A Continuous Time Model for Election Timing

D. Lesmono, E.J. Tonkes, and K. Burrage

Abstract

We consider a continuous time model for election timing in a Majoritarian Parliamentary System where the government maintains a constitutional right to call an early election. Our model is based on the two-party-preferred data that measure the popularity of the government and the opposition over time. We describe the poll process by a Stochastic Differential Equation (SDE) and use a martingale approach to derive a Partial Differential Equation (PDE) for the government's expected remaining life in office. A comparison is made between a three-year and a four-year maximum term and we also provide the exercise boundary for calling an election. Impacts on changes in parameters in the SDE, the probability of winning the election and maximum terms on the call exercise boundaries are discussed and analysed. An application of our model to the Australian Federal Election for House of Representatives is also given.

1 Introduction

In Majoritarian Parliamentary Systems such as in Australia, United Kingdom, or New Zealand, Prime Ministers have the discretion to call an early election. This discretion may give governments an advantage to remain in power by calling elections at the most favourable time, especially when their popularity is high. In those countries, governments' and oppositions'popularity are determined by poll data. Both governments and oppositions are very concerned about these data and when they are behind in the poll, they will try through their policies to improve their popularity in the next poll. This decision to call an election at the most favourable time is interesting to model from a mathematical point of view. Mathematical techniques such as the optimal stopping problem, dynamic programming, game theory, stochastic and partial differential equations have been used to describe and analyse this problem.

Smith [8] modelled optimal election timing by considering the government's competency and outcomes. Voters gain information about the government's competency via its outcomes and then they can judge whether or not the government is competent. The decision to call an election is based on the difference in the expected utility between calling and not calling an election given the government's competency and voter beliefs about this competency. His analysis also included political business cycles where the government can manipulate economic instruments to its favour and the role of the opposition's campaign.

Kayser [6] considered election timing as a finite horizon optimal stopping problem to model the government's decision explicitly. The government's decision at each time is to maximize the utility of office holding by considering an option to call an election. He used the term *surfing* to indicate the ability of the government to time the election and *manipulation* to indicate where the government manipulates its policies for its advantage. He found that changes in factors such as government's competency, utility, discount factor and maximum terms will impact on the degree of *surfing* and *manipulation*.

Balke [2] derived a mathematical model for optimal timing by considering the benefits and costs of calling an election. His model allows the government to choose a stopping time that maximizes the expected utility given a government's current popularity and the amount of time in power since the last election. The popularity is governed by a SDE with constant drift and volatility. His model started by maximizing expected utility for some given conditions and ended by solving a PDE with some boundary conditions. He gave analysis on election boundaries for different values of drift and volatility coefficients, maximum terms in office, alternative popularity processes, winning probability functions and discount rates of being in office until the next election.

A discrete time model of the early election problem with a constant *lead time*, a period between announcing and holding an election, was solved in [7]. A mean reverting SDE was assumed to describe the popularity of the government and the opposition from the poll data. Instead of using a given drift and volatility coefficient as in [2], we estimated our SDE parameters from the poll data using Maximum Likelihood Estimation (MLE). Also, we derived the probability of winning the election from any level of popularity based on the last 22 Australian Federal Election since 1949.

In this paper, we develop a continuous version of our model in [7]. We start by fitting a mean reverting SDE to the poll data and use a martingale approach and Ito's Lemma to derive a PDE for the expected remaining life in power with some boundary conditions. A Crank-Nicolson method is employed to solve the PDE numerically and yield results in terms of the expected remaining life in power and the call exercise boundary. The call exercise boundary gives the government an optimal policy to use its control, that is a decision whether to call an election or not. Comparisons for a three-year and a four-year maximum term are also given.

The rest of this paper is organised as follows. Section 2 gives the formulation of our model. Parameter fitting that includes the probability of winning the election and parameter estimation using MLE are given in section 3. The numerical scheme to solve our PDE along with the algorithm is given in section 4. Section 5 deals with results in terms of the expected remaining life in power and the call exercise boundary. Conclusions are presented in the last section.

