^{18}} \right] \) and \( \left[ \sqrt[7]{2^{18}} \right] \), which are 8, 3, 2 and 2 respectively. Consequently \( S_2(x) = 17 + 8 + 3 + 2 + 2 = 32 \) and so finally adding in the alternating sum of binomial coefficients in (7) yields
\[
N(x) = 600 - 32 + 35 - 35 + 21 - 7 + 1 = 583,
\]
as required.
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School of Mathematics and Geospatial Science, RMIT University, GPO Box 2476V, Melbourne VIC 3001, E-mail: michael.nyblom@rmit.edu.au

Received 27 July 2006, accepted for publication 10 August 2006.
The Shoelace Book
A Mathematical Guide to the Best (and Worst) Ways to Lace your Shoes
Burkard Polster
The American Mathematical Society
ISBN 0821839330

This little gem is precisely what its subtitle indicates. It provides analyses of how much string is required to tie shoelaces in various canonical patterns. These are mathematical, frictionless, inelastic shoelaces, and the analyses are combinatorial and geometric. That is, it is explicitly not a book of (topological) knot theory. To date only a handful of research papers on this topic have been published; this book constitutes a major statement of current knowledge.

The analyses are elegant, simple, and should be accessible to a reader with a basic understanding of calculus. The book has a formal mathematical layout, and is very readable. Beyond that, it must be mentioned that it is beautiful! Apart from many diagrams, it contains photos of real shoelaces and even cartoons relevant to the topic — recall Charlie Brown’s problems. For good measure, it provides appendices containing cultural and historical notes.

This book is valuable in that it may motivate high school and undergraduate students to consider studying more mathematics for its own sake, as it illustrates that high school mathematics may be satisfactorily applied to practical design problems. I recommend it highly to school and public libraries.

David De Wit
E-mail: Dr_David_De_Wit@yahoo.com.au

99 Points of Intersection
Hans Walser & Jean Pedersen
Mathematical Association of America
ISBN 0883855534

99 Points of Intersection is largely composed of a collection of 99 geometric figures where 3 or more lines or circles have a common point of intersection, with many of the figures having a certain aesthetic appeal. It has recently been translated from the original German, and is marketed at high school and undergraduate students.

The book is divided into three chapters. The first chapter serves as an introduction, and has some brief discussion of examples of intersection, ranging from concurrency of diagonals of a regular dodecagon, to interesting interference patterns generated by various families of functions.

The second chapter is presented largely visually, with a single page key at the beginning. Here 99 separate constructions are presented as a sequence of 3 diagrams outlining the construction process, with a fourth larger diagram for clarity. Each of these figures appear to have points of concurrency, the challenge is to demonstrate that these actually are points of concurrency. A motivated high school student could likely provide proofs for the first 11 diagrams, with the complexity of the problems increasing through the chapter.

The third chapter is a discussion of some general methods of proof, including software aided proofs, Ceva’s theorem and invariance under various transformations. Some remarks on selected figures that appear in Chapter two are also provided.

Many readers will find sources of frustration in this book. The student is likely
to be frustrated by sections such as §1.1.2 and §1.1.3, page 4. In §1.1.2 a simple puzzle is presented using a regular dodecagon that can be solved by analysing angles and applying the law of sines. In §1.1.3 the following sentence appears:

Precisely, we can interpret the circumcircle of the dodecagon with the diagonals as chords as the Klein model of this geometry, and the same circle with the arcs orthogonal to it in its interior as the Poincaré model.

I fear that even the keen student may feel as though, in half a page, the book suddenly changed gears on them.

The more advanced reader may be frustrated by Section §1.3.3, where the first 30 Chebyshev polynomials are superimposed. Comment is made that various curves appear in the interference pattern generated, such as a parabola and a Lissajous curve, however the comment is limited to two brief paragraphs with no discussion on why such curves may appear.

The book has some interesting puzzles and does try to present some results in Euclidean geometry that the target audience would most likely not have had exposure to. My main criticism would be that in places the material is perhaps not self contained enough, particularly for high school students who would probably not have the means to chase down all the references given. The translators also note that the majority of the references are in German, however they have added some publications that are in English. The book would be best suited to a student with ready access to someone with a more advanced background, or perhaps as a reference for someone building a short interest course for high school or early undergraduate students.

