



Book reviews

Fortran 95/2003 Explained

M. Metcalf, J. Reid, M. Cohen
Oxford University Press 2004
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In the 50 or so years since its inception, the Fortran programming language has evolved to accommodate remarkable changes in computer hardware as well as the experiences and insights gained by generations of programmers. For a striking impression of this evolution, one need only compare *Fortran 95/2003 Explained* with the earliest Fortran manual; the latter is readily available via the world wide web (go to [2] and follow the links “Index” → “IBM Fortran Manual for the 704”). The language standardisation process has given us Fortran 66, Fortran 77, Fortran 90 and Fortran 95. At present, the approval process for the next standard, known as Fortran 2003, is in its final stages [4].

Metcalf and Reid, the first two authors of the work reviewed here, are well known for a series of books on Fortran 90/95 that they have updated progressively over the past 15 years. Their latest offering adds a third author, Malcolm Cohen. *Fortran 95/2003 Explained* contains 20 chapters and 6 appendices. Chapters 1–10 describe Fortran 95 and Chapters 11–20 describe the language features added in Fortran 2003. Actually, the features in Chapters 11–12 were fixed several years ago in two Technical Reports and are already widely implemented. Thus, Chapters 1–12 repeat pretty much the contents of the previous book. At 416 pages, *Fortran 95/2003 Explained* is about 20% longer than *Fortran 90/95 Explained*, which

perhaps gives a fair idea of scale of the new features of the language.

Fortran 2003 is fully backwards compatible with Fortran 95, so neophobes have the option of ignoring the new standard. However, although some of the new features take considerable effort to understand, others make the language simpler and allow better performance. A relatively trivial example is that it becomes legal to say

```
real, parameter :: PI=4*atan(1.0)
```

because Fortran 2003 permits any intrinsic function in an initialization expression. More significantly, the allocatable array extensions and the associate construct reduce the need for using pointers. Another important simplification is interoperability with the C programming language. Calling C procedures from Fortran, or vice versa, has always been a portability nightmare. Providing standardised support for mixed-language programming will certainly be welcomed by anyone wanting to provide C bindings for a Fortran library. Fortran 2003 also improves portability by providing new intrinsic procedures for accessing command-line arguments and environment variables, thereby standardising what is currently provided only via compiler-specific extensions. Other new intrinsics relate to IEEE arithmetic and floating-point exception handling.

Although very useful, none of the above features changes the language in any essential way. More interesting are the enhanced facilities for derived types and object-oriented programming. Fortran 2003 allows derived types to be parameterized in the same way as the intrinsic data types. For example, we might define

```

type, public :: point(dim,wp)
  integer, len  :: dim
  integer, kind :: wp
  real(kind=wp) :: coord(dim)
end type point

```

and then declare a two-dimensional, single-precision point P and a three-dimensional, double-precision point Q by

```

type(point(dim=2, kind=SP)) :: P
type(point(dim=3, kind=DP)) :: Q

```

where the constants SP and DP are the kind type parameters for single and double precision reals, respectively. Support for object-oriented programming is provided by introducing procedure pointers, type extension (simple inheritance), polymorphic variables, abstract interfaces and type-bound procedures. As an example of a type-bound procedure we might define

```

type, public :: triangle
  real :: corner(3,3)
contains
  procedure :: centroid
end type triangle

```

where `centroid` is a function that returns the centroid of the triangle. After declaring `type(triangle) :: t`

and initialising `t`, we may use either the component-selection syntax `t%centroid()` or the traditional `centroid(t)`.

Space does not allow further elaboration of these and other new features of Fortran 2003. Short of buying the book, I can recommend a 38 page article by the second author [1], available via the world wide web (go to Fortran standards web page [4] and follow the link to “Electronic Archives”).

I have used the earlier books by Metcalf and Reid intensively for many years, and fully expect *Fortran 95/2003 Explained* to take their place as my reference on Fortran. The great strengths of the book are its attention to detail, careful explanation of fine points and comprehensive coverage. Although not suited to a complete beginner, the text does a good job of illustrating language features with examples and also

provides numerous exercises (with solutions at the back of the book). I do not know of any better reference work for the experienced Fortran programmer.

