Book reviews

Elements of Mathematics
Functions of a Real Variable:
Elementary Theory

N. Bourbaki
Translated by Philip Spain
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The book under review is the latest installment in the translation into English of the voluminous Bourbaki exercise. Most readers will be aware of the activities of les bourbakistes, a group of French pure mathematicians including André Weil (1906–98), Henri Cartan (born 1904), Jean Dieudonné (1906–92), Jean Delsarte (1903–68) and Claude Chevalley (1909–84), who set about after the First World War to reconstruct what they saw as the glories of the French intellectual tradition in mathematics [1, 2]. Their perceived negative attitude towards the intellectual standing of appliers of mathematics and the style of mathematical presentation they inspired has gained them both admiration and condemnation from the wider mathematical sciences community. It is difficult to approach the present volume without preconceptions, but even those who might not enjoy the presentation ought to acknowledge the heroic nature of the enterprise to which this volume belongs.

The book presupposes the reader to have a knowledge of topology and algebra sufficient for comfort with such matters as Zorn's Lemma, topological vector spaces, filters, Ascoli's Theorem, continuous group isomorphisms, and so on. Having made such demands, the authors consequently phrase results wherever possible at the most general and abstract level. There is no attempt to motivate most results in any way, very few illustrative examples are worked in detail, and little reference is made to mathematically rich disciplines beyond pure mathematics *per se*. These features make the book totally unsuitable, either as a text or even as a reference book, for Australian undergraduates, although sections of it might make sense to local pure mathematics honours students. (The suggestion in the opening note to the reader that it is directed to those who have a good knowledge of the first year or two of a university mathematics course refers to a distant land in a distant time.) However, mathematics lecturers with catholic tastes may find the presentation of some familiar topics interesting, and some of the proofs, rewritten in a less general context, could be profitably adapted for classroom use in higher-level courses. Respectable mathematics libraries should have this book on their shelves.

There are seven chapters, each of which I shall briefly address. Chapters I to IV cover material commonly found in the first two years of undergraduate courses, but more abstractly and more generally, as well as more deeply conceptually. Chapter I addresses the first derivative of $f : I \subset \mathbb{R} \mapsto E$, where E is a topological vector space, narrowing the context of E when unavoidable; the Mean Value Theorem and its relatives; higher derivatives and Taylor series; and convexity and its consequences. In the Mean Value Theorem section, I enjoyed the painless extension of some classic results to functions with a well-defined infinite derivative. The extended discussion of convexity was especially welcome.

Chapter II covers integration, starting from the primitive function or antiderivative viewpoint, and builds, via step functions, what is equivalent to a decent coverage of what most of us would call Riemann integration, including improper Riemann integrals, although that terminology is not used. Commendable care is taken on interchange of limit process issues, as one would expect.

Chapter III addresses the elementary transcendental functions. The reviewer has always preferred a brutally efficient approach to these functions based directly on complex power series. The group-theoretic perspective adopted in this chapter therefore induces a certain discomfort, although the exposition is generally clear and at times elegant.

Chapter IV covers differential equations, starting with $\mathbf{x}' = \mathbf{f}(t, \mathbf{x})$ when $\mathbf{x}$ lives in a complete normed real vector space. There is a full discussion of existence both of strict solutions, of solutions in a widened sense, and of approximate solutions. Continuity of solutions with respect to a parameter and dependence on initial conditions are also addressed, before the focus turns to linear equations and linear systems, covering at an exalted level many topics covered at a pedestrian level in undergraduate courses. As in previous chapters, the scarcity of applications greatly increases the efficiency of the exposition, but may induce a certain frustration in readers with less ascetic tastes.

The final three chapters cover material that most lecturers would be reluctant to describe as belonging to the elementary theory of functions of a real variable. Chapter V, entitled "Local study of functions", includes an extensive discussion of asymptotic expansions, though one largely devoid of practical techniques that one might use for finding them in any interesting context. There is more pleasure to be had in reading any dozen pages of Olver's magnificent text [4] than in this immaculately sterile account.

Chapter VI gives an account of generalized Taylor expansions and the Euler–Maclaurin summation formula, which is less forbidding than earlier chapters, perhaps because the inevitable prominent role of the Bernoulli numbers and polynomials gives the text a more concrete flavour.

Chapter VII follows the comforting precedent set by Chapter VI, and contains a straightforward and accessible account of the theory of the gamma function in the real domain, and the complex domain. The expected material is all there: the Bohr–Mollerup Theorem, limit and product formulae, functional equations and Stirling's formula.

