Math matters

Don Taylor

Firmness, Commodity and Delight

Mathematics has been in the world a long time; it emerged in Babylonia in the 3rd millennium BC and has permeated science and civilization ever since. It crosses both national boundaries and subject boundaries. In the words of R. W. Hamming [1], "*Our main tool for carrying out the long chains of tight reasoning required by science is mathematics*".

Today, the application of mathematics to problems in science, engineering, finance, economics and medicine requires ever greater numbers of mathematically literate members of the community. Given all that, why is the following comment still all too typical: "*I was forced to do mathematics at school but I have never needed any of it. Why should we teach children algebra and calculus?*" Such a question was put to me recently, and on camera! I provided the generic skills response: "*Mathematics promotes strong problem solving skills, clear thinking and logical analysis.*" I came away feeling that a lot more needed to be said. But claiming that calculus could help balance your cheque book did not seem to do justice either to mathematics or to the intelligence of the reporter.

I believe that the question asked by the reporter is an excellent one and illustrates the need to promote a better awareness of the importance of mathematics in the general community. And it would not hurt to establish a positive image of mathematics in the minds of our scientific colleagues, our political masters and our students as well.

Another question germane to this column is "What is Mathematics?" Of course there is a famous book with this title [2] and recently it has been asked by Ian Roberts [3] requesting a more succinct answer.

While pondering this question I recalled an article by Mitch Kapor [4] on software design. He asserts that good software exhibits *firmness*, *commodity* and *delight*. This is a reference to the Roman architect Vitruvius, who flourished in the first century BC and who is attributed with the notion that well-designed buildings are those which exhibit firmness, commodity and delight [5].

I believe that mathematics exhibits the same qualities and to varying degrees it is these qualities that attract people to mathematics. Furthermore, these notions provide a useful framework on which to weave an account of mathematics suitable for a general audience.

Firmness:

Mathematics has a strong logical foundation. Moreover it remains remarkably stable over time. The exemplar is Euclidean geometry [1]. In this connection, it is worth noting that many mathematicians and engineers were attracted to mathematics by their first encounter with Euclidean geometry at school; sadly it is no longer a central part of the high school curriculum for many good students.

This aspect of mathematics is reflected in its hierarchical nature and its long history. This in turn contributes to the fact that the study of mathematics, like the study of foreign languages, is inherently difficult.

Commodity:

Mathematics is inextricably linked with applications in engineering, science, economics and, more recently, biology. In this it is phenomenally successful. This interaction cuts both ways: recent advances in particle physics have provided new techniques in geometry and topology [6].

Delight:

For many mathematicians the most compelling feature of mathematics is its beauty and the pleasure one gets from its pursuit. In the words of G. H. Hardy [7]: "*The mathematician's patterns, like the painter's or the poet's, must be* beautiful*; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.*" Of course Hardy also said that the best mathematics is *serious* as well as beautiful. Indeed, the fact that good mathematics is both hard and beautiful is often the challenge that attracts the best students to mathematics.

So where do these ideas lead us? The key issue is the need to improve the public perception of mathematics in order to ensure its long term support. Mathematicians have not always done well at this but fortunately, as Michael Cowling points out in his President's column, there are signs that we are getting better at promoting our discipline.

In previous contributions to the *Math matters* series there has been an interesting mix of pessimism and optimism about the state of mathematics in Australia. The pessimism has been accurately captured by Peter Hall [8] and stems from poor funding and the general decline in the number of continuing positions for mathematicians in Australian universities (39 at Sydney in 2005 compared with 65 ten years ago; somewhat offset by an increase in research-only positions).

But as the series progresses a note of cautious optimism has crept in. In particular Peter Taylor [9], expanding on Tony Dooley's [10] exhortation to "*take greater control of the mysterious process between theory and applications*", observes that "*our destiny is in our own hands*" and therefore "*the health of the mathematical sciences lies with the mathematical community*".

To some extent the mix of pessimism and optimism reflects global trends. The 2004 review of UK mathematics [11] listed concerns with the loss of mathematicians in key areas such as statistics and at the same time noted that advanced technology relies on sophisticated mathematical content. The 1997 review of Canadian mathematics [12] also noted many challenges and their effects on the funding of mathematics.

A theme that permeates these reports is the need for the mathematical community to develop linkages with other disciplines and with industry.

Following Garth Gaudry's [13] account of the establishment of the Australian Mathematical Sciences Institute (AMSI) and the International Centre of Excellence for Education in Mathematics (ICE-EM) it was especially welcome to read the column by the Minister, Dr Brendan Nelson [14] acknowledging the importance of mathematics throughout primary, secondary and tertiary education. It is equally pleasing to see that Dr Nelson highlighted the role that ICE-EM will play in strengthening mathematics education and that there is recognition that discipline specific criteria should be used in any assessment exercise.