As a practical matter, it will be some time before any compilers implement the whole of Fortran 2003. This fact dictates the structure of the book, with Chapters 1–12 providing a self-contained description of Fortran 95 and the remaining Chapters 13–20 covering the new features of Fortran 2003. Such a structure is also convenient for anyone already familiar with Fortran 95. One point to note is that the new book does not explain the differences between Fortran 90 and Fortran 95. For this reason, if your Fortran compiler does not fully implement the Fortran 95 standard then the previous book will be a more suitable reference. Alternatively, you might consider using the open-source compiler [3] which has undergone very rapid development in the past year.

References

- [1] John Reid, The new features of Fortran 2003, WG5 document N1579.
- [2] <http://www.fortran.com/>.
- [3] <http://www.g95.org/>.
- [4] <http://www.nag.co.uk/sc22wg5/>.

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Curve Ball

J. Albert and J. Bennett
 Springer-Verlag Heidelberg 2003
 ISBN 0-387-00193-X

This is a very good, fun and highly interesting book, applying some straightforward, and some more difficult, statistical estimation and modeling concepts to baseball. I

should mention here that I am a statistician, and mostly Bayesian at that, and this definitely enhanced my interest and enjoyment of the book. The book is not only for baseball fans, although it helps to be a sports lover and a pre-requisite is knowing the rules, and language, of baseball in the US. In a few instances, even an avid sports watcher like myself was stumped by baseball terminology used; this is the main drawback of this book. A further drawback for mathematicians may be the lack of any non-statistical mathematical concepts, this was not a problem for me! This book is for sports minded numerate people. In addition the book contains no real modern or complex statistics, despite a certain quote on the inside cover.

The book starts off by sneakily introducing basic concepts of statistical modeling and probability through an analysis of various baseball tabletop board games. These games use probability and simulation, either through the use of dice or a spinning disk, to simulate outcomes from a baseball match. To a statistician this reads like an introduction to discrete data distributions, probability and independence, concepts well used throughout the book. The concept of an interactive effect between pitcher and batter is also well presented. Next, current player performance statistics, such as batting average, On Base Percentage and SLuGging percentage for batters, plus strikeouts and walk rates for pitchers, are considered while introducing the concepts of continuous distributions (and comparing these) and confidence intervals for unknown true percentages. These are introduced from a well-disguised Bayesian point of view. The book actually contains many hidden jibes at frequentist statisticians, although I challenge anyone but a Bayesian to find them! This is consistently very subtly done and added to my enjoyment of the book.

The question of whether players really have different abilities, whether some players or teams really have hot or cold streaks

and what factors (apart from ability) really affect performance is well presented and discussed. These questions are discussed at great length in the media and by fans, who will be relieved to know that baseball is probably not driven by random variation alone. Linear regression is discussed next, searching for the 'best' batting statistic for predicting runs per game. Surprisingly a lot of research has been done in this area, well past the simple OBP or SLG. The comparison of these statistics is very interesting, especially comparing additive to multiplicative models. Much of the rest of the book is concerned with measuring the effects of certain 'plays' at certain stages of the game and innings. For instance, what is the effect of a home run at a certain stage of a game, depending on number of outs, difference in runs scored, number of completed innings, runners on base, etc. Or, whether it is really ever worth the risk of stealing a base. These questions draw on most of the data ever collected about baseball and again are interesting and fun to look at. The analysis of the effects of certain influential plays, so called 'clutch' plays, in the world series of baseball, and how they changed the probability of a certain team winning or losing should be intriguing to sports fan and statistician alike. Wrapping up the book is an interesting simulation illustrating how often the team with the best ability actually wins the annual world series of baseball.

Initially this book starts out as a sneaky introduction to statistics and Bayesian concepts, however, it turns into a delight for sports fans and statisticians alike. Highly recommended if you are either or both (as I am) of these.