Bob Scealy
Centre for Mathematics and its Applications
Mathematical Science Institute
Australian National University
Canberra ACT 0200.
E-mail: bob.scealy@maths.anu.edu.au

Graph Algebras
CBMS regional conference series in mathematics

Iain Raeburn
American Mathematical Society 2005
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C*-algebras arise in a wide variety of contexts. The present book, expanding on a set of lectures presented by the author in 2004, describes how they can arise from directed graphs. The first four chapters provide a general introduction to the theory, while the final six chapters cover more specialist aspects and generalizations.

The book is aimed at "a reader who has taken a first course in C*-algebras, covering the Gelfand-Naimark Theorems, the continuous functional calculus, and positivity". Such is the influence of the group at Newcastle on operator theory in Australia, that most readers of the Gazette who meet this criterion are probably already familiar with the book. Indeed, given that over 50 of the articles on graph C*-algebras from the bibliography are written by current or former Newcastle staff members, it may be that a majority of prospective Australian readers are already experts in the field. For this reason, my review will be aimed at those with no familiarity with C*-algebras, to give them a very general introduction to the topic of the book.

Before this general introduction, I should point out that the author has clearly thought carefully about how to present the basic theory, using the simplest and clearest approaches. Then, in the final six specialist chapters, he has highlighted many interesting aspects of the field. Overall, the book is strongly recommended to all those with a basic background in C*-algebras who want a general introduction to the theory of graph algebras.

Graph algebras arise from the geometry of inner product spaces. If \( K \) is a subspace of the complex \( n \)-dimensional Euclidean space \( H = \mathbb{C}^n \) (or, more generally if
If \( K \) is a closed subspace of a Hilbert space \( H \), then every element \( h \) of \( H \) can be written uniquely as \( h = h_1 + h_2 \), where \( h_1 \) belongs to \( K \) and \( h_2 \) is orthogonal to \( K \). The mapping \( P \) defined by \( P(h) = h_1 \) satisfies \( P(h_1) = h_1 \) for all \( h \) and therefore \( P = P^* \). A simple calculation, using the orthogonality of \( h_1 \) and \( h_2 \), shows that it also satisfies \( P = P^* \), where \( P^* \) is the adjoint of \( P \) (determined by the property \( \langle Ph, k \rangle = \langle h, P^*k \rangle \) for all \( h, k \) in \( H \)). \( P \) is called the orthogonal projection onto \( K \); its image is \( K \) and its kernel is the orthogonal complement of \( K \).

If \( K_1 \) and \( K_2 \) are subspaces of \( H = \mathbb{C}^n \) with the same dimension and \( K_2 \) is isometrically isomorphic to the orthogonal complement of \( K_1 \), then there is a linear map \( S \) mapping the orthogonal complement of \( K_1 \) to 0 and mapping \( K_1 \) isometrically onto \( K_2 \). \( S \) is known as a partial isometry; it is characterized algebraically in terms of the orthogonal projections \( P_1, P_2 \) onto \( K_1, K_2 \) by the equations \( S^*S = P_1 \) and \( SS^* = P_2 \). \( P_1 \) is known as the initial projection of \( S \) and \( P_2 \) as its range projection.

Graph algebras are generated by families of partial isometries, with mutually orthogonal ranges, each associated with an edge of a directed graph. Infinite families are allowed, with some difficulties to overcome, but here I will just consider finite ones. So let \( G \) be a finite directed graph and, for each vertex \( v \) of \( G \), let \( P_v \) be an orthogonal projection, with the property that each pair of vertex projections \( P_v, P_w \) satisfy \( P_vP_w = 0 \); this corresponds to the associated subspaces being orthogonal. For each (directed) edge \( e \) of \( G \), with source vertex \( s(e) \) and range vertex \( r(e) \), let \( S_e \) be a partial isometry satisfying

\[
S_e^*S_e = P_{s(e)}. \tag{CK1}
\]

The graphical structure impacts on the algebra by means of the additional condition, applied whenever any edges arrive at \( v \),

\[
P_v = \sum S_eS_e^*, \tag{CK2}
\]

where the sum is taken over all the edges \( e \) with range \( r(e) = v \). In recognition of the writers of the fundamental paper underlying the theory of graph algebras, collections of partial isometries and orthogonal projections obeying the relations (CK1) and (CK2) for some directed graph \( G \) are known as Cuntz-Krieger families.