2 The Model

In Australia, there are many polls to measure the voting intention of the public towards the government or the opposition. Polls usually are taken fortnightly but once an election is announced, they may be taken more frequently, weekly or even daily in days leading to the election date. Both the government and the opposition take these polls seriously as they believe that poll data reflects the intention of the public and indicates who will win if the election is held on the day when the polls were taken. In this paper we just use one of the popular polls, namely Morgan Polls (www.roymorgan.com).

There are two major opposing political blocs in Australia, the Liberal and National Party (Coalition) and the Australian Labor Party (ALP). At the moment, the Coalition is in government while the ALP is the opposition. The data we use from Morgan Polls is two-party-preferred data, which measures the popularity of the government and the opposition. Our variable of interest is S, the difference between the government's popularity and the opposition's popularity. For example, when the two-party-preferred data gives 50%-50% for the government and the opposition's popularity, S will be zero. We maintain the condition $-1 < S < 1$, however later in the computation we shrink the interval to $-0.5 < S < 0.5$ as it is very unlikely to assume values outside of this interval.

We propose that the poll process is governed by the following mean reverting SDE:

$$dS = -\mu \frac{S}{1 - S^2} dt + \sigma dW. \tag{1}$$

The above SDE is mean reverting toward $S = 0$, which means that when one party is behind in the poll then it will react in such a way through its policy to make its popularity higher in the next poll. The rate it will move to $S = 0$ depends on the value of μ and S. We assume that μ and σ are positive constants. If $|S_0| < 1$ and $\mu > 2\sigma^2$ then there exists a unique solution to 1 (by applying Theorem 2.3 of [4]).

Let $t = 0$ represents the time when the current government was first elected to power and L be a random variable describing the length of time that the current government spends in office from $t = 0$. Also, let $Q_t = \mathbb{E}(L \mid \mathcal{F}_t)$ be the expected total life of a government given the filtration to time t, that is, all information about t and S_s with $s < t$. Then $V_t = Q_t - t = \mathbb{E}(L - t \mid \mathcal{F}_t)$ is the expected remaining life in power. By the phrase "remaining life in power", we mean the length of time from the present, until the next time that the opposition wins an election. Clearly, V_t is not a martingale, but Q_t is a martingale (from the *Tower Property* of expectation):

$$\mathbb{E}(Q_t \mid \mathcal{F}_s) = \mathbb{E}(\mathbb{E}(L \mid \mathcal{F}_t) \mid \mathcal{F}_s) = \mathbb{E}(L \mid \mathcal{F}_s) = Q_s$$

Applying Ito's Formula to $Q = Q(t, S)$ gives:

$$\begin{aligned} dQ &= \frac{\partial Q}{\partial t} dt + \frac{\partial Q}{\partial S} dS + \frac{1}{2}\sigma^2 \frac{\partial^2 Q}{\partial S^2} dt \\ &= \frac{\partial Q}{\partial t} dt + \frac{\partial Q}{\partial S} \left(-\mu \frac{S}{1 - S^2} dt + \sigma dW \right) + \frac{1}{2}\sigma^2 \frac{\partial^2 Q}{\partial S^2} dt \\ &= \left(\frac{\partial Q}{\partial t} - \mu \frac{S}{1 - S^2} \frac{\partial Q}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 Q}{\partial S^2} \right) dt + \sigma \frac{\partial Q}{\partial S} dW. \end{aligned}$$

Because Q_t is a martingale, the drift of the last equation above is zero and we obtain a PDE for Q.

$$\frac{\partial Q}{\partial t} - \mu \frac{S}{1 - S^2} \frac{\partial Q}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 Q}{\partial S^2} = 0.$$

Substituting $V_t = Q_t - t$, we have a PDE for V, namely

$$\frac{\partial V}{\partial t} - \mu \frac{S}{1 - S^2} \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial S^2} = -1. \tag{2}$$

The boundary conditions are given by

$$\frac{\partial V}{\partial S}(-1, t) = \frac{\partial V}{\partial S}(1, t) = 0. \tag{3}$$

These boundary conditions indicate that when the government is at the lowest (highest) level of popularity it can not become more unpopular (popular).