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Cyclic Homology in Non-Commutative Geometry

J. Cuntz, G. Skandalis, and B. Tsygan
Enc. Math. Sci. **121**
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The Springer Encyclopedia of Mathematical Sciences is a well-known series of volumes, containing concise but in-depth overviews of many of the main topics of Pure Mathematics. Therefore, the volume under review, which is part of a sub-series on Operator Algebras and Non-Commutative Geometry, raises high expectations. Over the past few decades Non-Commutative Geometry and Cyclic Theory have established themselves as important research areas. The central idea behind Non-Commutative Geometry is to treat non-commutative (associative) algebras as if they were algebras of functions on a manifold. In particular, cyclic cohomology then is the analogue of de Rham cohomology. This approach has shed new light on Non-Commutative Algebra in general, and in particular on the theory of operator algebras. On the other hand, it has also found applications to problems in classical Geometry, for example the study of foliations and group actions on manifolds, which both give rise to a non-commutative algebra that can serve as a model for the algebra of functions on the quotient space, even if the set of orbits no longer has a reasonable topology.

Since Non-Commutative geometry is not part of mainstream undergraduate or graduate education, good overviews of the area are essential for interested outsiders. There are two well-known introductions, one by Connes and one by Loday. Given the fact that they are about ten years old, a new overview of the field is more than welcome.

The volume under review offers a collection of three separate survey articles. The first article, by Cuntz, offers an introduction to cyclic cohomology, with many improvements over the approach presented in

Loday's book. It also gives a quick introduction to bivariant K-theory and the Chern-Connes character for locally convex algebras as developed by the author, and a short section on further variations, such as entire cyclic cohomology and local cyclic cohomology. The second article, by Tsygan, starts again with basic definitions of cyclic homology, but in a different set-up than Cuntz. It then proceeds with 'Non-Commutative Differential Calculus', which generalises the calculus of differential forms and vector fields on manifolds to the non-commutative setting, using the framework of homotopical algebra. The final sections briefly give some examples and a very rough sketch of Index Theory for deformations and its application to the proof by Bressler-Nest-Tsygan of a conjecture by Schapira and Schneider concerning a Riemann-Roch type theorem for so-called elliptic pairs, which generalised Riemann-Roch for complex manifolds, the Atiyah-Singer index theorem, the Boutet de Monvel index theorem for boundary value problems, and the Kashiwara index theorem for holonomic D-modules. The final article in the volume is a translation of the Séminaire Bourbaki talk by Skandalis on the work by Connes-Moscovici on Index Theory in the context of manifolds with group actions.

Unfortunately, the pace, the level of detail and the motivation provided is very uneven amongst and even within the articles. The part of Cuntz' article concerning cyclic homology is an excellent introduction for the non-expert; it is elegantly written and well-motivated. On the other hand, the part dealing with K-theory and the Chern character does give all the necessary definitions, but with hardly any motivation, or general background philosophy. The article by Tsygan is very strong when it comes to motivation, philosophy, providing a broad view and stimulating ideas. However, it is too short: only forty pages. This shows especially towards the end, where detail and exposition are suppressed to such an extent

that at least the reviewer found it impossible to get any grip on even the main ideas. Skandalis' Bourbaki talk, on the other hand, is of a highly technical nature, and for that reason essentially inaccessible for the average interested outsider.

In conclusion, I feel this volume is a missed opportunity. In the hands of a critical editor with a strong commitment to non-expert readers it could have become an excellent survey of the area. However, in its current form I find it hard to recommend, especially when taking into account the price of about AU\$150 for only 137 pages.

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Lectures on Partial Differential Equations

Vladimir I. Arnold
Universitext

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This book contains a series of lectures delivered by Professor Arnold to third year students at the Mathematical College of the Independent University of Moscow throughout the fall semester of the 1994/1995 academic year. Many important examples of partial differential equations are found in the continuous - medium models of mathematical and theoretical physics. These equations, mainly of the second order, and the boundary value problems associated with them are discussed in these lectures.

As the author points out, there is no unified theory of partial differential equations. Some classes of equations have their own theory, for example elliptic and parabolic equations, whereas others have no theory

at all. The reason for this, according to the author, is a very complicated geometry intertwined with partial differential equations. There is a complete theory for equations of the first order. These equations are thoroughly discussed in the first two lectures. The main topics are their characteristics and the Cauchy problem, as well as the geometry associated with these.

