Book reviews

Einstein's Heroes: Imagining the World Through the Language of Mathematics

Robyn Arianrhod
University of Queensland Press 2003
ISBN 0-7022-3408-7

Robyn Arianrhod's Einstein's Heroes is a popular-level book describing the work of some of the main intellectual influences on Einstein. The scientist that figures most prominently in the book is James Clerk Maxwell. Indeed, I think the book could also have been called something like: The Life and Ideas of Maxwell. But the book is more than just a biography: Arianrhod also has a larger aim in the book. Her aim, roughly speaking, is to explore the idea that there is "something special" about the language of mathematics that reveals hitherto inaccessible levels of reality to us. So Arianrhod is also examining broadly "philosophical" ideas in her book. Einstein's Heroes begins with a brief chapter in which the larger themes of the book are introduced. She starts with the story of a white child raised as an aboriginal, who had come to see and think about the world in a different way through learning to speak a different language. She then moves on to an account of Newton's main ideas. This takes up roughly the first quarter of the book. In the bulk of the book, Maxwell is the central figure. Arianrhod does not merely give us an exposition of his scientific ideas, but also describes his life and career, and his interactions with his contemporaries. The later chapters describe the influence that Maxwell's ideas had on Einstein, and on more recent developments. She also returns to the more general themes of language, representation and reality. Any popular book on science must avoid two potential dangers. On the one hand, it must avoid excessive dryness and technicality: that will only turn potential readers away. But on the other hand it must also avoid treating the material in such a loose, vague or merely metaphorical way that readers come away without any real understanding of the science discussed. Arianrhod has avoided both these dangers very well indeed. She explains concepts of mathematics and geometry, and many aspects of the ideas of Newton and Maxwell, with crystal clarity and a high level of rigour. But she is also careful to frequently leaven the more technical material with anecdotes about, for example, Maxwell's personal life and interesting asides in to the history of ideas. In these respects her book compares rather favourably, I think, with some other popular books about physics. This achievement is particularly impressive when one notes that the primary focus of her book – Maxwell's electromagnetism equations – do not on the face of it seem like promising material for a book of interest to the general reader. Despite all this, I do have one very minor quibble with the book. I'm afraid I found some of her claims about the nature of the relationship between language and the world, and about the special status of mathematical language, a little unconvincing. But this is a very minor blemish. If the way this book manages to combine clarity and rigour with “general interestingness” is any indication of how Arianrhod conducts her teaching, then her students at Monash are very fortunate indeed.

John Wright
School of Liberal Arts, University of Newcastle, Callaghan, NSW 2308
E-mail: John.Wright@newcastle.edu.au

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Automatic Sequences: Theory, Applications, Generalizations

Jean-Paul Allouche and Jeffrey Shallit
Cambridge University Press 2003
ISBN 0-521-82332-3

Beautifully presented in a concise and scholarly manner, this book develops the fascinating theory of sequences generated by one of the most basic models of computation; namely, finite automata. Generalizations of such sequences, including Sturmian words and k-regular sequences, are also considered, and the strength of the theory is made evident through selected applications in number theory (in particular, formal power series and transcendence in positive characteristic), physics, and computer science. A topic such as this incorporates results from both mathematics and computer science, and consequently, papers on the subject are widespread in the literature, having been studied under different guises and with inconsistent notation. Allouche and Shallit, however, manage to successfully combine a myriad of concepts from a range of seemingly disparate disciplines to form a coherent and extremely informative resource for anyone from the professional researcher to the inquisitive undergraduate student.

Chapters 1 through to 5 provide us with the required background knowledge on stringology, number theory and algebra, numeration systems, finite automata, and automatic sequences. The book then delves into interesting generalizations of automatic sequences, such as the class of morphic sequences, of which automatic sequences form a sub-class. Other generalizations include characteristic words, multi-dimensional sequences, and sequences over infinite alphabets. Of particular interest to experts in this field are the relatively new results on transcendence of formal power series and automatic real numbers, given in Chapters 12 and 13. And the enthusiastic reader is sure to revel in the total of 460 exercises and 85 open problems, which, together with a very comprehensive list of references and bibliographical notes, certainly invoke the urge for further exploration.