For example, the directed graph

\[ \text{\includegraphics[width=0.2\textwidth]{graph.png}} \]

has an associated Cuntz-Krieger family of \( 2 \times 2 \) matrices: \( S_e = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \), \( S_f = e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \), \( P_v = S_v^*S_v = e_{21}e_{12} = e_{22} \) and \( P_w = S_f^*S_f = e_{11} \). Notice that (CK2) is satisfied because the only edge entering \( v \) is \( f \), with \( P_f = S_f^*S_f \), and the only edge entering \( w \) is \( e \), with \( P_w = S_v^*S_v \). The Cuntz-Krieger family contains each of the matrix units \( e_{11}, e_{12}, e_{21}, e_{22} \), so generates the \( * \)-algebra of all two by two matrices.

The directed graph above is also associated with the following Cuntz-Krieger family, consisting of pairs of \( 2 \times 2 \) matrices: \( S_e = (e_{12}, e_{12}) = (e_{21}, -e_{21}) \), with \( P_v = (e_{22}, e_{22}) \) and \( P_w = (e_{11}, e_{11}) \). The \( * \)-algebra generated by this family is not isomorphic to the algebra \( M_2(\mathbb{C}) \) of all \( 2 \times 2 \) matrices, but is instead the direct sum of two copies of \( M_2(\mathbb{C}) \): for example the element \( (e_{22}, 0) \) can be realized as \( \frac{1}{2}(S_f^*S_f + S_fS_f) \).

The graph algebra \( A \) associated with a directed graph is not only generated by a Cuntz-Krieger family for the graph, but also has a universal property: every other \( C^* \)-algebra generated by a Cuntz-Krieger family for the graph is a quotient of \( A \). For the graph above, constructing \( A \) involves moving outside the finite dimensional world and taking the norm closure to produce the \( C^* \)-algebra \( A = C(S^4, M_2(\mathbb{C})) \) of continuous \( 2 \times 2 \) matrix-valued functions on the unit circle (as is explained in the book). As required by the universal property, both the two algebras \( M_2(\mathbb{C}) \) and \( M_2(\mathbb{C}) \oplus M_2(\mathbb{C}) \)
above are homomorphic images of \( A \), obtained by evaluating at a single point or at a pair of points.

Other directed graphs, such as a single vertex with \( n \) loops, require all associated Cuntz-Krieger families to be infinite-dimensional. For this example, all Cuntz-Krieger families generate isomorphic \( C^* \)-algebras (known as \( O_n \)) and these are simple, i.e. have no non-trivial norm closed ideals. The reasons for these facts, together with many other fascinating results linking the structure of the graphs to those of the algebras, can be found in the book under review.

Peter Stacey
Department of Mathematics and Statistics
La Trobe University, VIC 3086
E-mail: P.Stacey@latrobe.edu.au

103 Trigonometry Problems: From the Training of the USA IMO Team

Titu Andreescu and Zuming Feng
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103 Trigonometry Problems is the third of its nature by the two Authors. The first two being 101 Problems in Algebra and 102 Combinatorial Problems. All three books contain instructive problems and techniques used in the training and testing of the USA International Mathematical Olympiad (IMO) team.

As such the book is appropriate for any highly capable secondary school student. Of course it is suitable for any person who is interested in problem solving after the style of the IMO. As it turns out most Olympiad type mathematics problems are not trigonometric in nature, yet many can be cracked by using trigonometry in clever ways.

The book is divided into five sections and a glossary. The first being on trigonometric fundamentals, the next two sections containing the 103 problems and the last two sections containing solutions to those problems.

The first section gives the basic definitions, properties and proofs of the trigonometric functions. This includes the addition and subtraction formulas, the sum-to-product and difference-to-product formulas, the extended sine rule and the cosine rule. These are applied to derive the theorems of Ptolemy, Ceva, Menelaus, Stewart, Heron, Brahmagupta and De Moivre. There are a few worked examples of applications to problems in geometry and algebra, in particular inequalities. In a number of instances diversions outside of trigonometry are discussed. For example, the section “Think Outside the Box”, shows how by considering a dilation one is led to the construction, using only straight edge and compass, of a square inscribed inside a triangle. There is a section on vectors leading up to the Cauchy-Schwartz inequality which is interpreted as \(|u||v|/|u \cdot v| = |\cos \theta| \leq 1.\n
A section on complex numbers connects to De Moivre’s formula. The important limit \(\lim_{\theta \to 0}(\sin \theta)/\theta \) is discussed. Although this section gets off the ground quite quickly, there are some parts that are treated perhaps a little too carefully. For example, because the proofs of the addition formulas are appropriately geometric in nature, there is much paying of attention to noting that their proofs only apply to restricted intervals in the beginning. Yet this over attention to detail gets overlooked on page 9 by the use of the subtraction formula before it is developed.