Lectures from 3 to 12 are devoted to partial differential equations of the second order. Lectures 3 - 5 cover Huygen's Principle in the theory of wave propagation. Solutions to main boundary value problems (Cauchy and D'Alembert) are constructed. Emphasis is made on the importance of the Fourier method which is very effective for these equations. Many laws of physics can be described by the variational principles. Lectures 6 - 7 contain a discussion of the principle of least action going back to Hamilton. This results in the derivation of the Euler - Lagrange equations. As an illustration the boundary value problems for the wave and Laplace equations are studied. Lecture 8 gives detailed studies of harmonic functions: mean value property and maximum principle amongst others. Lectures 9 -10 are concerned with potential theory for harmonic functions and the role of fundamental solutions in this theory. This culminates with a theorem on removable singularities for harmonic functions, which in a colloquial language can be expressed by saying: "a stretched membrane cannot be propped up at one point by a needle". The final lecture is devoted to the Dirichlet and Neumann problems in bounded and exterior domains for the Laplace equation.

The book is closed with Appendix A offering the proof of Maxwell's theorem on the multifold representation of spherical functions and Appendix B containing examination problems.

In brief, this book contains beautifully structured lectures on classical theory of linear partial equations of mathematical

physics. Professor Arnold stresses the importance of physical intuitions and offers in his lecture a deep geometric insight into these equations. The book is highly recommended to anybody interested in partial differential equations as well as those involved in lecturing on these topics. I encourage readers of this book to take note of the Preface which contains very interesting comments on the role of Bourbaki's group in mathematics, a theme which resurfaces many times in these lectures.

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Symmetric Functions and Combinatorial Operations on Polynomials

Alain Lascoux
 CBMS Reg. Conf. Series in Math. 99
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 ISBN 0-8218-2871-1

The theory of symmetric functions is a rich and beautiful subject which has applications ranging from the representation theory of the symmetric and general linear groups, to orthogonal polynomials, knot theory, combinatorics and algebraic geometry.

By definition, a *symmetric function* of an alphabet \mathbb{A} is any function of the elements of \mathbb{A} which is fixed by all permutations of \mathbb{A} . Least this definition seems too specialized, suppose that $f(x)$ is a polynomial in one variable with (distinct) roots $\mathbb{A} = \{\alpha_1, \dots, \alpha_n\}$. Then

$$f(x) = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n)$$

$$= \sum_{k=0}^n (-1)^k \left(\sum_{1 \leq i_1 < \dots < i_k \leq n} \alpha_{i_1} \dots \alpha_{i_k} \right) x^{n-k}.$$

Thus, the coefficient $e_k(\mathbb{A})$ of $(-1)^{n-k} x^k$ is a symmetric polynomial in the roots of $f(x)$. In fact, $e_k(\mathbb{A})$ is the k th *elementary symmetric* polynomial in the alphabet \mathbb{A} . It is a celebrated theorem of Newton's that the ring of symmetric functions in \mathbb{A} is generated by the elementary symmetric functions $\{e_k(\mathbb{A}) \mid 1 \leq k \leq \#\mathbb{A}\}$.

This book develops and applies the theory of symmetric functions to the study of polynomials. This is possible because, as illustrated above, the coefficients of any polynomial are symmetric functions of the roots of the polynomial. The first chapter introduces symmetric functions, with the Cauchy kernel playing a central role both here, and throughout the book, as it used to define an inner product on the space of symmetric polynomials. Chapter 2 looks at λ -rings and Lagrange inversion. In chapter 3 the Euclidean algorithm is reinterpreted in terms of symmetric functions, with remainders and resultants being described in terms of Schur functions. Later chapters consider, among other topics, continued fractions, Wronkians, the division algorithm, Padé approximation, divided differences and orthogonal polynomials. The last three chapters switch to the non-commutative setting and introduce Schubert polynomials, a non-commutative Cauchy kernel and the plactic algebra.

There is a wealth of information in this book, as well an extensive bibliography and an abundance of exercises (with solutions!) for the conscientious reader. The major drawback of this book is that it is written for experts with many details being left to the reader. Moreover, important concepts are often introduced hastily without being fully explained or motivated. For example, one of the stated aims of the book is to use the theory of symmetric functions to "describe the technique of λ -rings"; yet, λ -rings are never formally defined and, instead, one must pick this up by osmosis.

