Report on the Mathematics-in-Industry Study Group 2005

Graeme Wake

1 Introduction

The second of the ANZIAM Mathematics-in-Industry Study Groups to be held in New Zealand (MISG2005) took place at Massey University at Auckland, 24–28 January 2005. Hosted by the Centre for Mathematics in Industry, based there, it was directed by Professor Graeme Wake, Adjunct Professor of Industrial Mathematics. Administrative support was provided by the Institute of Information and Mathematical Sciences (headed by Professor Robert McKibbin) and the MISG2005 Administrator was Nikki Luke. Seven problems were presented, six from New Zealand and one from Australia. Industry based in Australia seems very reluctant to bring their problems off-shore, in spite of considerable effort being made to lure them to a New Zealand-based MISG. With a strong following from New Zealand it points to a need for a MISG-type of activity in both countries with synergy maintained between them. The Centre for Mathematics in Industry was formed to provide a national base for MISG and has also built links with emerging similar activities, in South Korea and Thailand in 2004. This year's MISG was fortunate in attracting Professor Sam Howison, Director of the Oxford Centre for Industrial and Applied Mathematics in the United Kingdom as an overview facilitator. The Deputy Prime Minister for New Zealand, the Honourable Dr Michael Cullen provided a keynote opening address providing welcome, but controversial, publicity for MISG2005. Student workshops were held and addressed by Dr Howison, Mr Paul Milliken (an entrepreneurial consultant) and MISG2005 Director Professor Graeme Wake. We were fortunate in obtaining a significant grant from Technology New Zealand under their "Smart Start" programme, which is gratefully acknowledged. Last but not least we acknowledge the immense work provided by the problem moderating team — which this year included a postgraduate student in each case. Their contributions—often beyond the call of duty—is warmly acknowledged. Without this input MISGs just would not happen. The seven problems are described below along with the outcomes from the group meetings. A full technical report will be published by the CMI as "Proceedings of MISG2005" in due course. In most cases ongoing work is in progress. DVD recordings of the problem outlines are available for $NZ27 (incl GST) from the undersigned. Likewise the Full Proceedings of the MISG2004 (last year) are available for $NZ22.50 (incl GST) from the same place.

Predicting off-site depostion of spray drift from horticultural spraying through porous barriers on soil and plant surfaces (Lincoln Ventures Ltd/Plant Protection Chemistry of NZ)

The task set the MISG team was to develop and investigate a mathematical model of shelterbelt efficiency. Factors such as wind profiles through and above the shelterbelts, release height of the spray drift, capture efficiency of different droplet sizes and evaporation rates all need to be considered. The object is to either produce a better working model or to clearly define the deficiencies in the existing models. Any model that is developed would need to be usable at the farm level. That is, any inputs to the model need to be easily measured or estimated quantities such as free stream wind velocity, optical porosity of the shelterbelt and typical vegetation element

size of the shelterbelt. In practice barriers effective at trapping spray drift must have some airflow through them, solid barriers will direct airflow with spray droplets upward and over the barrier. During the week, the MISG team have verified that an existing model was suitable for use in determining the efficiency of a shelterbelt at collecting spray drift. The model is relatively simple to program and uses as inputs easily obtainable variables such as the free stream wind velocity, the optical porosity of the shelterbelt and the structure of the shelterbelt. With allowances for settling and evaporation the model was found to be valid over the range of inputs typically found for droplet distribution, wind velocity and vegetation element size. Numerical simulations of the flow field over and through the shelterbelt have justified some of the assumptions used in the model and given insight into the flow characteristics that are important to consider. Although these models are never perfect representations of the real world, we believe they are suitably robust for inclusion in a larger spray drift management system. Although care must be taken to ensure that some of the original assumptions are not overly breached.

Development of empirical relationships for metallurgical design of hot-rolled steel products (New Zealand Steel Ltd, Glenbrook)