The second section contains 52 introductory problems. The third section contains 51 advanced problems. Few of the problems are very geometric, in fact most are algebraic in nature. More than half of the advanced problems are inequalities. Nonetheless a wide array of problems that can be
tackled with a geometric approach are to be found. The fourth section contains solutions to the introductory problems. The fifth section contains solutions to the advanced problems. In both instances multiple solutions are often offered. The glossary contains supplementary material needed in the solutions to some of the problems.

Angelo Di Pasquale
Department of Mathematics and Statistics
The University of Melbourne, VIC 3010
E-mail: A.DiPasquale@ms.unimelb.edu.au

Research Problems in Discrete Geometry
Peter Brass, William Moser and János Pach
Springer
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When asked to review this book, my first thought was ‘What is discrete geometry?’ In attempting to answer this question I can do no better than quote the book’s preface by Paul Erdős:

As a matter of fact, I cannot even give a reasonable definition of the subject. Perhaps it is not inappropriate to recall the following old anecdote. Some years ago, when pornography was still illegal in America, a judge was asked to define pornography. He answered: “I cannot do this, but I sure can recognize it when I see it.”

After reading the book I too can recognize discrete geometry when I see it.

The book has its origins in a list of 14 problems originally proposed by Leo Moser and distributed by William Moser under the same title in 1977. At least seven further editions of problems have appeared since in various forms, culminating in the book under review.

This book contains a wealth of problems and conjectures divided into 11 chapters, with each chapter divided into many sections. Each section outlines the history of the problems included, details any progress made and has its own extensive bibliography.

The authors begin with problems concerning packings and coverings and there are three chapters of such problems. A packing is a collection of subsets of a domain such that no two subsets have an interior point in common, while a covering is a collection of subsets such that every point of the domain is contained in at least one subset. Restrictions may be placed on the type of subset used, for example spheres, or on the domain. A common theme is to determine the optimal density of the packing/covering and which configurations achieve this. The most famous problem in this area is Kepler’s Conjecture that the maximum density of a packing of equal balls in \( \mathbb{R}^3 \) is \( \pi/\sqrt{18} \). On the recent highly computational proof of this conjecture the authors state that ‘so far no one has found any serious gap in the approach of Hales and Ferguson, although no one has been able to fully verify it either.’ There is also a chapter on tilings, that is, a collection of subsets which is both a packing and a covering.

Another fertile problem area arises from configurations of points. Some of the main themes are: What is the maximum number of occurrences of the same distance between \( n \) points? The minimum number of distinct distances? The minimum number of distinct directions determined by the points? The minimum number of lines passing through precisely two of the points? There are also chapters on graph drawings, lattice point problems, and geometric inequalities.

All in all, the book under review is very comprehensive, almost encyclopedic in its treatment, and will make a very good reference, especially for those starting out in the field and looking for problems to work on.

Michael Giudici
The University of Western Australia
35 Stirling Highway, Crawley WA 6009
E-mail: giudici@maths.uwa.edu.au
Completed PhDs

Murdoch University:

University of Melbourne:
- Dr Christopher Fricke, *Applications of integer programming in open pit mining*, supervisor: Natashia Boland.
- Dr Maya Ramakrishnan, *Distributed approaches to capacity reallocation in networks*, supervisors: Peter Taylor and Andre Costa.

University of Western Australia:
- Dr Jing Xu, *On closures of finite permutation groups*, supervisors: Cheryl Praeger, Cai Heng Li and Michael Giudici.

Appointments

Swinburne University of Technology:
- Professor William Phillips has been appointed to Professor of Engineering Mathematics in the Faculty of Engineering & Industrial Sciences. He will become head of the Mathematics Discipline.
- Dr Geoff Brooks has been appointed Professor of Engineering Mathematics in the Faculty of Engineering & Industrial Sciences.

University of Melbourne:
- Dr Peter Milley and Dr Stephan Tillmann have been appointed Research Fellows.