Applicable to practically all areas of mathematics and computer science, this book is sure to become a much celebrated text on infinite sequences of symbols and their applications. A worthy addition to every mathematician’s bookcase!

Amy Glen
School of Mathematical Sciences, Discipline of Pure Mathematics, University of Adelaide, SA 5005
E-mail: aglen@maths.adelaide.edu.au

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Option Theory with Stochastic Analysis: An Introduction to Mathematical Finance

F.E. Benth
Springer Heidelberg 2004
ISBN 3-540-40502-X

The book under review was written as a text for an introductory level course (although we read in the Preface that it was used “in a course for students… preparing for a *master* in finance and insurance mathematics”) on option theory (in continuous time and mostly within the Black–Scholes framework), with an objective to "relax the mathematical rigour to focus on ideas and techniques". There already exist several rather good books devoted to that topic and compiled with similar intentions. One could still argue, however, that financial mathematics students with various backgrounds might benefit from new approaches and simplified presentations, especially of the continuous time theory.

The main body of the text consists of five chapters: Introduction, Statistical Analysis of Data from the Stock Market, An Introduction to Stochastic Analysis, Pricing and Hedging of Contingent Claims, and Numerical Pricing and Hedging of Contingent Claims. Unfortunately, one can find rather serious deficiencies in all of them. The overall impression of the reviewer is that, despite some positive features (e.g. its small volume), the book is not well written and contains quite a few misleading and even wrong statements.

It would take too much space to give a detailed analysis of the exposition or to list all the deficiencies noticed by the reviewer in the text. The following small sample, however, could give you some flavour of what one can find there.

On p. 8, in the introduction to probability theory, we read: "We can find the expectation of a random variable X conditional on the event $A \subset \Omega$ as

$$\mathbf{E}\,[X|A] = \mathbf{E}\,[1_A X].\text{"}$$

First the reviewer decided that that must have been just a typo. However, after having seen (on p. 47) that "from Jensen's inequality (see [47, Thm. 19, p. 12]) it holds that

$$|\mathbf{E}\,[Z|\mathcal{F}_s]| \le \mathbf{E}\,[|Z|\,|\mathcal{F}_s] \le \mathbf{E}\,[|Z|],$$

which shows that the conditional expectation is finite under this moment condition", the reviewer is not so sure about that.

One could say a lot about the way the author introduces covariance/correlation and also makes claims about normally distributed random variables (*before* giving the definition of the univariate normal distribution — and that of the multivariate normal distribution he doesn't give at all). We just note that the author refers to

$$X \overset{d}{=} \mu + \sigma Y$$

as a "factorization (sic!) of a normal variable into a linear combination of a constant and a standard normal variable" (p. 9) and then talks about "powerful statistical distributions" (p. 11). And, referring to the sample mean and variance, the author says that "it is standard to use these two estimators for the *empirical* mean and variance of the sample".

On p. 36 we read that "the limit in (3.3) converges in variance, and thus for every $\omega \in \Omega$" (sic!). [In fact, there is a footnote commenting on the last statement, but it doesn't make things look much better.] On p. 39 we discover that, for any twice differentiable function, one can write down Taylor's expansion formula with a cubic remainder term. Furthermore, we learn on p. 41 that a semimartingale is a process that "can be decomposed into an Itô integral and a standard integral" (sic! and no further comments), and all this happens *prior* to the introduction of the notion of martingale (which is also done in quite a dangerous way).

From the author's discussion of completeness/incompleteness and arbitrage on pp. 86–91, we learn that "completeness comes from the fact that we need to be able to trade in every source of noise" and also that "the Lévy process introduces noise that cannot be traded", and also read that an $n \times m$ matrix is "non-singular exactly when $n = m$ (being quadratic)" (?!). Further, we also learn that "if we had completeness in the markets for derivatives, options and claims would not exist simply because they would be redundant. We could achieve exactly the same by entering into the claim's replicating portfolio". The last is as meaningful as the claim that, if the flour were available in all food stores, bakeries would become redundant as everybody could bake bread for themselves.

