My brilliant career

Peter Gould

Where can a mathematics education lead you? In this series, mathematics and statistics graduates from Australian universities write about their careers, proving that the world is their oyster. Peter Gould is a teacher, and Chief Education Officer in Mathematics at the NSW Department of Education and Training. Below he details the joys and pitfalls of effective teaching in mathematics, and the fight against the Brussels Sprouts effect — most people believe that mathematics is good for you yet very few want to consume it themselves.

Teaching mathematics

"The title which I most covet is that of teacher…Happy is the person who comes to understand something and then gets to explain it." (Marshall M. Cohen)

It is 4.30 a.m. on a Friday morning in August. The alarm has just gone off and I recall that I set the alarm 30 minutes earlier than usual so that I would not be late for breakfast. This morning, breakfast is a Mathematics Roundtable discussion at Sydney University, facilitated by Adam Spencer. As I drive to the roundtable breakfast I think about the workshop I ran yesterday for the Newcastle Mathematics Association. The enthusiasm and professionalism of the 50 mathematics teachers who attended the workshop is fresh in my mind. I am welcomed to the roundtable discussion by Professor Gavin Brown and Professor David Day. The topics under discussion include the shortage of secondary mathematics teachers and changes in enrolment patterns in mathematics courses in New South Wales.

One of the questions that people want to know the answer to is, do we have a shortage of secondary mathematics teachers in NSW and if so, how large is the shortfall? I have discussed this question with colleagues from the area of workforce planning within the NSW Department of Education and Training, and now at least I understand the nature of the question. One of the advantages of having a background in mathematics is that it gives you an appreciation of just how difficult it can be to answer apparently easy questions.

As much as we may wish otherwise, the challenge of staffing secondary mathematics classrooms in NSW does not lend itself to an application of the Dirichlet Principle. *If m pigeons are put into m pigeonholes, there is an empty hole if and only if there's a hole with more than one pigeon.* There may well be more secondary mathematics teachers than teaching positions, yet the pigeons are not prepared to travel to the pigeonholes! If you add in the age of the teaching population, you also have some aging pigeons. The projections as to whether supply will meet demand are at best "seen through a glass darkly", and the uncertainty surrounding projections does not allay anyone's fears.

My current occupation is the state manager of the mathematics learning area, also described as the Chief Education Officer, for NSW government schools. It is a role, like many others, well described by the final sentence of the role statement; "and other duties as required". I should mention from the

outset that although I have the word mathematics in my job description, I do not call myself a mathematician. The problems that I deal with each day are sometimes assisted by mathematics but they are not of themselves mathematical.

By inclination and training, I am a teacher. I started teaching mathematics to high school students in the Western suburbs of Sydney in 1978 and completed a Masters degree in Mathematics (part-time) by 1982. I was promoted to head of department in 1988 and spent five years as the Mathematics head teacher at Canley Vale High School, a comprehensive high school of 1200 students in the outer suburbs of Sydney. In the three years following this, I worked as a Regional Mathematics Consultant, supporting teachers and heads of departments in the South West of Sydney. It was during this time that I started to learn about the way that young children develop their understanding of mathematics concepts. I came to this task with an immense treasure, my ignorance. In 1995 I moved to my current role with state responsibility for support in the teaching of mathematics. I provide advice on the teaching of mathematics. I also manage support for teachers in implementing the mathematics syllabus from Kindergarten to Year 12 in government schools.

Why teaching?

Some students have a clear vision of what occupation they will pursue when they leave school. I was not one of those students. Two factors led to my career choice. The first was that I was offered a teacher's scholarship and the second was some pragmatic advice from a good friend. Having some financial support to gain a degree was important and the guarantee of a job when I finished was a bonus. It could be said that I stumbled into mathematics teaching, yet it was a happy stumble.

When I first started teaching, I remember being surrounded by other mathematics teachers who also loved mathematics. Perhaps that is why mathematics teachers were sometimes considered a little strange. They enjoyed conquering mathematical challenges. They thrived on the excitement of finding something that you did not know and coming to know it as irrefutably proven! Strange indeed are the things that excite mathematics teachers. Even today, mathematics teachers are often believed to be a little different.