University of Sydney:
- Dr Gordon Monro and Associate Professor Bill Gibson have recently retired.
- Dr Bartosz Trojan has been appointed as a postdoctoral fellow and Mr Yuezhu Wu as a research associate.
- Dr Clio Cresswell has been reappointed as Senior Lecturer for another five years.

University of Western Australia:
- Mr Kai Zhang has commenced as a Research Associate.
Awards and other achievements

Professor Tony Guttmann from the University of Melbourne was awarded the Szekeres Medal for 2006.

Dr Andrew Mathas from the University of Sydney was awarded the Australian Mathematical Society Medal for 2006.

Ben Wilson from the University of Sydney has been awarded the BH Neumann prize for the best talk by a student at the Annual Meeting of Australian Mathematical Society.

Professor Eugene Seneta, Emeritus Professor of Statistics of the University of Sydney, has been awarded the Moyal Medal for 2006. The Moyal Medal is awarded annually by Macquarie University and recognizes distinguished contributions to research in mathematics, physics or statistics.

New Books


Conferences

5th Ballarat workshop on Global and Non-Smooth Optimization: Theory, Methods and Applications
28–30 November 2006, University of Ballarat, Victoria
Conference Co-ordinator: Maxine Kingston
Enquiries: Maxine Kingston and Adil Bagirov
E-mail: m.kingston@ballarat.edu.au, a.bagirov@ballarat.edu.au

Summer School and Workshop on Granular Materials
4–8 December 2006, Australian National University, Canberra, VIC
Organisers: Peter Arnold

8th Pacific Rim Geometry Conference
11–15 December 2006, Murramarang, South Durras, NSW
Organisers: Ben Andrews, Alan Carey, Alexander Isaev, Neil Trudinger and Xu-Jia Wang
Amenability of Groups and Algebras
4–8 January 2007, Australian National University, Canberra, ACT
Organisers: H.G. Dales, A. Kepert, R.J. Loy and G.A. Willis

From Statistical Mechanics to Conformal and Quantum Field Theory (a Special Theme Program)
8 January–8 February 2007, Australian Mathematical Sciences Institute Headquarters, University of Melbourne, VIC
Organisers: Paul Pearce (Australia), Giuseppe Mussardo (Italy) and Chaiho Rim (Korea)

28 January–1 February 2007 (Statistics Day: 29 Jan), Esplanade Hotel, Fremantle, Western Australia
Conference Co-ordinator: Graeme Hocking.
Enquiries: Graeme Hocking, Duncan Farrow, Marty Firth (for Statistics Day)
E-mail: G.Hocking@murdoch.edu.au, D.Farrow@murdoch.edu.au, martyf@ichr.uwa.edu.au.
Web: http://www.anziam07.murdoch.edu.au

Invited speakers are:
Adrian Baddeley - University of Western Australia
Nigel Bean - University of Adelaide
Peter Clarkson - University of Kent, UK
Phil Howlett - University of South Australia
Ian James - Murdoch University
Greg Kriegsmann - New Jersey Institute of Technology
Mark M. Meerschaert - The University of Otago, NZ
Hinke Osinga - University of Bristol, UK
Graeme Wake - Massey University, NZ

Mathematics in Industry Study Group
5–9 February 2007, University of Wollongong, Wollongong Campus, NSW.
Organisers: Maureen Edwards and Tim Marchant
E-mail: maureen.edwards@uow.edu.au
Web: http://www.misg.math.uow.edu.au
Visiting mathematicians

Visitors are listed in the order of the last date of their visit and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