On the other hand, an impediment to the public recognition of the importance of mathematics has been ably pointed out by Cheryl Praeger and Ian Enting [15]. This is the lack of visibility of mathematics and mathematicians in the workplace.

Only a very small percentage of our students go on to become academic mathematicians. For example, at Sydney we see more than 2500 students in first year but after four years there are only 25 honours students. Of the other 99% of the cohort, about 200 or so graduate with a major in mathematics or statistics but then pursue (well-paid) careers in areas that are not readily identified with mathematics.

Most of these students go out into the world without thinking of themselves as mathematicians or statisticians, but they will use their mathematical skills in their jobs as engineers, scientists or economists. We should be pleased that we see such a large number of students and make every effort to maintain contact with them.

It is also the case that a lot of mathematics, both teaching and research, is conducted outside of mathematics departments. This is not necessarily a bad thing but there is a danger that the successes of mathematics will be labelled by the application area, ignoring the underlying mathematics. While it is wonderful that our students gain employment in diverse fields, the prospect of ever shrinking mathematics departments is not a pleasant one. In order for the profession to renew itself and to continue producing students we need departments of mathematics; the application areas are not set up to train new generations of mathematically skilled graduates.

Perhaps a way of addressing this problem is to engage in joint teaching ventures with our scientific and engineering colleagues. There is already a heartening trend in this direction but we should endeavour to be present wherever mathematics is taught. For example: applied mathematics for chemical engineers; statistics for psychologists; cryptography and network security; bioinformatics; computational geometry. In many cases funding concerns may make it difficult to negotiate such arrangements at the department level and therefore the general principle needs to be established at an institutional level.

Another way to increase the visibility of mathematics is to increase the range of public lectures and to support colleagues who are good expositors and communicators. Not long ago I thought it next to impossible to illustrate abstract mathematics with compelling images. Fortunately, Bill Casselman's excellent cover art for the Notices of the American Mathematical Society has shown me otherwise.

Recreational mathematics [16] can also play a rôle but we need to be careful that we emphasize logical thinking.

Conclusion

This is a critical time for mathematics and statistics in Australia. Following the lead in Canada and the United Kingdom we are embarking on a review of the mathematical sciences. It is essential that the profession widen its focus and establish an image of mathematics in the general community that is positive and conducive to ongoing support.

To do this, we as a community should

- Use every opportunity to work with school teachers and to take an active part in curriculum issues. The ICE-EM Mathematics program is a good example; it involves "*professional development for teachers and provides materials that are clearly written, mathematically correct and consistent*" [17].
- Increase the public awareness of mathematics.
- Ensure that there are mathematicians on the key committees in our organizations. (Do not invite a 'Rochester incident' [18].)
- Actively seek to collaborate with science and engineering colleagues by teaching joint courses in application areas.

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References

[1] R.W. Hamming, *The unreasonable effectiveness of mathematics*, Amer. Math. Monthly **87** (1980).
[2] Courant and Robbins, *What is mathematics*, Oxford University Press, 1941.
[3] I. Roberts, AustMS Gazette **32** (2005), 146–147.
[4] M. Kapor, *A software design manifesto*, Dr Dobb's Journal **172** (1991), 62–65.
[5] The phrase appears in the book *The Elements of Architecture*, the 1624 rendering of Vitruvius' *De Architectura* by Henry Wotton.
[6] E.P. Wigner, *The unreasonable effectiveness of mathematics in the natural sciences*, Comm. in Pure and Appl. Math. **13** (1960).
[7] G.H. Hardy, *A Mathematician's Apology* (Cambridge University Press 1967), p. 85.
[8] P. Hall, AustMS Gazette **31** (2004), 6–11.
[9] P. Taylor, AustMS Gazette **31** (2004), 285–287.
[10] T. Dooley, AustMS Gazette **31** (2004), 76–78.
[11] An International Review of UK Research in Mathematics, 8 March 2004. A Review undertaken on behalf of the Engineering and Physical Sciences Research Council (EPSRC) and the Council for the Mathematical Sciences (CMS).
[12] The National Review of the Canadian Mathematical Community, 13 May 1997.
[13] G. Gaudry, AustMS Gazette **31** (2004), 145–146.
[14] B. Nelson, AustMS Gazette **32** (2005), 227–232.
[15] C. Praeger and I. Enting, AustMS Gazette **31** (2004), 217–221; 239–243.
[16] See David Singmaster's article *The unreasonable utility of recreational mathematics*, presented to the First European Congress of Mathematics, Paris, July 1992.
[17] Mike Evans, ICE-EM Schools Project Manager.
[18] A. Jackson, Notices of the Amer. Math. Soc. **43** (1996), 300–306; 447.

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