The above-mentioned examples of poor handling of even relatively basic material are typical for the book. The overall level and logic of exposition are scarcely any better.

Note in conclusion that the book appeared in the Springer *Universitext* series, that (unlike, say, the *Springer Undergraduate Mathematics Series*) apparently lacks an advisory board—at least, the reviewer failed to find anything in the book about who was responsible for selecting the text for publication.

K. Borovkov
Department of Mathematics and Statistics, University of Melbourne, Parkville VIC 3010
E-mail: K.Borovkov@ms.unimelb.edu.au

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Mathematics for Finance: An Introduction to Financial Engineering

M. Capiński and T. Zastawniak
Springer Heidelberg 2003
ISBN 1-85233-330-8

As the authors modestly announce at the very beginning of the Preface, the book "is an excellent financial investment. For the price of one volume it teaches two Nobel Prize winning theories"—and these are the arbitrage-free pricing of derivative securities and Markowitz portfolio optimization (and the Capital Asset Pricing Model). There are eleven chapters in the book: Introduction; A Simple Market Model; Risk-Free Assets; Risky Assets; Discrete Time Market Models; Portfolio Management; Forward and Futures Contracts; Options: General Properties; Option Pricing; Financial Engineering; Variable Interest Rates; Stochastic Interest Rates. So one can see that the bulk of the text is devoted to the former theory, although, in the reviewer's opinion, the 35 pages devoted to portfolio management are quite instructive and constitute a valuable part of the book.

The level of exposition is pretty basic, with results for continuous time being mostly just outlined. That makes the book accessible to second year undergraduate students (and not only for students of mathematics, but hopefully also for students of business management, finance and economics). According to the authors, its contents could be covered in about 100 class hours. Prerequisites include elementary calculus (mostly used to find extrema of differentiable functions), some probability theory ("familiarity with the CLT would be a bonus") and elements of linear algebra (operations with matrices, solving systems of linear equations).

To keep the exposition at a low level, the authors had to state some key results (e.g. the Fundamental Theorem of Asset Pricing, even in the case of a simple discrete time market) without proving (or even properly explaining) them. This makes the task of (really) "understanding the underlying theory" pretty hard for the reader, but, on the other hand, with its rather extensive discussion of the general properties of options (Chapter 7) and a careful explanation of a large number of other important concepts, the book can still serve as a valuable introduction into the area.

There are a lot of (mostly numerical) simple examples and about 190 "doable" exercises dispersed throughout the book, solutions to all of the exercises occupying about 40 pages at the end of the text—which makes the book suitable for self-study. On the other hand, the reviewer has got an impression that the authors may have overdone it here (as the abundance of examples/exercises sometimes creates a situation where one can't see the forest for the trees).

Having said that, the overall impression of the book is quite positive. The reviewer can only congratulate the authors with successful completion of a difficult task of writing a useful textbook on a traditionally hard topic.

K. Borovkov
Department of Mathematics and Statistics, University of Melbourne, Parkville VIC 3010
E-mail: K.Borovkov@ms.unimelb.edu.au

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Analysis on Lie Groups with Polynomial Growth

N. Dungey, A. ter Elst and D. Robinson
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Consider the heat equation on $\mathbb{R}$, $\partial_t \varphi_t + H\varphi_t = 0$, where H denotes the Laplacian $\sum_{i=1}^{d} \partial_t^2$. One of the greatest achievements of Joseph Fourier was the solution of this equation to obtain $\varphi_t(x) = (G_t * \varphi_0)(x)$. Here, G is the Gaussian $(4\pi t)^{-d/2} e^{-|x|^2/(4t)}$. As is well-known, G_t can be considered as the semi-group kernel generated by the semi-group e^{tH}. Since $||G_t||_\infty = (4\pi t)^{-d/2}$, it follows that heat disperses at a rate proportional to $V(t)^{-1/2}$, where $V(t)$ is the volume of the ball of radius t.