Prof. William Messing; University of Minnesota; 1 to 30 November 2006; p-adic Shimura varieties; USN; K.F. Lai
Dr Colva Mary Roney-Dougal; University of St. Andrews; 13 November to 1 December 2006; Algorithms for groups and Lie groups; USN; J.J. Cannon
Prof. Eric Rains; University of California Davis; 5 September to 3 December 2006; –; UMB;
Prof P.J Forrester
Dr Alex Kitaev; Steklov Institute; 11 August to 6 December 2006; Singularities and integrable systems; USN; N. Joshi
Dr Eamonn O’Brien; University of Auckland; 26 November - 6 December 2006; –; UWA; Prof Cheryl Praeger
Prof. Krzysztof Kurdyka; –; 24 to 9 December 2006; Hyperpolic polynomials; USN; L. Paunescu
Dr Peter Brooksbank; Bucknell University; 26 November - 10 December 2006; –; UWA; Prof Cheryl Praeger
Dr Dorette Pronk; Dalhousie University; 10 September to 15 December; Category Theory; MQU; Prof. Ross Street
Dr Natalia Kopteva; Université de Provence; 15 October to 15 December 2006; –; UMB; –
Prof. Fima Klebaner; Monash University; 15 October to 15 December 2006; –; UMB; –
Jonathan James; Uni of Cambridge; 1 November to 21 December 2006; –; UWA; Prof Cheryl Praeger and Dr Michael Giudici
Akos Seress; Ohio State University; 1 to 21 December 2006; Group theory and combinatorics; UWA; Cheryl Praeger
Dr Andreas Bley; Konrad-Zuse-Zentrum für Informationstechnik; 19 September to 22 December 2006; –; UMB; –
Mr Thomas Houtmann; Lix Ecole Polytechnique; 14 to 23 December 2006; Complex multiplication of genus 2 curves; USN; D.R. Kohel
Dr Zheng-Xue Tang; Deakin University; 1 January 2005 to 31 December 2006; Computer simulation of textile yarn spinning; USN; W.B. Fraser
Marta Remmenga; New Mexico State University; January to December 2006; –; UWA; Prof Adrian Baddeley
Prof. Dan Schafer; Oregon State University; 1 February to 31 December 2006; –; UWA; Dr Berwin Turlach
Mr Danker Adriaan Roozemond; Eindhoven University; 30 April to 31 December 2006; Algorithms for Lie Theory; USN; J.J. Cannon
Dr Duncan Farrow; Murdoch University; July to December 2006; –; UWA; Prof. Andrew Bassom
Dr Jesse Johnson; University of California Davis; 1 August to 31 December 2006; –; UMB; –
Dr Paul Hammerton; University of East Anglia; September to December 2006; –; UWA; Prof. Andrew Bassom
Prof. Satoshi Koike; Hyogo University; 22 December 2006 to 14 January 2006; Bilipshitz homeomorphisms; USN; L. Paunescu
Dr Damien Stehle; French Ministry of Education; 29 January 2006 to 28 January 2007; Lattice reduction; USN; J.J. Cannon
Mrs Horanage Chandrika Fernando; Sri Lanka Institute of Information Technology; 1 July 2006 to 31 January 2007; –; UMB; –
Prof. Eva Vedel Jensen; University of Aarhus; December 2006 to January 2007; Stereology; USN; Prof Adrian Baddeley
Dr Mark Watkins; University of Bristol; 26 November 2006 to 4 February 2007; Analytic number theory; USN; J.J. Cannon
Prof. Hechun Zhang; Tsingua University; 29 January to 25 February 2007; Lie algebras and quantum groups; USN; R. Zhang
Dr Pramod Achar; Louisiana State University; 18 to 26 February 2007; Geometric representation theory; USN; A. Henderson
Frederick Vercauteren; Catholic University Leuven; 2 September 2006 to 28 February 2007; Number Theory and Cryptography; USN; J.J. Cannon
Prof. Wieslaw Krawcewicz; University of Montreal; 15 January to 31 March 2006; Symmetric topological invariants and their application to non-linear elliptic partial differential equations; USN; E.N. Dancer
Dominic Schuhmacher; University of Zurich; 1 April 2006 to 31 March 2007; –; UWA; Prof. Adrian Baddeley
Dr Hidekazu Nagahata; Okayama University; 27 September 2006 to 31 March 2007; –; UMB; –
Mr Sergei Haller; Justus-Liebig-UniversitÃ¤t, Gießen; 16 January 2006 to 11 May 2007; Algorithmic methods for Lie groups; USN; S. Murray
Stephen Glasby; Central Washington University; Late February to June 2007; –; UWA; Cheryl Praeger
Mr Ioannis Soultatos, PhD Student; UCLA; 1 September 2006 to 31 July 2007; –; UMB; –
Prof. Buyung-Moo Kim; Chungju National; 31 July 2006 to 31 July 2007; Integral Theory; USN; D.E. Taylor
Prof. Nicholas Fisher; International Statistics Institute; 1 September 2004 to 31 August 2007; Statistics; J. Robinson
Jan Saxl; Cambridge University; mid-November 2006 to December 2007; –; UWA; Cheryl Praeger
Prof. Richard Cowan; University of Sydney; 1 January 2005 to 31 December 2007; Stochastic models; USN; J.Robinson
Dr Youyun Li; Hunan Changsha University; 1 May 2006 to 1 May 2008; –; UWA; Dr Song Wang
AustMS Accreditation

The secretary has announced the accreditation of:

Dr Ejanul Haque of RMIT University as an Accredited Member (MAustMS).