3 Parameter Fitting

3.1 Probability of Winning

We assume that the probabilities of winning the election at each level of S, which are depicted in figure 1, are given by:

$$P(W|S) = 10(S + 0.5)^3 - 15(S + 0.5)^4 + 6(S + 0.5)^5 \tag{4}$$

$$P(W|S) = 3(S + 0.5)^2 - 2(S + 0.5)^3. \tag{5}$$

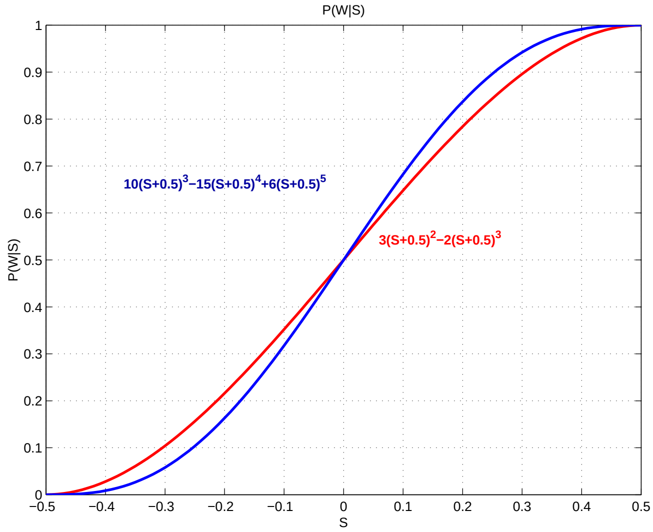

Figure 1. Probability of Winning the Election

These probabilities are modified from those used in [2] to accommodate a situation where $-0.5 < S < 0.5$. These probabilities of winning the election increase as S increases, but they increase at a decreasing rate for high values of S. For low values of S, $P(W|S)$ increases at an increasing rate. In our numerical computation we use $-0.5 < S < 0.5$ since in the real poll data it is unlikely to have the value of S outside that interval. Also we have $P(W|0.5) = 1$, $P'(W|0.5) = 0$, $P(W|-0.5) = 0$ and $P'(W|-0.5) = 0$ to indicate that at the highest level of S, the government will certainly win the election while at the lowest level of S, the government will lose office with certainty. Note that in reality even if $S > 0.5$ the party is not guaranteed to win the election due to exaggerated majority effect (a party can win more than 50% of the votes and yet still lose the election). Equation (4) gives a higher probability of winning the election from state $S > 0$, but lower if $S < 0$ than equation (5).

3.2 Parameter Estimation

Parameters for the SDE in (1) were derived using MLE. This method is quite robust for time series data with unequal increments. Based on Morgan Polls data from April 1993 - October 2004, the estimated value for $\hat{\mu}$ and $\hat{\sigma}$ in (1) are 4.17 and 0.28, respectively.

Since there is no explicit analytical solution to (1), we start by taking Euler-Maruyama discretization to our SDE in (1), which is:

$$S_{i+1} = S_i - \mu \left(\frac{S_i}{1 - S_i^2} \right) \Delta t_i + \varepsilon_{i+1}; \qquad \varepsilon_{i+1} \sim N(0, \sigma^2 \Delta t_i) \qquad i = 1, 2, ..., N-1.$$