Much effort has been expended over the past few decades to exploring these simple observations for Lie groups other than $\mathbb{R}$ and for operators other than the Laplacian. It has been understood for some time that Lie groups of polynomial growth i.e. groups where there is a G-invariant metric such that measure of the balls grows in a polynomial fashion, are the natural setting for generalisation of many of these theorems. Groups such as non-compact semisimple Lie groups, where the balls grow exponentially, require different methods.

Recent work, starting with the French school of Nicholas Varopoulos, Laurent Saloff-Coste and Thierry Coulhon, investigated the situation first for nilpotent groups, and later for more general solvable groups. Derek Robinson and his collaborators Tom ter Elst, Georgios Alexopoulos, and Nick Dungey, have made significant progress in recent years, and have given characterisations of groups where the kernels associated with general strongly elliptic second order operators satisfy Gaussian bounds, as well as giving estimates for Riesz kernels and other derivatives. This monograph is aimed at providing an up-to-date comprehensive survey of this work. It is a natural companion piece to Davies' *Heat kernels and spectral theory* (1989), to the book of Varopoulos, Saloff-Coste and Coulhon *Analysis and Geometry on Groups* (1992), or to Saloff-Coste's *Aspects of Sobolev-type inequalities* (2002).

After a brief introductory chapter, the book gives a careful outline of the theory of Lie groups, derivations, elliptic, subelliptic and strongly elliptic operators and their associated kernels. The important techniques of analysis which will be used in the text: the Carnot-Carathéodory metric, the method of transference and the de Giorgi estimates are introduced (although the proof of the latter is put off to an Appendix.) The point of the de Giorgi estimates is to use energy estimates on the stationary solutions of the heat operator associated with a subelliptic operator, in order to deduce Gaussian bounds.

Chapter III gives an analysis of the structure theory of solvable groups, introducing the nilshadow Q_N and the semi-direct shadow $G_N = Q_N \rtimes M$ associated with a solvable group G. It is proved that G can be realised as a quotient of a larger group $\tilde{G}$, whose nilshadow is a stratified group. For groups of polynomial growth, M is a compact group, and the strategy of proof is to use transference from the (stratified) nilpotent groups to get Gaussian bounds for G.

The next chapter contains, in some senses, the heart of the matter. Homogenisation theory in $\mathbb{R}^n$ is a classical method for treating subelliptic operators with oscillatory coefficients, estimating them by dilation followed by a form of averaging of the spectrum. Alexopoulos saw how to extend this to solvable groups, starting from a subelliptic operator H on G to which one associates a subelliptic operator $\hat{H}_0$ on $L^2(G_N)$: actually, this is constant in the M directions, and can be reduced to an operator $\hat{H}$ on Q_N. It turns out that $\hat{H}$ is a limit of dilates of H, and this enables one to establish Gaussian bounds, first for the nilshadow, and then for G itself. The fundamental equality is of the form

$$|K_t(g)| \leq Ct^{-D/2}e^{-b(|g|')^2/t}$$

where K_t is the kernel associated to the subelliptic operator H, c and d are constants, and $|\cdot|'$ is the Carnot-Carathéodory distance on G. This is an exact analogue of the results of Fourier quoted above! To a great extent, the methods of this chapter are based on work of Alexopoulos, who first obtained these results for sublaplacians on groups of polynomial growth, and then generalised them to solvable groups.

The stage is then set to extend this elegant theory. In Chapter V, the authors show how to obtain bounds like

$$|A^\alpha K_t(g)| \leq Ct^{-|\alpha|/2}V'(t)^{-1/2}e^{\omega t}e^{-b(|g|')^2/t}$$

where A^α is a derivative of order $|\alpha|$ of K_t. If $|\alpha| = 1$, the bound is optimal, although it can be improved to a Gaussian bound if G is near-nilpotent, i.e., a semi-direct product of a compact group and a nilpotent group. The authors prove their main structure theorem: Gaussian bounds hold for derivatives of K_t if and only if G is near-nilpotent. There are also other equivalent conditions, which I shall not detail here. In particular, this implies that it is not possible to get Gaussian bounds on the derivatives of K for a general solvable group.