New Zealand Steel Ltd asked the Study Group to develop empirical relationships for their hot-rolled coil and plate products. These empirical formulae are intended to describe the relationship between various mechanical properties of the coil and plate products and input parameters such as processing temperatures at various stages of the operation and steel chemistry. NZ Steel Ltd provided the Study Group with a large collection of data relating mechanical properties to the various input parameters of the hot-rolling process, which was analysed using multiple linear regression. A key measure of the analysis is the value of $R2$, which should be as close to unity as possible. This is a measure of how well a variation of the input variables explains a variation in the mechanical properties. Analyses were performed which showed that the mechanical properties do indeed depend linearly on the hot-rolling variables. Separate models were developed for each of the metallurgical properties. The model for Ultimate Tensile Strength (UTS) had the largest $R2$ value of 0.94, Yield Strength (YS) was next with a value of 0.78, and Elongation had a value of 0.57. The multiple linear regression model was used to determine how much the YS could be increased by varying the steel chemistry and processing temperatures within the allowed ranges. It was found that the mean YS could be increased to about two standard deviations above the test minimum, an outcome which would dramatically reduce test failures for this product. In reality, the optimisation problem is more complicated than this as more than one steel product uses the same chemical grade of steel. Hence optimisation of the relevant mechanical properties over a whole class of steel products needs to be done.

Optimising the relationship of the electricity spot price to real-time input data (Transpower Ltd, Wellington)

Electrical power is paid for at a marginal price calculated by an optimisation to minimise the total cost of generation based on bids made by the power generation companies and consumer requirements. Generation companies are paid on the marginal rate (the level of the highest bid accepted) determined at their location. Similarly bulk power consumers are charged on the marginal price of supply at their location, which includes costs related to delivery to the user's location. There are well known

laws that determine the amount of power delivered along transmission lines. In the case where power lines form a loop, when a power limit is reached on one of the lines, it is necessary to bring into use more expensive generators to allow an increase in power consumption. This creates a step change in the marginal rates charged to consumers. Further it becomes more difficult to deliver power to one end of the limiting line, which is reflected in increased rates at that end. This sudden change in prices is known as a spring washer, with prices increasing on one side of the limiting line, and decreasing on the other side. This contrasts with the usual conditions where consumer costs are constant (when line costs are negligible). Under some circumstances the change in power costs can become extreme. Transpower wanted to determine when large spring washers could occur, and also determine when the spring washer is sensitive to the physical parameters of the network. The MISG group proposed two methods to determine the closeness of a possible spring washer. The first is an investigation of near optimal vertices in the linear programming optimisation. The sensitivity figures from the linear programming optimisation can be used as the basis for this calculation. The number of vertices near to the optimum and the proportion of these investigated, will determine the reliability of this method.

Factors associated with trends in bare ground in high country (Environment Canterbury, Christchurch)

The problem posed at MISG was to analyse the monitoring programme dataset to determine the factors associated with improvement or degradation in vegetative cover. A model resulting from this analysis would assist Environment Canterbury in recommending appropriate management strategies for different land types. Percent bare ground has been monitored at approximately 140 sites throughout the high country, at intervals of one to seven years. Record length varies from 12 to 27 years. Site characteristics specified in the dataset include soil type, topographic position and general management history. Initial analysis at Environment Canterbury suggested that soil fertility and altitude were important factors in recovery of vegetation, but that removal of the already low level of grazing had little effect. Each management factor was studied separately. For each, records were selected from the database where both levels of the factor were present in the same environment, e.g., sites with and without fertiliser application in a similar geographic area and on the same soil type. Five data blocks (regions/soil types) were available for fertiliser analysis and two for grazing analysis. Two-way analysis of variance (ANOVA) showed that fertilising/oversowing was effective in increasing vegetative cover on all soil types, though the magnitude of that change was greater at low altitude than at high. No difference in revegetation rate could be detected between low intensity grazing (less than one stock unit/ha) and no grazing. No comparison was available between "high" intensity grazing (1–4 stock unit/ha) and no grazing. The conclusions were:

- A general model was developed for change in percent bare ground, where the significant factors include fertiliser application, starting percent bare ground, annual average temperature and winter rainfall.
- Soil chemistry and physical properties also appear to be important. Further data gathering and analysis is needed to include these in the model.
- Fertiliser application and oversowing has a strong positive effect on revegetation on all soils tested, with the effect strongest at low altitude.
- Little effect on revegetation was observed from de-stocking (from low intensity grazing to none).

- The effects of rabbit control were difficult to interpret, though there seemed to be some extra positive effect on the better soils that were also fertilised.