The discretization follows the method used by Broze, Scaillet and Zakoïan ([3]) in modelling the short-term interest rate. We use S_i as a short notation for S_{t_i} to indicate the difference in popularity at time t at the i^{th} level. The log-likelihood function, which we want to be maximized is :

$$L_i = -\frac{N-1}{2}\ln(2\pi\sigma^2) - \frac{1}{2}\sum_{i=1}^{N-1}\ln(\Delta t_i) - \frac{1}{2\sigma^2}\sum_{i=1}^{N-1}\left\{\frac{S_{i+1}-S_i+\mu\left(\frac{S_i}{1-S_i^2}\right)\Delta t_i}{(\Delta t_i)^{1/2}}\right\}^2$$

where N is the number of data. Taking the first partial derivatives of L_i with respect to σ^2 and μ, and setting them to zero will give the maximum likelihood estimates for μ and σ^2:

$$\widehat{\mu} = -\frac{\sum_{i=1}^{N-1}(S_{i+1}-S_i)\left(\frac{S_i}{1-S_i^2}\right)}{\sum_{i=1}^{N-1}\left(\frac{S_i}{1-S_i^2}\right)^2\Delta t_i}$$

$$\widehat{\sigma}^2 = \frac{1}{(N-1)}\sum_{i=1}^{N-1}\left\{\frac{S_{i+1}-S_i+\widehat{\mu}\left(\frac{S_i}{1-S_i^2}\right)\Delta t_i}{(\Delta t_i)^{1/2}}\right\}^2.$$

We note that the Euler-Maruyama method has strong order 0.5 for general nonlinear SDEs, but for additive noise problems it is strong oreder 1 and so is more than adequate for our purposes.

4 Numerical Scheme

4.1 Finite Difference Scheme

For the numerical solution, we use a weighted average (θ) method, that reduces to the Crank-Nicolson Method when $\theta = 0.5$. First, we divide the time T into n partitions, $j = 1, 2, .., n$ and the popularity level S into m partitions, $i = 1, 2, ..., m$ and approximate the partial derivatives using the finite difference scheme below.

$$\frac{\partial V}{\partial t} \approx \frac{V(i,j+1)-V(i,j)}{\delta t}$$

$$\frac{\partial V}{\partial S} \approx \theta\frac{V(i-1,j)-V(i+1,j)}{2\delta S} + (1-\theta)\frac{V(i-1,j+1)-V(i+1,j+1)}{2\delta S}$$

$$\frac{\partial^2 V}{\partial S^2} \approx \theta\frac{V(i-1,j)-2V(i,j)+V(i+1,j)}{(\delta S)^2} + (1-\theta)\frac{V(i-1,j+1)-2V(i,j+1)+V(i+1,j+1)}{(\delta S)^2}.$$

At the boundary, we use a reflecting boundary condition by assuming $V(0,j) = V(2,j)$ and $V(m+1,j) = V(m-1,j)$. Substituting these approximations into (2) and considering the boundary conditions (3) for the top and lowest level of S, lead us to the following tridiagonal system of equations :

$$\begin{bmatrix} B & C1 & 0 & \cdots & \cdots & 0 \\ A & B & C & 0 & \cdots & 0 \\ 0 & A & B & C & \ddots & 0 \\ \vdots & \cdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \cdots & 0 & A & B & C \\ 0 & \cdots & \cdots & 0 & A1 & B \end{bmatrix} \begin{bmatrix} V(1,j) \\ V(2,j) \\ \vdots \\ \vdots \\ \vdots \\ V(m,j) \end{bmatrix} = \begin{bmatrix} \beta & \gamma 1 & 0 & \cdots & \cdots & 0 \\ \alpha & \beta & \gamma & 0 & \cdots & 0 \\ 0 & \alpha & \beta & \gamma & \ddots & 0 \\ \vdots & \cdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \cdots & 0 & \alpha & \beta & \gamma \\ 0 & \cdots & \cdots & 0 & \alpha 1 & \beta \end{bmatrix} \begin{bmatrix} V(1,j+1) \\ V(2,j+1) \\ \vdots \\ \vdots \\ \vdots \\ V(m,j+1) \end{bmatrix} + \begin{bmatrix} \delta t \\ \delta t \\ \vdots \\ \vdots \\ \vdots \\ \delta t \end{bmatrix}$$