The last chapter extends the theory to study the asymptotics of semigroups, using homogenisation theory and Gaussian bounds. The main theorems are recent work of Robinson, Dungey, Duong and other collaborators, in the basic direction of establishing a functional calculus for K_t. Again, the central idea is to reduce the proof to certain estimates on Q_N. The main theorems give L^p-multiplier bounds, both from above and below, on fractional powers of H. Again, there is a classification theorem: L^p bounds hold for derivatives of the semi-group S_t associated to H if and only of Gaussian bounds hold.

It is a good moment for this theory to be given a decent exposition, and who better than these authors to do it? The book contains a veritable wealth of examples, and a thorough exploration of the formidable array of analysis which has been assembled to attack these problems. It will be an invaluable research tool, and a wonderful textbook for anyone wishing to get a handle on the area.

Anthony H. Dooley
School of Mathematics, University of New South Wales, Sydney 2052
E-mail: a.dooley@unsw.edu.au

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Graphs on Surfaces and Their Applications

S. Lando and A. Zvonkin
Encycl. Math. Sci. **141**
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This fascinating book is concerned with a modern approach to topological graph theory, with a particular focus on the numerous unexpected applications to, and interrelationships with, other fields of mathematics and also quantum physics, guiding the reader to the cutting edge of current research.

The basic objects of study in the book are: constellations, which are finite sequences of permutations; Riemann surfaces and their representations as ramified covering spaces of the two dimensional sphere; various classes of embedded graphs such as trees, cacti, etc.

There is a nice discussion of the combinatorial and geometric consequences of the profound Belyi theorem, relating Riemann surfaces that are defined over the algebraic numbers, to meromorphic functions having three critical values.

The method of matrix integrals turns out to have an unexpected relevance to the enumeration of graphs, and is the focus of a couple of chapters in the book. The method has its origins in certain matrix models of quantum physics, where the fields are matrix valued. It turns out that in such matrix models, the partition function is the generating function for certain classes of graphs. Some hints are given on methods of calculating these matrix integrals.

The book contains an account of the work of Harer and Zagier, which used the method of matrix integrals as a tool for computing the Euler characteristic of moduli spaces of algebraic curves. There is also a useful sketch of Kontsevich's deep proof of Witten's remarkable conjecture relating matrix models, the KdV hierarchy and the intersection theory of moduli spaces of algebraic curves.

The book also relates the enumeration of graphs to algebraic geometry and singularity theory via the Lyashko-Looijenga mapping. A beautiful extension of the Belyi theorem is discussed, involving an action of the braid group on constellations and the topological classification of meromorphic functions having four critical values.

The final chapter deals with the relationship of graphs to Vassiliev knot invariants and link invariants, via the structure of Hopf algebras on chord diagrams.

The book ends with a useful crash course on the representation theory of finite groups and their relevance to the enumeration of constellations, in the form of an appendix written by Don Zagier.

The book contains numerous diagrams, examples and exercises, making it appealing to both students and researchers.

Mathai Varghese
School of Pure Mathematics, University of Adelaide, Adelaide, SA 5005
E-mail: mathai.varghese@adelaide.edu.au

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Chaos, A Mathematical Introduction

J. Banks, V. Dragan and A. Jones
Cambridge University Press 2003
ISBN 0-521-531047

The study of one-dimensional discrete dynamics, or first order difference equations, has shown that profound complexity can be derived from simple models. The field has matured considerably since pioneering work by Metropolis, Stein and Stein, May, Feigenbaum and others in the 1970s. One-dimensional dynamics is treated in most of the many books on dynamical systems, a classic example being [1]. Several books totally devoted to one-dimensional discrete dynamics also exist, including [2] and [3]. These two research monographs give an expert coverage of the extremely deep results in the field which have attracted the interest of many prominent mathematicians.