There are extensive exercises at the ends of chapters, many of which would frighten most students and some of their lecturers. These are perhaps the most valuable part of the present book to the mathematics educator, who may well find there attractive problems for cutting the teeth of honours students or beginning research students.

The final drafts of most French originals in the Bourbaki series and the bulk of the historical notes were apparently prepared by Jean Dieudonné [3], and in the present volume the historical notes give real pleasure and significant enlightenment. The lengthy historical notes to Chapters I–III (pp. 129–162, with many references) trace the origins of calculus from the ancient Greeks to Newton and Leibniz and are highly recommended.

I have not spot-checked the quality of the translation against the original, but the translated text is well-phrased and attractively presented. Apart from a typesetting command accidentally printed atop the index, I noted no misprints or production flaws. Fidelity to the French original in notation gives us $f_g'(x)$ and $f_d'(x)$ for left and right derivatives, respectively, and a few other unidiomatic notations, but nothing that will deeply offend.

If one is prepared to draw a distinction between the way mathematics is most elegantly presented and the way in which it is most sensibly taught, then the book may be judged as an unqualified intellectual success and the publisher and translator are to be congratulated on making it available in English.

References

[1] Frédéric Bayart, biography of Jean Dieudonné, http://www.bibmath.net/bios.

[2] La France, terre des mathématiciens, http://www.diplomatie.gouv.fr/label~france/FRANCE/SCIENCES/mathematiciens/mathematiciens.html.

[3] J.J. O'Connor and E.F. Robertson, biography of Henri Cartan, http://turnbull.mcs.st-and.ac.uk/~history/Mathematicians/Cartan_Henri.html.

[4] F.W.J. Olver, *Asymptotics and Special Functions.* (Academic Press New York 1974).

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Topology II
Homotopy and Homology
Classical Manifolds

D.B. Fuchs and O.Ya. Viro
Springer-Verlag Heidelberg 2004
ISBN 3-540-51996-3

A good survey must include enough details to represent an area accurately and exclude enough details to remain accessible to readers. A third desirable ingredient in a survey of an area that already contains many good books is that it understand its place among books around it. This clearly written book on algebraic topology possesses each of these three ingredients.

Only brief treatments of each topic are given, referring the reader to further details in other books more often than in research papers. A reasonably complete account of basic algebraic topology is given. A sense of what the book contains and omits is demonstrated by its treatment of characteristic classes which is essentially contained in 20 pages, plus a brief description of classifying spaces of classical groups. It does not come close to the beautiful treatment by Milnor and Stasheff in the 300 page book *Characteristic Classes* yet it contains, for example, a description of cohomology operations which is not in Milnor and Stasheff. (More detailed accounts of cohomology operations do appear in other texts.) Furthermore, it is nice to have Eilenberg-Maclane spaces $K(\Pi, n)$ described in the same book as the theory of characteristic classes where classifying spaces are used much like Eilenberg-Maclane spaces. The book lists all known explicit Eilenberg-Maclane spaces $K(\Pi, n)$ which gives one way to understand Eilenberg-Maclane spaces more generally.

The book is organised in in the typical style of a text book on algebraic topology. The first 100 pages are devoted to an introduction to homotopy theory, followed by 100 pages on homology and cohomology, and finishing with 50 pages on calculations of the homotopy and homology groups of manifolds such as spheres and classifying spaces that are fundamental to the rest of algebraic topology. The authors' comment "...the book may be regarded as the synopsis of a textbook on topology" reflects the fact that many proofs are only sketched or readers are referred elsewhere for more details. For someone familiar with the proofs and other books, I found this appealing rather than annoying. It is ideal as a good reference manual if you know what you are looking for.

This book is translated from Russian and this process has affected the text. The three sections have been put together with three separate tables of contents, two bibliographies and fortunately one index. The third section—the final 50 pages mentioned above—is an earlier article by Fuchs that was included with the first two in the English translation, and that is why there is a second bibliography. The first bibliography cites the third section as a separate article, and the third section does not refer to the first two. Finally the process of translation may be the cause of the occasional typos in the text.

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The Calculus of Variations

Bruce van Brunt
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The request to write a review of this book came to me at an opportune moment. The author describes this book as suitable for a one semester course for advance undergraduate students in math, physics or engineering. I was just about to embark on teaching a course on this distinguished subject, not having used it myself for quite some number of years, so I felt that I was in the ideal position to assess how well written the book was for someone not already an expert in the field. Accordingly, I chose to use this book as my primary reference for presenting the course, and that turned out to be an excellent decision.

