The number of wins required to qualify for the AFL finals

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Abstract

We calculate best possible bounds on the number of wins required to qualify for the major round in the current AFL system.

Recently [2], Neil Craig, coach of the AFL team The Adelaide Crows, commented that his team had a long way to go before qualifying for the major round: "As we know you've got to win 12, so we're a long way away from that." In this note we will consider how right (or wrong) Craig was.

The AFL minor round consists of 16 teams playing each other in an incomplete double round robin. The teams play a complete round robin of 15 rounds, then each team plays 7 more matches. After the completion of 22 rounds, the top 8 teams play off in the major round. Teams are graded according to their number of wins and draws (scoring 4 for a win, 2 for a draw), ties being broken on scoring percentage. Two obvious questions arise:

- How many matches must a team win to guarantee a place in the major round?
- What is the least number of wins a team may have to qualify for the major round?

Here we count each draw as half of a win. Table 1 shows the numbers of wins achieved by the teams finishing 8th and 9th at the end of the minor round in each of the years 1995–2004.[1]This table indicates that 12 wins usually, but not always, suffices.

We will prove the following somewhat surprising result.

Theorem 1 *Any team winning 17.5 or more matches will qualify for the major round. No team winning 4.5 or fewer matches can qualify for the major round. These results are best possible.*

Before proving this result we give the equivalent result for a single round robin of 15 rounds.

Theorem 2 *If the AFL minor round were reduced to a single round robin, then any team winning 11.5 or more matches would qualify for the major round and no team winning 3.5 or fewer matches could qualify for the major round. These results are best possible.*

Proof. Consider the 9 teams finishing highest (the "top" teams) and the 7 finishing lowest (the "bottom" teams). The 7 bottom teams play $\binom{7}{2} = 21$ games against each other, accounting for 21 wins. There are $\binom{16}{2} = 120$ games in all, so the top 9 teams can win at most $120 - 21 = 99$ games between them. Hence any team winning more than $\frac{99}{9} = 11$ games will finish in the top 8 and qualify for the finals. Similarly, by considering the top 7 and bottom 9 teams we see that any team losing more than 11 games will finish in the bottom 8 and fail to qualify. This proves the first result.

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[1]Thanks to Janice Gaffney for collecting and supplying this information.

	95	96	97	98	99	00	01	02	03	04
8th team	10	11.5	10.5	12	11	12	12	11	13	12
9th team	9	11	10.5	12	10.5	11	11	11	12	11

Table 1

To prove the second result consider the following scenario. Suppose that each of the bottom 7 teams lose all of its matches against the top 9 and wins 3 of its 6 matches against other bottom 7 teams. Suppose that each of the top 9 teams wins 4 of its 8 matches against other top 9 teams. Then each bottom team finishes with 3 wins and each top team finishes with $7 + 4 = 11$ wins. Hence some team with 11 wins will not qualify. □

Proof of Theorem 1. Suppose that the 9 top teams are $\{1, 2, \ldots, 9\}$ and the 7 bottom teams are $\{10, \ldots, 16\}$. Again the 7 bottom teams play $\binom{7}{2} = 21$ games against each other, accounting for 21 wins. There are $\frac{16.22}{2} = 176$ games in all, so the top 9 teams can win at most $176 - 21 = 155$ games between them. Hence any team winning more than $\frac{155}{9} = 17\frac{2}{9}$ games will finish in the top 8 and qualify for the finals. Similarly, any team losing more than 17 games will finish in the bottom 8 and fail to qualify.

Now we exhibit a scenario whereby a team winning 17 matches fails to qualify. Suppose that in the first round robin results are as in the proof of Theorem 2, so that each bottom team finishes with 3 wins and each top team finishes with 11 wins.

We give in table 2 an explicit schedule for the extra 7 rounds. Here, the entry **4** in row 1, column 10 indicates that team 1 plays team 10 in round 4 and the first named team (1) wins. The entry 4 in row 5, column 9 indicates that team 5 plays team 9 in round 4 and the first named team (5) loses.

Thus teams 1 and 2 will finish on 18 wins and teams 3, ..., 9 finish on 17 wins. One of these teams will fail to qualify. □

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		**1**								**4**		**5**	**6**	**7**	**2**	**3**
2	1										**4**	**2**	**7**	**3**	**6**	**5**
3				**2**							**5**	**4**	**3**	**1**	**7**	**6**
4			2							**7**	**1**	**6**	**4**	**5**	**3**	
5						**3**			4	**5**	**6**	**7**	**1**	**2**		
6					3		**5**			**6**	**7**	**1**	**2**			**4**
7						5		**6**		**1**	**2**	**3**			**4**	**7**
8							6		**7**	**2**	**3**			**4**	**5**	**1**
9					**4**			7		**3**			**5**	**6**	**1**	**2**

Table 2

More generally, one can show the following result.

Theorem 3 *If the AFL minor round consisted of a single round robin together with an extra* m *rounds, where* $0 \le m \le 15$*, then the number of wins need for a team to be guaranteed a place in the major round would be*

$$w(m) = \begin{cases} \frac{1}{2}\lceil 22 + \frac{16}{9}m \rceil & \text{if } m \le 9, \\ 15 + \frac{1}{2}m & \text{if } m > 9. \end{cases}$$

We have not shown this result is best possible, i.e., we have not tried to find explicit schedules attaining these bounds. We leave this (presumably straightforward) task to the reader.

Note that the current AFL system can lead to a requirement that is nearly as large as possible in terms of wins required over games played: the maximum value of $w(m)/(15+m)$ is 0.804, attained for $m = 8$; for the current system with $m = 7$, $w(7)/22 = 0.795$.

The perceived unfairness of the AFL finals system itself was considered in [1].

References

[1] George A. Christos, *New AFL finals system is unfair*, AustMS Gazette **27** (2000).
[2] Neil Craig, http://afc.com.au/default.asp?pg=news\&spg=display\&articleid=196771.

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