On the other hand, the beauty of one-dimensional dynamics is that some of its concepts and results are accessible at an elementary level. This explains why some coverage of the area is now standard in the undergraduate curriculum. Like many topics in dynamical systems, the excitement and significance of cutting-edge research results can be conveyed without an enormous amount of preparatory material. The present book fits into this category. It is pitched at the undergraduate level, for second and third year students, and is based on a course given at La Trobe University. As the authors state in the preface, chaos theory "uses many of the mathematical concepts and techniques from other parts of undergraduate mathematics". In my opinion, this is one of the great strengths of this book: it is a wonderful example of how first year calculus results can be employed to obtain nontrivial results in one-dimensional dynamics. It simultaneously reinforces an appreciation and understanding of the results themselves whilst teaching the student about the dynamics.

The first six chapters deal with standard material, introducing (periodic) orbits and their stability, cobweb diagrams and iteration. In the process, the ideas of limits, convergence and differentiability are reinforced. In particular, the treatment in Chapter 5 of stability of periodic orbits emphasizes how, around a periodic point, the map f^n is locally dominated by an affine map, the stability of which is treated separately in Chapter 4.

The unique strength of the book is revealed in its second half. In Chapter 7, the important concept of *wiggly iterates* for a one-dimensional mapping f is introduced. It means that f^n, $n \geq 1$, has 2^{n-1} humps with the base of each hump decreasing to 0 as $n \to \infty$. The case of the logistic map $x \mapsto 4x(1-x)$ is an example. The idea of wiggly iterates forms the basis of a significant discussion of the ingredients of chaos in Chapters 8 and 9. The three-faceted definition of chaos used here follows that of Devaney in [1] (although it should be noted that La Trobe mathematicians showed over a decade ago that one of these facets is redundant and implied by the other two). It is shown in Chapter 9 that wiggly iterates for a map implies sensitive dependence on initial conditions everywhere, transitivity and a dense set of periodic points. Conversely, the presence of any one of these properties implies a (symmetric) one-hump map has wiggly iterates.

Chapter 10 shows that the most significant way that f fails to have wiggly iterates is because it, or some power of it, has a *woggle*. A woggle is a fat wiggle (and so, in time, one may wonder if this term will also be appropriated to describe ageing children's entertainers). A sufficient condition to not have a woggle is for the so-called Schwarzian derivative $S(f)$ to be negative. This is a lovely Chapter which motivates the usually mysterious concept of $S(f)$ and also shows how it is preserved under composition. The mean value theorem is used extensively in the proofs.

Chapters 11 and 12 give a nice coverage of (topological) conjugacy, i.e. the idea of relating one map to another in terms of an invertible continuous coordinate transformation. This idea is very important in dynamics (and, of course, in mathematics more generally). It allows a choice of a canonical representative for the conjugacy class, or a normal form. To illustrate this, the authors prove that the tent map is representative of all one hump maps with wiggly iterates. Along the way in Chapter 12, the concepts of Cauchy sequences, completeness and uniform convergence are introduced and used.

Finally, in Chapter 13, the notion of an invariant set for a one-hump map is introduced. This allows extensions of the results of previous chapters to one-hump maps that expansively map the unit interval into a range bigger than the domain. Again, having wiggly iterates turns out to be a key concept as it implies that the largest invariant set for the one-hump map is a Cantor set and that the dynamics on this Cantor set is chaotic.

The authors should be commended for managing to steer a course through simple mathematics so as to recover some of the important results in one-dimensional dynamics. The book is well-written with a great set of illuminating and searching problems. Plus there are many great illustrations. The book is consistent with the pedagogical approach of learning mathematics through extensive problem-solving that is a hallmark of the La Trobe teaching method. I can see various uses for this book. The obvious one, consistent with its origins, is to form the basis of a stand-alone second or third year one-semester course in one-dimensional chaos. Parts of it could also be used to make nice learning modules that could be inserted into the advanced stream of a calculus course, nicely reinforcing those results with dynamical applications. Finally, for bright undergraduates who need to be extended, parts of it could be given as a reading course for independent study.

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John A.G. Roberts
School of Mathematics, University of New South Wales, Sydney NSW 2052
E-mail: jagr@maths.unsw.edu.au