For the reasons above, before attempting this book I recommend that beginning readers first read Sagan [2] or Stanley [3]. The bible in this subject is of course Macdonald's classic book [1]. This said, Lascoux does cover many topics which are not easily found elsewhere in the literature, so the reader who is willing to invest the necessary time will find this book a great adventure.

References

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- [2] B. E. Sagan, *The symmetric group*, Graduate Texts in Mathematics **203**, second ed. (Springer-Verlag New York 2001).
- [3] R. P. Stanley, *Enumerative combinatorics, Vol. II*, (CUP Cambridge 1999).

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Russian Mathematicians in the 20th Century

Yakov Sinai (ed.)

World Scientific Singapore 2003

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This large book (700 pages) has chapters about 33 Russian mathematicians, each of whom was one of the important mathematicians of the 20th century — but there is no perceptible ordering in that list, neither chronological nor alphabetical. The mathematicians considered are (in alphabetic order): A. D. Aleksandrov, P. S. Aleksandrov, S. N. Bernstein, N. N. Bogoliubov, N. G. Chebotaryov, B. N. Delone, D. F. Egorov, D. K. Faddeev, I. M. Gelfand, A. O. Gelfond, L. V. Kantorovich, M. V. Keldysh, A. Ya. Khinchin, A. N. Kolmogorov, M. G. Krein, M. A. Lavrentev, Yu. V. Linnik, L.

A. Liusternik, N. I. Luzin, A. M. Lyapunov, A. I. Malcev, A. A. Markov, D. E. Menshov, P. S. Novikov, I. G. Petrovsky, L. S. Pontryagin, V. A. Rokhlin, V. I. Smirnov, S. L. Sobolev, V. A. Steklov, A. N. Tikhonov, P. S. Urysohn, I. M. Vinogradov. Israil Moiseevich Gelfand is the only one still living. Remarkably, no women are included — I would certainly have expected a chapter about O. A. Ladyzhenskaya; and each of V. N. Faddeeva, P. Ya. Kochina, L. V. Keldysh and O. A. Oleinik had a strong claim to be included.

Each chapter starts with a brief biography by the Editor, with photograph and dates of birth and death. None of those biographical articles cites any source, except for D. K. Faddeev. (But there is no photograph of D. K. Faddeev, and no date is given for his death.) The texts of the biographies mostly occupy 1 to 2 pages: but the entire biographical text for L. A. Liusternik (p. 469) consists of the sentence “Lazar Liusternik was a corresponding Member of the Division of Physical-Mathematical Sciences since 4 Dec 1946”.

The Editor's biography is followed by a biographical article (in Russian) for V. A. Steklov, and by 3 biographical articles (1 in Russian, 2 in English) for D. F. Egorov. Each of the other 31 chapters reprints one or more articles by that mathematician (in Russian or French or German or English, or in English translation from Russian or from French), and in some chapters another biographical article is reprinted (in English translation from Russian, or in English).

This anthology of mathematical writings contains many very significant works, including I. M. Vinogradov's renowned papers on primes, A. O. Gelfond on Hilbert's 7th problem, A. Ya. Khinchin's classic little book *Three Pearls of Number Theory*, A. N. Kolmogorov on turbulent fluids, S. L. Sobolev on functional analysis, I. G. Petrovsky on partial differential equations, L. V. Kantorovich's pioneering papers (written in

English) on linear programming, and A. A. Markov (*junior*) on algorithms.

Many of the reprints of articles lack bibliographic details, and so the reader needs to consult the list of Contents (pp. *vii-xi*) — but some bibliographic information is missing there. The articles by A. M. Lyapunov are printed in English translation (typewritten), but the Contents ascribes those translations only to *Collected Papers*, with no further information. The article about V. A. Steklov is a speech (in Russian) given by N. M. Gyunter at a memorial meeting of the Leningrad Physical-Mathematical Society on 9 October 1926, and the Contents identify the source only as *Uspekhi*, Vol. 1, new series, No. 4. In fact, Gyunter's speech was first published as pp. 49–77 in the book *In Memory of V. A. Steklov* (in Russian), a collection of papers published by the Academy of Sciences of the USSR at Leningrad in 1928. It was reprinted in *Uspekhi Matem. Nauk* (new series) **1**, No. 3–4 (1946), 23–43, and that reprint is reproduced in this book. I. M. Vinogradov's renowned papers on primes are published in English translations, which are not identified as coming from his *Selected Works*[1].