With the arrogance of youth, a trait I sometimes miss, after about four years I thought that I knew everything that I needed to about teaching. In the following years I revised my assessment of what I knew. I subsequently came to believe that I had not even managed to get the questions right. Although I could keep a class of students busy and quiet for significant periods of time, I was not certain that they were all learning. In fact, when I got around to asking myself this question, my honest answer would have been that I did not really expect them all to learn everything that I taught. If I gave a test at the end of a topic and everyone had managed to get close to 100%, I would have concluded that I had made the test too easy!

Spending time being both a student and a teacher helped to bring some of my beliefs about teaching and learning into the light. Perhaps the most poignant memory is of discovering the difference between what a good assessment looks like from the point of the assessor compared to the one who is assessed. As a student, I found I was far less fond of the "good assessment to really sort them out" than I might otherwise have been as a teacher. The words "fair and balanced" were more likely to come to mind when assessment was viewed from the perspective of the learner. This time in the dual role of teacher and student also helped to shape one of my central beliefs about teaching. Expressed simply, I believe that it is

almost impossible to be an effective teacher if you have forgotten what it is like to be a learner. I have watched teachers, usually early in their careers, struggle to understand why their students could not see what to them was obvious. These teachers had forgotten what it was like "not to know".

Good mathematics teaching is not necessarily a new mode of teaching. In an old-fashioned mathematics classroom, the junior students talk quietly to each other as they await the arrival of their teacher. The teacher enters the room quietly, walks to the chalkboard, lifts his right foot and stamps it firmly on the chalkboard. You can clearly see the outline of the teacher's boot on the board. The sound and the unusual action have silenced the class. The teacher slowly scans the faces of the students to be certain that he has their undivided attention. Into that moment of anticipation he states, "How are you going to find the area of that?" A lesson on finding the area of irregular figures has begun.

Although, in many ways, effective teaching is a performing art, it is not simply a series of gimmicks or a bag of tricks. A good mathematics lesson is often like a good story. A good story is highly organised; it has a beginning, a middle, and an end. The story often follows a protagonist who meets challenges and resolves problems that arise along the way.

Just as good literature never goes out of fashion, neither does good teaching. The use of technology can enhance mathematics teaching but good teaching is not dependent upon digital technology. I have taught mathematics to children and adults using drinking straws, pieces of string, golf balls, glass marbles and pieces of paper. Whether the mathematics was basic or advanced was not a function of the materials used but rather the questions asked.

Which mathematics?

One of the enduring enquiries of mathematics students is, "When are you ever going to use this?'" Mathematics teachers hear the question so often that they become quite creative in their answers. First there is the pragmatic answer, "In your mathematics test." Then there is the earnest answer, "You are learning about quadratic functions because the reflector inside a car's headlight is parabolic!" And finally, there is the plain silly: "Sine curves will be useful when you become a hair dresser in creating natural looking waves."

Clearly knowing something of the applications of mathematics is of value to mathematics teachers. It is the 'applications component' of your studies that ages most rapidly. I have heard a mathematics teacher explain how pyramidal numbers are useful in determining the number of cannon balls you have! Historically, this may well have been an application of figurate numbers but it is not one that students relate to easily today. More relevant may be an appreciation of the different types of functions associated with wavelets because of their use in compressing image data, or the use of prime factorisation as the trapdoor one-way function in public-key encryption codes.

You can study mathematics simply for the joy of exploring new ideas. However, students do need to see the significance of what they are learning. "Trust me, it is good for you" does not convince a large number of students.

Beyond the mathematics underpinning the content I teach, the form and essence of the mathematics I have learnt permeates my day-to-day work. Rather than the detail, it is a number of general principles and fundamental theorems that influence my work. For example, although I have had no need to calculate the Jacobian of a transformation in my work, I frequently draw on the idea of metric spaces, or more precisely, creating a metric. This basic idea underpins my interpretation of the measurement principles involved in the Simple Logistic Model of Item Response Theory and my limited knowledge of psychometrics. Modern test development

often makes use of Item Response Theory, the probabilistic model used with international comparisons of performance in mathematics, such as the Trends in International Mathematics and Science Study (TIMSS).