AustMS Special Interest Meeting Grants: call for applications

The Australian Mathematical Society sponsors Special Interest Meetings on specialist topics at diverse geographical locations around Australia. This activity is seen as a means of generating a stronger professional profile for the Society within the Australian mathematical community, and of stimulating better communication between mathematicians with similar interests who are scattered throughout the country.

These grants are intended for once-off meetings and not for regular meetings. Such meetings with a large student involvement are encouraged. If it is intended to hold regular meetings on a specific subject area, the organisers should consider forming a Special Interest Group of the Society. If there is widespread interest in a subject area, there is also the mechanism for forming a Division within the Society.

The rules governing the approval of grants are:

(a) each Special Interest Meeting must be clearly advertised as an activity supported by the Australian Mathematical Society;
(b) the organizer must be a member of the Society;
(c) the meeting must be open to all members of the Society;
(d) registration fees should be charged, with at least a 20% reduction for members of the Society. A further reduction should be made for members of the Society who pay the reduced rate subscription (i.e. research students, those not in full time employment and retired members);
(e) a financial statement must be submitted on completion of the Meeting;
(f) any profits up to the value of the grant are to be returned to the Australian Mathematical Society;
(g) on completion, a Meeting Report should be prepared, in a form suitable for publication in the Australian Mathematical Society Gazette, and sent to the Secretary;
(h) a list of those attending and a copy of the conference Proceedings (if applicable) must be submitted to the Society;
(i) only in exceptional circumstances will support be provided near the time of the Annual Conference for a Special Interest Meeting being held in another city.

In its consideration of applications, Council will take into account locations around Australia of the various mathematical meetings during the period in question. Preference will be given to Meetings of at least two days duration. The maximum allocation for any one Meeting will be $$(1,000 + 150n)$$ where $$n$$ is the number of AustMS members registered for and
attending the meeting, and with an upper limit of about $5,000. A total of up to $12,000 is available in 2007. There will be six-monthly calls for applications for Special Interest Meeting Grants, each to cover a period of eighteen months commencing six months after consideration of applications. Please email Secretary@austms.org.au for an application form.

Elizabeth J. Billington
AustMS Secretary

Nominations sought for the 2007 AustMS Medal

The Medal Committee for the 2007 Australian Mathematical Society Medal is now seeking nominations and recommendations for possible candidates for this Medal. This is one of two Medals awarded by the Society, the other being the George Szekeres Medal, which is awarded in even numbered years. The Australian Mathematical Society Medal will be awarded to a member of the Society for distinguished research in the Mathematical Sciences. Council have recently resolved that the age limit of 40 on 1st January 2007 may be relaxed in some circumstances; see rule (i) below.

For further information, please contact (preferably by email) the Chair of the 2007 Medal Committee, Professor W.W.L. Chen, Department of Mathematics, Macquarie University, NSW 2109, (wchen@maths.mq.edu.au). Nominations should be received by 28th February 2007.

The other three members of the 2007 Medal Committee are Professor A. McIntosh (Outgoing Chair), Professor H. Possingham (Incoming Chair) and Professor M. Varghese (one year).


Rules for the Australian Mathematical Society Medal

1. There shall be a Medal known as “The Australian Mathematical Society Medal”.
2. (i) This will be awarded annually to a Member of the Society, under the age of 40 on 1st January of the year in which the Medal is awarded, for distinguished research in the Mathematical Sciences. The AustMS Medal Committee may, in cases where there have been significant interruptions to a mathematical career, waive this age limit by normally up to five years.
   (ii) A significant proportion of the research work should have been carried out in Australia.
   (iii) In order to be eligible, a nominee for the Medal has to have been a member of the Society for the calendar year preceding the year of the award; back dating of membership to the previous year is not acceptable.
3. The award will be approved by the President on behalf of the Council of the Society on the recommendation of a Selection Committee appointed by the Council.
4. The Selection Committee shall consist of 3 persons each appointed for a period of 3 years and known as “Incoming Chair”, “Chair” and “Outgoing Chair” respectively, together with a fourth person appointed each year for one year only.
5. The Selection Committee will consult with appropriate assessors.
6. The award of the Medal shall be recorded in one of the Society’s Journals along with the citation and photograph.
(7) The Selection Committee shall also prepare an additional citation in a form suitable for newspaper publication. This is to be embargoed until the Medal winner has been announced to the Society.