where

$$A = \left[\frac{\mu S\theta\delta t}{2\delta S\left(1-S^2\right)} - \frac{\sigma^2\theta\delta t}{2\left(\delta S\right)^2}\right];\ B = \left[1 + \frac{\sigma^2\theta\delta t}{\left(\delta S\right)^2}\right];\ C = -\left[\frac{\mu S\theta\delta t}{2\left(1-S^2\right)\delta S} + \frac{\sigma^2\theta\delta t}{2\left(\delta S\right)^2}\right]$$

$$\alpha=\left[\frac{\sigma^{2}\left(1-\theta\right)\delta t}{2\left(\delta S\right)^{2}}-\frac{\mu S\left(1-\theta\right)\delta t}{2\left(1-S^{2}\right)\delta S}\right];\ \beta=\left[1-\frac{\sigma^{2}\left(1-\theta\right)\delta t}{\left(\delta S\right)^{2}}\right];\ \gamma=\left[\frac{\mu S\left(1-\theta\right)\delta t}{2\left(1-S^{2}\right)\delta S}+\frac{\sigma^{2}\left(1-\theta\right)\delta t}{2\left(\delta S\right)^{2}}\right]$$

$$A1=C1=-\frac{\sigma^{2}\theta\delta t}{\left(\delta S\right)^{2}};\ \alpha 1=\gamma 1=\frac{\sigma^{2}\left(1-\theta\right)\delta t}{\left(\delta S\right)^{2}}.$$

We use a Crank-Nicolson method, which is an implicit method and then solve the tridiagonal system above. Since, the Crank-Nicolson method is an implicit method, it is unconditionally stable and so there is no stability restriction on the size of δt.

4.2 Algorithm

The algorithm for our solution method is as follows. We start with an initial estimate for the expected remaining life in power at time t_1, which is $V(S_i,t_1), i=1,2,...,m$. Then we calculate the expected remaining life at the election date at $t=3$ years using the following formula:

$$V(S_i,T)=\frac{P\left(W\mid S_i\right)}{K}\int_{-1}^{1}V(S_i,t_1)\left[1-\left(S_iq^2+q-S_i\right)^2\right]^{\mu/\sigma^2}dq \qquad (6)$$

where K is a probability normalizing constant given by:

$$K=\int_{-1}^{1}\left[1-\left(S_iq^2+q-S_i\right)^2\right]^{\mu/\sigma^2}dq.$$

In (6), the expected remaining life at the final time depends on the expected remaining life at the beginning of the term, the probability of winning the election and some sampling and response errors and diffusion of the poll process. The sampling and response errors basically measure the deviation of the voting intentions over some periods of time. The term $(1-(S_iq^2+q-S_i)^2)^{\mu/\sigma^2}$ in $V(S_i,T)$ and K is intended to represent the sampling and response errors and the diffusion of the poll process and to capture the evolution of this process from the beginning of term to the steady state distribution.

After calculating the expected remaining life at the election date, the algorithm moves backwards by solving the expected remaining life at time $t_{n-1},t_{n-2},...,t_1$ according to the PDE in (2) and boundary condition (3) for the lowest and the highest values of S. At each time step, the algorithm checks whether or not an election should be called. If $V(S_i,t_j)<V(S_i,T)$ then the value of calling an election is greater than the value of not calling an election; therefore an election should be called. We then update the expected remaining life at the beginning of the period and repeat the procedure until the difference between $V(S_i,t_1)$ in two consecutive iterations is less than some tolerance value, ϵ. In this algorithm there exists a fixed point that guarantees the convergence, by Brouwer's fixed point theorem. This procedure is represented in Algorithm 1.