The role of information and communication technologies (ICT) in the learning of mathematics also plays a significant role in my work. This can range from advising on representing mathematical ideas for young children in television programs on the ABC to the design of interactive content for websites, such as HSC-Online. It has also involved investigating the advantages of using vector-based animations to provide online mathematics content compared to bit-mapped raster images. Indeed, the question of what may be the best teaching applications of emerging technologies contributes to an ongoing exploration.

Basic principles of information theory and coding theory are used in designing components of online learning in mathematics. They also assist in appreciating the opportunities provided by an effective implementation of MathML. At a more basic level, questions surrounding the use of technology in teaching mathematics often translate into questions of access. When should students be able to use four function calculators, scientific calculators, graphing calculators or computer algebra systems in learning mathematics?

My involvement in curriculum development means that the history of the development of mathematical ideas is particularly important to me. In NSW, we are due to review our senior mathematics syllabuses and this will be accompanied by active debate as to which mathematics should be in these courses. Without understanding the history of the development of mathematics, it is easy to fall prey to simplistic arguments that "it was ever thus".

Mathematics is a dynamic body of knowledge. It has also been described as a universal language. Although the appellation of universal language is used to justify the messages sent into space in search of extraterrestrial life, in its most basic interpretation it is not literally true. Anything described by language will be subject to the dynamic nature of language. As an example of the dynamic nature of language, I point to the use of the term trapezium. From a purely mathematical perspective it does not matter what we take a trapezium to be as long as we clearly define this at the start of any argument we make involving this term. However, mathematics textbooks and syllabus documents like to define terms. Added to this is the weight of opinion of parents and some teachers that mathematics is first and foremost about correct answers. We know that Americans take a trapezium to be a quadrilateral with no sides parallel and a trapezoid to be a quadrilateral with two sides parallel. Australia tends to follow the British use of the term trapezium.

It would be comforting to leave the discussion of the term trapezium at this point. My word processor accepts three "dialects" of English: AUS, UK and US. Why not accept that when you write the word trapezium, what you mean depends upon which dialect of English you speak? However, those imbued with the belief that you need to have a correct answer in mathematics are not satisfied with what they perceive to be an anomaly. They want an authoritative and definitive answer!

In seeking to understand mathematics, I have often been exhorted to read the originals. As Euclid's *Elements* was, for a long time, the "bible" of Euclidian geometry, I consider this to be an adequate original source. Indeed, at one time, Euclid's *Elements* was the second most frequently printed publication following the Bible.

In Euclid's *Elements* Book I, Definition 22 we have, "Of quadrilateral figures, a *square* is that which is both equilateral and right-angled; an *oblong* that which is right-angled but not equilateral; a *rhombus* that which is equilateral but not right-angled;

and a *rhomboid* that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called *trapezia.*"

What I discovered by examining Euclid's *Elements* is simply that our current definitions have evolved. Definition 22 does not allow a rhombus to be a square, something that would be at odds with the geometry we currently teach in schools. Euclid's classification system does provide the rationale for the American definitions of trapezium and trapezoid.

The emperor's new clothes

My mathematical studies have contributed to my sense of what constitutes a sound argument and an appreciation of scholarship. When I use the term scholarship, I mean the development of logically reasoned argument rather than argument solely by appeal to authority. One of the reasons I enjoy mathematics and teaching it is that you do not need to seek a priest to tell you what to believe. You can, with care, use reasoning to determine the truth or falsehood of a statement in mathematics.

This same need for critical reasoning is essential to appreciate the decisions that are being made by governments. Public policy discourse increasingly involves the formulation and critique of arguments based on data. Yet data sense is a relatively rare form of reasoning, especially in the political domain.