The book is written with the student in mind, and reaches out to grab you immediately with a number of problems to be solved using variational methods: classical problems such as the Brachystochrone, Catenary, simple geodesics, and Dido's problem, as well as a simple optimal control problem. The first Chapter also uses these problems to illustrate the idea of optimising a functional, the problem that lies at the heart of the Calculus of Variations. Presenting problems from a range of areas can be tricky. One cannot, these days, assume that the students will all have strong backgrounds in physics, math, and engineering, and so one cannot go into much depth in the problems. I found that the author generally presented these examples at about the right level, so that a student can grasp the essential details of the problem, without necessarily learning all of the ins and outs of each area in which the calculus might be applied.

The book then proceeds to describe, in Chapter 2, the first variation, and the Euler-Lagrange equations, and their derivation. In Chapters 3 and 4 the methods are extended first to allow multiple independent and dependent variables, and then to problems such as Dido's which include isoperimetric constraints. In Chapter 5 he shows the application of these techniques to eigenvalue problems, such as the Sturm-Liouville problem, and in Chapter 6, he extends the methods to include other forms of constraints. In Chapter 7, he brings in one final extension to allow free endpoints, and presents the transversality conditions. He then moves into Hamilton's formulation, and the Hamilton-Jacobi equation in Chapter 8 and extends this material by considering conservation laws, and Noether's theorem in Chapter 9, finishing the book with two appendices covering some of the more theoretical material required elsewhere in the book.

From my perspective, the book was pitched at a good level for the students I was teaching, apart from a few segments of the course which I believe the students found heavy going. The theorem proofs are typically clear, and generally not too formal, which I approve of in this context – they are at the right standard for what the students could absorb. Also, the frequent use of examples to illustrate results made the material much more approachable.

In any such work, particularly under the constraint that it be equivalent to a semester course, there are compromises. There are areas ommitted. The author has made careful efforts to make this pruning as painless as possible, but, for instance, I would have been pleased to see more material on approximation approaches to finding extremals. Also I found that, from a personal point of view, the book concentrated too exclusively on classical problems such as those from mechanics. It is fair to say, that these problems are essential to the Calculus of Variations, and bring the student in contact with the wonderful history of this subject. However, I found also, in teaching the course, that some more modern problems, such as those found in optimal control theory can bring yet further life into the subject, and it would have been the icing on the cake for the book to have focussed on a few such problems (it does present simple cases in at a couple of points, but does not dwell on these problems). However, this ommision is in keeping with the author's desire to make the book consistent with a one semester course, and he has to my mind succeeded in getting the material down to just the right level for this purpose.

Overall I enjoyed this book, and would unreservedly recommend it for its stated purpose (teaching an advanced undergraduate course), though, one might also find that supplementing it with a book on optimal control, or another application area would provide some additional perspective to students. The book really brought home to me the elagance of this subject, and its long history, while also teaching the more practical aspects of the work.

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A Course in Modern Analysis and its Applications

Graeme Cohen
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At the undergraduate teaching level, mathematical analysis is a certain amount of trouble. Its proofs are demanding for students, requiring a subtlety and attention to detail in argument which is greater than some other parts of mathematics at the undergraduate level, let alone comparisons with other subjects. The attention needed to do justice to these subtleties takes up considerable time and effort, and students are not invariably well equipped to deal with the necessary types of argument and often tend to see analysis as a series of unrelenting and purely technical demands. The effect is that the likelihood of seeing some really interesting results in analysis is reduced, and such results often have to be deferred to honours or even postgraduate level. So, perhaps a second year course in analysis may end with the result that in considering a uniformly convergent sequence of continuous functions on a bounded interval, the operation of integration may be interchanged with the operation of taking limits—but there is little room for building on this to obtain results of major interest. Undergraduate analysis often tends to be a journey from which most students have disembarked before they arrive at an interesting destination. The problem is serious because mathematics is now competing with many other subjects for the interest and commitment of students, so that thinking of undergraduate pure mathematics primarily as a preparation for research is a luxury which can no longer be afforded, if indeed it ever could have been.