Note that from the PDE in (2), we can calculate its steady state distribution by letting $\partial V/\partial t=0$ and assumming a symmetry condition $\partial V/\partial S=0$ at $S=0$. The steady state distribution is given by:

$$V_\infty=\frac{\left(1-S^2\right)^{\frac{\mu}{\sigma^2}}}{\int_{-1}^{1}\left(1-S^2\right)^{\frac{\mu}{\sigma^2}}dS}.$$

Algorithm 1
1: Start with initial estimate $V(S_i, t_1)$, $i = 1, 2, \cdots m$.
2: Calculate $V(S_i, T)$.
3: Calculate $V(S_i, t_j)$, $j = n-1, n-2, \cdots, 1$ according to the PDE.
4: If $V(S_i, t_j) < V(S_i, T)$ then set $V(S_i, t_j) = V(S_i, T)$.
5: Stop if $\|V(S_i, t_1)_{(3)} - V(S_i, t_1)_{(1)}\| \leq \epsilon$, otherwise go to (2).

5 Results

The numerical results of the continuous model are presented in terms of the expected remaining life in government and the call exercise boundaries for a three-year and four-year term. Effects on the call exercise boundary with changes in μ and σ are also given.

5.1 Expected Remaining Life

The expected remaining life for a three-year and a four-year term are depicted in figure 2(a). It has been debated whether the Australian people prefer a three or a four-year maximum term. There was a referendum proposed by the Hawke government in 1988 to alter the Constitution from a three-year maximum term to a four-year maximum term, but only 32.92% of voters were in favour of a four-year term (see [1] for details). From those two figures, we can see that at the beginning of the term, regardless of the level of popularity, the expected remaining life is almost constant with a four-year term having a larger value than a three-year term (around 10.8 and 7.6 years). Then as time elapses, the expected remaining life decreases, except for the highest level of popularity, where it remains constant. A four-year maximum term gives a longer expected remaining life in power than a three-year maximum term since the government has more time to choose the best time to call an election before its term expires. Having more time gives more possibilities for the government to wait until its popularity reaches certain level before calling an election and therefore the expected remaining life will be longer.

We also give in figure 2(b) the expected remaining life when the option to call an early election is removed. This situation relates to the constitutions of other countries with fixed terms. A comparison reveals that the option to call an early election significantly extend the government's life in office. The reason is quite obvious. The early election option gives the government flexibility to choose a favourable time for holding the election. A similar condition in finance is that the value of an American option is at least the same as a European option.

5.2 Exercise Boundary

We also provide a comparison between a three-year and a four-year maximum term in the call exercise boundaries in figure 3(a). These boundaries give an indication for the government of the best time to call an early election. In general, calling an election should be done when S is at least greater than zero. Calling an election earlier in the term requires a larger S than calling it later in the term. This finding is in line with one of the testable implications of Smith [8] where an early election is called when the government is popular. Also, the call exercise boundary for a four-year term is above that for a three-year term, which gives a smaller exercise region. So, for a four-year term, the government is less likely to call an early election. This finding is the same as one of Balke's findings (see Proposition 9 of [2]).

The call exercise boundary is monotonically decreasing in time. So, when at time t, an election is called at popularity level S, then it should also be called when the popularity

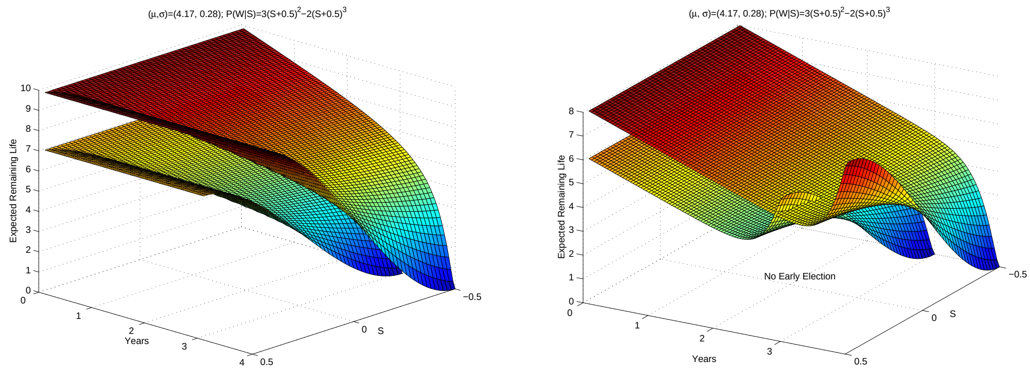

Figure 2. Expected Remaining Life (a) With Early Exercise Option (b) Without Early Exercise Option