At the July 2001 conference on global warming at Bonn, Australia argued to use "carbon sinks" as credit for cutting greenhouse gas emissions. Carbon sinks are forests that eliminate carbon dioxide from the atmosphere. There appears to be some kind of balance sheet logic behind this and the problem reduces to a measurement task. However, measuring things is rarely simple.

For example, a government report indicated that the area of forest in Australia had increased from 43 million hectares in 1992 to just under 157 million hectares in 1998. How could this be? Trees did not suddenly sprout up all over the country. Taking the numerical data to be correct, what could account for this rapid increase in the growth of forests in Australia? What constitutes a forest? An initial response is that a forest is a bunch of trees. How big do the trees need to be and how far apart could this bunch of trees be? Between 1992 and 1998 it was the definition of "forest" that changed. Now we count all our woodlands (where the trees are quite far apart) and most of our mallee (where the trees are not very tall) as forest. The principle of needing to compare like with like, can be easily overlooked.

The current Australian Government Department of the Environment and Heritage website suggests that "a forest of trees with a potential height of at least two metres and crown cover of at least 20 per cent" and "in patches greater than one hectare in area" may be counted as afforestation. Perhaps all of those practical activities getting students to measure the height of a tree might have been worthwhile after all! What are the implications of this definition of afforestation and does it matter? As a teacher, I believe that logical argument, which is at the very heart of mathematics, needs to be evident in public policy discussions.

Mathematics education and the media

Appreciating the significance of the study of mathematics to effective citizenship is often overlooked. One of the unintended consequences of mathematics schooling has been the creation of a fixed attitude towards mathematics evident in much of the community. People's attitudes appear to have been shaped by what I sometimes refer to as the Brussels Sprouts effect - most people believe that mathematics is good for you yet very few want to consume it themselves.

Mathematics tutoring colleges and programs abound in Australia, encouraged no

doubt, by the Brussels Sprouts effect. Parents want the best for their children and are happy to pay someone to "feed their children Brussels Sprouts." The Brussels Sprouts effect is not necessarily a recent phenomenon. Bertrand Russell may have identified an early form in his own education.

I was made to learn by heart: "The square of the sum of two numbers is equal to the sum of their square increased by twice their product". I had not the vaguest idea what this meant, and when I could not remember the words, my tutor threw the book at my head, which did not stimulate my intellect in any way. (Bertrand Russell, Autobiography, 1986, p. 34)

If you look at the way that mathematics education is portrayed in the media, there appears to be little reason to hope. Standards are almost always reported as falling as people hark back to a golden era that never existed. I read newspaper articles with titles like *Learning Curves. Teaching does not add up*, and *A teacher problem that is multiplying*, or my current favourite, *A cure for maths hysteria?* Viewing television, I am exhorted to learn how schools are failing our kids. The news reports and current affairs programs play a significant role in inoculating the community with a Brussels Sprouts view of mathematics.

Portraying occupations in television series can have a significant impact on career choices. Shows such as CSI have spawned an increase in applications to study forensic science in many universities, well beyond the actual needs of the community. Although it is currently too early to tell, it will be interesting to see what impact the show *NUMB3RS* has on enrolments in mathematics courses. At least the shows aired to date are based on real cases and the mathematics is correct.

Teaching mathematics

Teaching mathematics is not always easy. There are days when you feel that you are truly "advancing to the rear". These are the days when you might even start to believe Mencken's quip, "Those who can — do. Those who can't — teach".

Yet outweighing all of the challenges are the days when you see the light shine in the eyes of your students and you can hear "I get it" like a summer cicada chorus. I can still bring to mind a memory of one of my students smiling and nodding all the way through an explanation I was providing in introducing a new topic. He told me at the end of the lesson it was because it just made so much sense! These are the days when you know that, "The best teacher is not the one who knows most, but the one who is most capable of reducing knowledge to that simple compound of the obvious and the wonderful". As a mathematics teacher, I think that this time, unlike his earlier quip, H. L. Mencken got it right!

NSW Department of Education and Training, GPO Box 33, Sydney NSW 2001
E-mail: Peter.Gould@det.nsw.edu.au