Implementing Lanier's patents for stable, safe and economical ultra-short wing vacu- and para-planes (Backyard Technology, Queensland)

Backyard Technology are interested in aspects of aircraft design described by Edward H. Lanier in a series of six patents obtained from 1930 to 1933. Lanier's overall aim was to provide an exceptionally stable aeroplane that would both fly normally and recover from undesirable attitudes without pilot aid. Backyard Technology were specifically interested in Lanier's idea of creating a vacuum cavity in the wing by replacing a section of the upper skin of the wing with a series of angled slats, believing that this wing design would give superior lift and stability compared to typical wing designs. During our study, buoyancy calculations (using Fastflow), indicated that the effect of reducing air density within the wings would have an almost negligible effect, perhaps lightening the aeroplane by a few hundred grams. The other arguments provided by Lanier for additional lift similarly appear unconvincing.

Modelling the physics of high speed product-weighing (Compac Sorting Equipment Ltd, Auckland)

Compac Sorting Equipment Auckland (Compac) manufactures and exports high-speed, accurate sorting systems for fruit and vegetables. Their sizers operate at between 10-15 pieces of fruit per second per lane. They weigh each piece of fruit individually, using a pair of cantilever loadcells, in less than 1/10 of a second. Compac wanted a mathematical model of the weighing process, that will help them to accurately weigh heavier fruit (more than 250g) at higher speeds (in less than a tenth of a second). They also asked for help with easing back on the size and stability of the weighing assembly, which would reduce the physical size and manufacturing cost of the overall system. The signal from each load-cell is amplified and low-pass filtered. The tail end of the signal is averaged, to obtain a mass that is required to be accurate to less than $1g$. The MISG group studied the frequency components present in the output of load-cells, for various sized fruit running at various speeds. Apart from a high frequency which is of no concern to Compac, it was typically observed two lower frequencies, which reduce as fruit mass increases, causing difficulties with oscillations getting past the analogue filter. An option is to reduce the cutoff frequency of the low-pass filter. However, this might not help at higher operating speeds, as there may not be enough time for the filtered signal to level off. They developed models for simple harmonic motion in the vertical direction, as well as a side to side rocking motion between the two load-cells. The modelling suggests that the reduction in the low frequency is generally to be expected as mass increases. The key parameters are mass (and its distribution), effective spring constant, and effective damping. An option is to stiffen and reduce the effective mass of the load-cells, thereby increasing oscillation frequency and damping. However, stiffer load-cells require greater amplification of the signal from the load-cell and are more vulnerable to drift, thus potentially reducing the overall accuracy of weighing. A possible strategy is to use the understandings from the modelling, rather than just filtering out the oscillations. We showed that it is feasible to infer key parameter values from the oscillation frequency, damping rate and oscillation amplitude. A joint approach, digitally combining this information with filtered output, might be faster and more accurate than the present setup. In order to do this, a model that takes into account the structure of the loadcells and

attached plates has been proposed. This model is more complex than a damped oscillator and involves a few time-dependent frequencies but is a promising direction for continued work.

Determining temperature control of wash water in a laundry environment (Fisher & Paykel Ltd, Auckland)

Fisher and Paykel (F&P) are developing a new model of washing machine. One of its key features will be that it uses less water. It is important to regulate the operating temperature of washing machines since if they operate hotter than the user-selected temperature there is a risk of damage to clothes and if they operate below the user-selected temperature there is a risk of incompletely dissolved detergent being sprayed onto clothes, which is also undesirable. F&P seek to improve their temperature regulation strategies from the current state-of-the-art. Further, since the new machine will have a smaller mass of water relative to clothes load, the impact of abnormal clothes loads and of start-up disturbances in water supply temperatures (e.g. cold slugs in hot water supply) on the bulk temperature is greater. Thus a thorough review of temperature regulation strategies is well motivated. A simple control strategy was suggested which uses feedback from a sump temperature sensor was presented. The dynamic model and analysis thereof via the MATLAB code will determine if this strategy is sufficient. Preliminary simulations using this code suggest that in most situations the sump temperature can be controlled to within F&P's specifications (2 degrees Celsius).

Concluding remarks

MISG2005 was sponsored by the list on the web page which also has further details on the problems: http://misg2005.massey.ac.nz.

The Director's prize for the best remark "Overheard in passing" was awarded to Ron Thatcher (Manchester) and Ken Russell (Wollongong) who were heard to say "Have we seen the plot yet?" (Ron Thatcher) "No we have lost the plot." (Ken Russell)

The ANZIAM organisation has asked us to do MISG2006 which will be in the same style and location as MISG2005. The dates for this are: 30 January–3 February 2006, at Massey University, Auckland. See: http://misg2006.massey.ac.nz.

Centre for Mathematics-in-Industry, Massey University, Auckland, New Zealand
E-mail: g.c.wake@massey.ac.nz