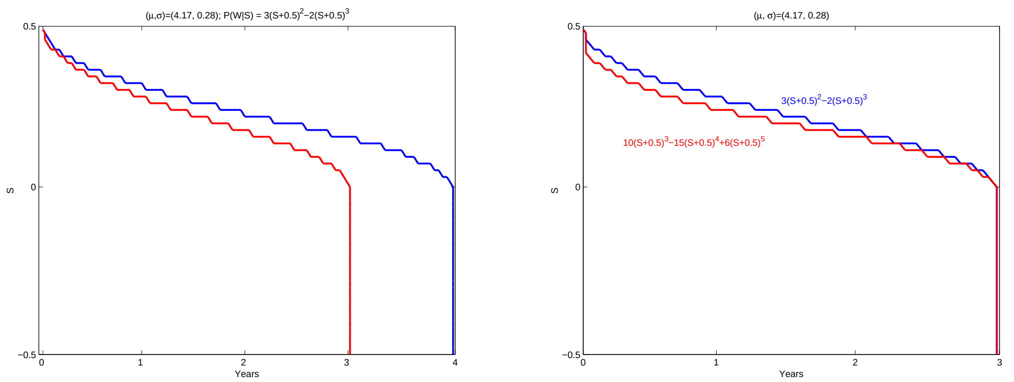

Figure 3. Call Exercise Boundary for (a) Different Maximum Terms (b) Different $P(W|S)$

level is higher than S. Also, at the popularity level of S, if an election is called at time t, it should also be called at time $t+1$. These two properties agree with the findings of by Ito [5], and are called *reservation growth rate property of election timing* and *declining reservation growth rate property of election timing*.

In figure 4 we give results for the call exercise boundaries in response to some changes in the values of μ and σ. When μ is fixed and σ varies, as in figure 4(a), the call exercise boundaries rise as σ gets larger. This means that the exercise region becomes smaller and the government is less likely to call an early election. Larger σ corresponds to a larger volatility and this result seems to contradict Balke's results in [2]. Balke found that as volatility increases, the call exercise boundary becomes lower. So, when the government's popularity is higher, the government should call an election to lock in its higher popularity and to increase the probability of winning the election. However, in our model, the drift coefficent is not constant as in Balke [2] but its value depends on S. It seems that this drift coefficeint has more control on the SDE than the volatility coefficient. As in figure 4(b), when σ is fixed and μ varies, a larger value of μ gives a larger exercise region. As μ indicates

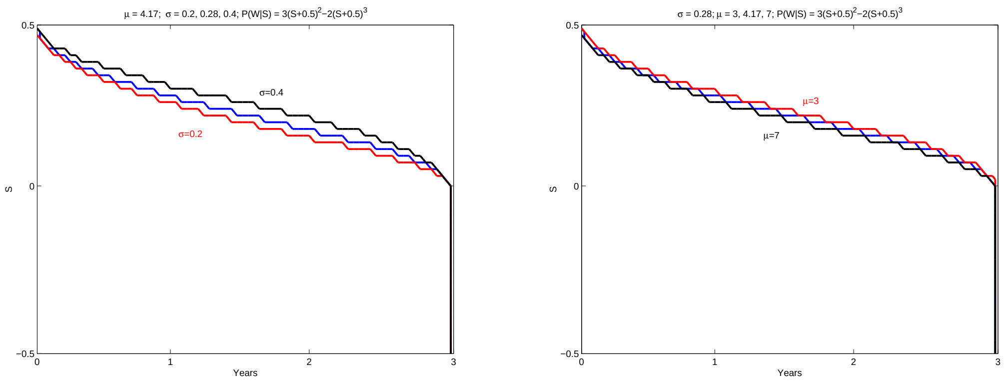

Figure 4. Call Exercise Boundary (a)μ fixed, σ varies (b) σ fixed, μ varies

the rate the SDE reverts to zero, larger μ corresponds to faster reversion to zero, so Balke's argument holds in this situation.