Graeme Cohen has produced a coherent book, aimed at a potentially wide readership, which draws together abstract, classical and numerical analysis and which has some new ideas which will contribute to the

teaching and appreciation of mathematical analysis. It is intended for future teachers of high school mathematics, as well as for students of engineering, finance or science and for those who may proceed to research. The book is well written and is complemented by appropriate exercises of varying levels of difficulty, a considerable number of which are fully worked out. The book is largely self contained and does not presuppose much earlier knowledge of analysis. However, some elementary and advanced calculus, integration, and differential equations are assumed, but these are necessary more for some of the applications rather than for the main text. Unlike J. E. Littlewood, when he claimed to be reading one of his own papers, a student studying this work should largely be able to do so without feeling it is "heavy going". Cohen illustrates his philosophy in writing the book by a self-deprecating quotation from John Ward's Preface to his "The Young Mathematician's Guide"(1719), which I quote in part: *'Tis true indeed, the Dress is but Plain and Homely, it being wholly intended to Instruct, and not to Amuse or Puzzle the young Learner with hard Words; nor is it my Ambitious Desire of being thought more Learned or Knowing than really I am...*
The opening chapter, "Prelude to modern analysis", gives a clear overview of sets, countability, sequences, series, uniform convergence and some linear algebra. This gives all the essential background for the remainder, which consists of familiar topics: metric spaces, the Banach fixed point theorem, compactness and Ascoli's Theorem, topological spaces, normed spaces and linear mappings, and inner product and Hilbert spaces. (The chapter on topological spaces can, in fact, be omitted from the main development.)

It is the applications which are a feature of the book and are what makes it distinctive, rather than any new treatment of the standard topics. Thus, perturbation mappings arise from Banach's Theorem; Chebychev approximation, the Weierstrass approximation theorem, integral equations, and ill conditioning in numerical analysis are discussed in the normed spaces context; and least squares approximation in inner product spaces is given more detailed treatment than is usual at this level, with discussion of some of the various families of classical orthogonal polynomials. An application to quantum mechanics is discussed, but not developed in any detail and, beyond complete orthonormal sets and generalized Fourier series, there is no serious application of the concept of Hilbert space. Overall, the applications are interesting, develop naturally from the material, and are dealt with successfully from the point of view of student approachability, especially bearing in mind the length of the work—typically, other comparable books which deal with applications to this extent are substantially longer and more difficult, often being based on measure theory. Many comparable books have no corresponding applications, and the concrete approach is an advantage in today's environment. The book is recommended as successful in meeting some of the changing expectations in what is, in Australia but also elsewhere, a ferment of challenges for traditional disciplines.

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Computational Techniques for the Summation of Series

Anthony Sofo
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This book collects in one volume the author's considerable results in the area of the summation of series and their representation in closed form, and details the techniques by which they have been obtained. His function theoretic method is based upon use of residue theory, and chiefly applied to series solutions which result when differential-difference equations are solved by means of transforms. Of course, the series can stand on their own, and indeed one gets the impression that this is where the author's passion lies. Identities and properties of the series are developed, and I can't begin to list all the connections with "named polynomials" that he describes.

The first chapter of the book surveys methods used by others in the summation of series, including very recent work, and includes a section on hypergeometric functions. The second part of this chapter investigates a particular binomial sum, and shows how the methods of a variety of authors give recurrences and hence closed form representations for it and some generalisations. Specific reference is made to particular *Mathematica* routines in the author's own work on this sum.

Chapter 2 describes the method by which a series solution to a delayed differential-difference equation (both homogeneous and with a forcing term) is firstly found using the Laplace transform, and then expressed in closed form in terms of the dominant zeros of its transcendental characteristic function. Various situations in which such equations arise are mentioned. The series in this chapter are non-hypergeometric.

That the representations obtained in Chapter 2 are not "high precision fraud" (to quote the author, who in turn is quoting someone else) relies on treatment of the remainder using Bürmann's Theorem. Consequently, Chapter 3 consists of a proof of Bürmann's Theorem, and its application to the series from Chapter 2.

Chapter 4, similarly, is an auxiliary chapter, the results derived in it being needed in Chapter 5. Particular finite binomial sums are expressed as polynomials, by determining recurrences for the coefficients. In Chapter 5, the results of Chapter 2 are generalised.

In Chapters 6–8 attention turns to difference equations, and the technique is analogous, though the appropriate transform is the Z-transform and the series which result are hypergeometric. Closed forms and identities are developed for these series, and again connections are made with related results in the literature.

As far as I have been able to check, almost all of the book has appeared elsewhere previously; readers who want a taste for the flavour of the results and the writing style need look no further than the very pages of this *Gazette* [1]. Basically, the bulk of each chapter corresponds verbatim to a published paper. (This is not stated clearly, and only at the end of chapters, rather than, say, in the Preface.) Hence the writing style is concise and technical, and the book did not convey to me a strong feeling of coherence or development. The Preface and the very brief introductory passages to each chapter contain a surprising number of typographical errors or grammatical lapses, given that they must have been written especially for the book. On the plus side, the calculations are given in plenty of detail, and closely related work which has appeared in a variety of places is conveniently collected together. That the author passes on his extensive knowledge of the literature of results for series will also be valued by the interested scholar.

References

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