The 3 chapters of A. Ya. Khinchin's classic little book *Three Pearls of Number Theory* are reproduced from the English translation published by Graylock Press — but the publication date (1952) is not stated. And Khinchin's very significant preface *A Letter to the Front: March 24, 1945* is not reproduced from pages 9 and 10. He addressed that to a former student (for one year) who had been wounded after 3 years of fighting against the Nazi invaders, and had written from a hospital to his former professor asking him to send “some little mathematical pearls”. After several days deliberation, Khinchin selected “the three theorems of arithmetic which I am sending you, to be genuine pearls of our science”. He explained that “They have all been solved quite recently, and there are two

remarkable common features in their history. First, all three problems have been solved by the most elementary arithmetical methods (do not, however, confuse elementary with simple: as you will see, the solutions of all three problems are not very simple, and it will require not a little effort on your part to understand them well and assimilate them). Secondly, all three problems have been solved by very young, beginning mathematicians, youths of hardly your age, after a series of unsuccessful attacks on the part of ‘venerable’ scholars. Isn't this a spur full of promise for future scholars like you? What an encouraging call to scientific daring!

The work of expounding these theorems compelled me to penetrate more deeply into the structure of their magnificent proofs, and gave me great pleasure.”

A. N. Kolmogorov's 4 papers on turbulence in fluids are published in English translations, which are not identified as coming from Volume 1 of his *Selected Works* [2]. L. S. Pontryagin's papers are published in English translations, which are not identified as coming from Volume 1 of his *Selected Works* [3].

The paper [4] by M. Krein and D. Milman (in English with Ukrainian summary, pp. 457–462) was published in the Polish journal *Studia Mathematica* t. **9**, but the highly significant date of publication (1940) is not indicated. L. A. Liusternik's survey article [5] is attributed to *Uspekhi*, new series, Vol.1, No.1 with no date given: actually it was published in Vol.1 No.11 (1946), 30–56. The very brief biographical article (p. 599) about Andrei Andreyevich Markov (1856–1922) tells that “Markov had a son (of the same name) who was born on September 9, 1903 and followed his father in also becoming a renowned mathematician”. But the following articles were both written by the younger Andrei Andreyevich Markov! M. A. Lavrentev's paper is attributed to *J. d'analyse Mathématique* **19**; which should be *J. d'Analyse Mathématique*

19 (1967), 217–225. A. N. Tikhonov’s paper is attributed to *Math. Ann.* **102**, but the date of publication (1930) is not indicated; and P. S. Aleksandrov’s paper *The principal topological discoveries of A. N. Tikhonov* is misnamed in the Contents (p.*xi*) as *The principal mathematical discoveries of A. N. Tikhonov*.

There are several further misprints, including (on p.*viii*) “*Scienes*” for “*Sciences*”, “P. S. Alexdrov” for “P. S. Aleksandrov” and “Petreovski” for “Petrovsky”.

This book is a valuable anthology of mathematical writings — but it should have been edited with greater care.

References

- [1] I.M. Vinogradov, *Selected Works* (translated by P. S. Naidu) (Springer-Verlag Berlin 1985).
- [2] A.N. Kolmogorov, *Selected Works* (translated by V. M. Volosov) (Kluwer Academic Publishers Dordrecht 1991).
- [3] L.S. Pontryagin, *Selected Works* (translated by P. S. Naidu) (Gordon & Breach New York 1986).
- [4] M.G. Krein & D.I. Milman, *On extreme points of regular convex sets*, *Studia Mathematica* **9** (1940), 133–138.
- [5] L.A. Liusternik, *Topology and the calculus of variations* (in Russian), *Uspekhi Matem. Nauk*, new series **1** No.11 (1946), 30–56.

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