The effect of the probability of winning the election on the call exercise boundary is given in figure 3(b). As the probability of winning the election becomes larger, especially for $S > 0$ (see figure 1 for equation (4)), the call exercise boundary decreases, giving a larger exercise region. Thus, the government is likely to call an early election. This result also corresponds to one of Balke's findings (see Proposition 6 of [2]).

6 Conclusions

We derived a continuous time model for the election problem in the Australian House of Representatives. Our model is based on the two-party preferred data that measure the popularity of the government and the opposition. Based on that data, we fitted a mean reverting SDE and used a martingale approach to derive a PDE for the expected remaining life in power along with some boundary conditions. The solution of the PDE is calculated numerically using the Crank-Nicolson method and the call exercise boundary was found for the optimal policy. It was found that a four-year maximum term gives a substantially longer expected remaining life in government due to more freedom owned by the government to time the election. A condition where the option to call an election is removed was also considered and we found that in this situation the expected remaining life is shorter than a situation where there is an option for early election.

We also gave results in terms of the call exercise boundary that is monotonically decreasing with time, meaning that an earlier and earlier election requires higher and higher government popularity. In a three-year maximum term, the government is more likely to call an early election than in a four-year maximum term. Impacts of the maximum term, parameter values in the SDE and the probability of winning the election on the call exercise boundaries were also discussed.

Some extensions to our model include the possibility for the government to use some controls such as policy announcement to raise its popularity in the poll and thus impact on the probability of being re-elected. Another extension is to consider the possibility for the opposition to do the same by introducing policy responses to raise its popularity. In those two situations, the election timing problem must be tackled using a game theory approach.

Acknowledgements. The first author acknowledges a PhD scholarship from AusAid and a top-up scholarship from the Australian Research Council Centre of Excellence for Mathematics and Statistics of Complex Systems (ARC MASCOS). The third author would like to thank the Australian Research Council for its funding of a Federation Fellowship.

References

[1] Australian Electoral Commission, *1988 Referendums: Statistics* (Canberra 1990).
[2] N.S. Balke, *The Rational Timing of Parliamentary Elections*, Public Choice **65** (1990), 201–216.
[3] L. Broze, O. Scaillet, and J-M Zakoïan, *Testing for continuous-time models of the short-term interest rate*, Journal of Empirical Finance **2** (1995), 1999–223.
[4] I. Gyöngy, *A Note on Euler's Approximation*, Potential Analysis **8** (1998), 205–216.
[5] T. Ito, *The Timing of Elections and Political Business Cycles in Japan*, Journal of Asian Economics **1** (1990), 135–156.
[6] M.A. Kayser, *Who Surfs, Who Manipulates? The Determinants of Opportunistic Election Timing and Electorally Motivated Economic Intervention*, American Political Science Review **99** (2005), 1–11.
[7] D. Lesmono, E.J. Tonkes, and K. Burrage, *An early political election problem*, ANZIAM J. **45** (2003), C16–C33.
[8] A. Smith, *Endogenous Election Timing in Majoritarian Parliamentary Systems*, Economics and Politics **8** (1996), 85–110.

Department of Mathematics, The University of Queensland, Brisbane, Australia; Department of Mathematics, Parahyangan Catholic University, Bandung, Indonesia
E-mail: dlesmono@maths.uq.edu.au; jdharma@home.unpar.ac.id

Advanced Computational Modelling Centre (ACMC), The University of Queensland, Brisbane, Australia

Advanced Computational Modelling Centre (ACMC), The University of Queensland, Brisbane, Australia
E-mail: kb@maths.uq.edu.au

November 22, 2005.