(8) One Medal shall be awarded each year, unless either no one of sufficient merit is found, in which case no Medal shall be awarded; or there is more than one candidate of equal (and sufficient) merit, in which case the committee can recommend the award of at most two Medals.

Elizabeth J. Billington
AustMS Secretary
The Australian Mathematical Society

President:  Prof. M. Cowling
School of Mathematics
University of New South Wales
NSW 2052, Australia.
President@austms.org.au

Secretary:  Dr E.J. Billington
Department of Mathematics
University of Queensland
QLD 4072, Australia.
ejb@maths.uq.edu.au

Treasurer:  Dr A. Howe
Department of Mathematics
Australian National University
ACT 0200, Australia.
algy.howe@maths.anu.edu.au

Business Manager:  Ms May Truong
Department of Mathematics
Australian National University
ACT 0200, Australia.
office@austms.org.au

Membership and Correspondence

Applications for membership, notices of change of address or title or position, members’ subscriptions, correspondence related to accounts, correspondence about the distribution of the Society’s publications, and orders for back numbers, should be sent to the Treasurer. All other correspondence should be sent to the Secretary. Membership rates and other details can be found at the Society web site: http://www.austms.org.au.

Local Correspondents

ANU:  J. Cossey
Aust. Catholic Univ.:  B. Franzsen
Aust. Defence Force:  R. Weber
Bond Univ.:  N. de Mestre
Central Queensland Univ.:  R. Stonier
Charles Darwin Univ.:  I. Roberts
Charles Sturt Univ.:  J. Louis
CSIRO:  T. McGinnes
Curtin Univ.:  J. Simpson
Deakin Univ.:  L. Batten
Edith Cowan Univ.:  L. Bloom
Flinders Univ.:  R.S. Booth
Griffith Univ.:  H.P.W. Gottlieb
James Cook Univ.:  S. Belward
La Trobe Univ. (Bundoora):  P. Stacey
La Trobe Univ. (Bendigo):  J. Schutz
Macquarie Univ.:  R. Street
Monash Univ.:  S. Clarke
Murdoch Univ.:  M. Lukas
Queensland Univ. Techn.:  G. Pettet
RMIT Univ.:  Y. Ding
Swinburne Univ. Techn.:  J. Sampson
Univ. Adelaide:  D. Parrott
Univ. Ballarat:  P. Manyem
Univ. Canberra:  P. Vassiliou
Univ. Melbourne:  B. Hughes
Univ. New England:  J. MacDougall
Univ. New South Wales:  I. Bokor
Univ. Queensland:  H. Thompson
Univ. South Australia:  J. Hewitt
Univ. Southern Queensland:  S. Suslov
Univ. Sydney:  M.R. Myerscough
Univ. Tasmania:  B. Gardner
Univ. Technology Sydney:  E. Lidums
Univ. Western Australia:  V. Stefanov
Univ. Western Sydney:  R. Ollerton
Univ. Wollongong:  R. Nillson
Victoria Univ. Techn.:  P. Cerone
Univ. Canterbury:  C. Price
Univ. Waikato:  W. Moors
Publications

The Journal of the Australian Mathematical Society
Editor: Prof. C.F. Miller
Department of Mathematics and Statistics
The University of Melbourne
VIC 3010
Australia

The ANZIAM Journal
Editor: Prof. C.E.M. Pearce
Department of Applied Mathematics
The University of Adelaide
SA 5005
Australia

Bulletin of the Australian Mathematical Society
Editor: Dr A.S. Jones
Bulletin of the Australian Mathematical Society
Department of Mathematics
The University of Queensland
QLD 4072
Australia

The Bulletin of the Australian Mathematical Society aims at quick publication of original research in all branches of mathematics. Two volumes of three numbers are published annually.

The Australian Mathematical Society Lecture Series
Editor: Prof. M. Murray
Department of Pure Mathematics
The University of Adelaide
SA 5005
Australia

The lecture series is a series of books, published by Cambridge University Press, containing both research monographs and textbooks suitable for graduate and undergraduate students.