Acceptance speech on the award of the George Szekeres medal

Bob Anderssen

1 From the heart and soul!

Let me first share the huge impact that this award has had on me. When one sits in an audience on such occasions, and the recipient of an award says how daunting it is to be in such a situation, it is difficult to appreciate; if one finds oneself in such a situation, it is easily understood.

For me, this award continues to be a great shock that I still have difficulty comprehending. When Elizabeth Billington telephoned to inform me of the fact, it was so unexpected I thought I might faint – I even sent her an email message asking for independent clarification. The emotional impact was so strong that I was nearly in tears when I told my wife. (She just patted me on the shoulder.)

For me, as like you, George Szekeres is very special, and, thereby, any medal named in his honour is very special. It is therefore with a mixture of great pride and humility that I accept this medal and honour.

When I say that I am incredibly shy, everyone laughs! It could be that that shyness has been important in me being the recipient of this singular honour. The shyness says, “Why me? There are clearly other people who deserve this honour.”

The pride says, “If ever there was an honour that I would treasure above all others, it is the George Szekeres medal. How can Lady Luck be so kind to me?”

Though my parents, wife, family and friends have played an important role in my being honoured in this manner, for which I am most grateful, it could not have happened without the trust and support that I received from key people such as teachers, peers and colleagues. All cannot be mentioned – it would be a cast of hundreds. Many are already acknowledged through joint publications. However, it is important for me to acknowledge those who, historically, were stepping stones in my professional development: Granger Morris, Clive Davis, Leo Howard and John Mahony in Queensland; Ren Potts, Rainer Radock and Rudolf Vyborny in Adelaide; Gordon Preston and John Miller at Monash; Mike Osborne, Bernhard Neumann and Neil Trudinger at the ANU; Joe Gani, Chris Heyde, Frank de Hoog, Ron Sandland, Bob Frater and Murray Cameron in CSIRO; John Giles, Bill Summerfield, Ren Potts, Neil Trudinger, Chris Heyde, Terry Speed, Walter Bloom and Reyn Keats while President of the Australian Mathematical Society.

The importance of the support and guidance that I and the colleagues of my generation received should not be underestimated. It is not now as strong as it was then. Rationalisms like “Everything must make a profit!” work against fostering and development of mathematical talent in our community, as well as the development of mathematicians. I sincerely hope that my award of the George Szekeres medal can be used to increase awareness of the importance of mathematics and statistics in the community, and in underpinning the rigour in science, technology and industry.

2 From the mathematician!

In accepting this medal and honour, I would like to share with you some of the beacons that have been most influential to me in the way that I do research and collaborate with colleagues. I would also like to briefly review some of my research over the last 15 years.

Von Bekesy's rules for experimentation

In 1961, von Bekesy received the Nobel Prize for Physiology and Medicine for his work on the functioning of the auditory system, which, among other things, established the role of the ear drum as a vibrating membrane and not as a set of strings like in a piano, as originally proposed by Helmholtz.

His rules for performing an experiment were:

vB-1. Never perform an experiment unless absolutely necessary.

vB-2. Any experiment must be as simple as possible.

vB-3. The first two rules are best implemented and applied by someone who could perform the most complicated of experiments if and when the need arose.

On reflection, these rules can be given an interesting mathematical interpretation and perspective:

MIP-1. Understand the problem context and conceptualize the matter under examination before initiating new research. Many questions have simple answers once viewed from the right perspective. In a way, George Polya's strategy [14] for problem solving represents a way of implementing this rule.

MIP-2. This is the major challenge of mathematical research: formulate a model; develop a proof or find an explanation that exposes the matter under consideration with lucidity and simplicity. This is the real test of mathematical knowledge and skill. It is easy to write down mathematics that is more complex than necessary. Finding a simple and perceptive explanation is the challenge.

MIP-3. This represents a non-trivial explanation of, and justification for, the importance of a thorough deep understanding of mathematics. It should be used by mathematicians as a marketing point for encouraging students to study mathematics - not because they should become mathematicians, but because they will be more successful in what ever they do in the future.

We now know something of how the brain works. It is clear that the study of mathematics helps build the circuitry in the brain that is important for problem solving, whatever the context.

There is an equally important counterpart of such rules in terms of how industrial mathematical modelling should be approached. It is the easiest thing in the world to formulate complex mathematical models of real-world phenomenon. The challenge is to formulate models that respect von Bekesy's rules. What is required, for a good model, is the simplest possible structure that answers the question under examination (viz. Occam's Razor).

Interestingly, this objective of simplicity often confuses the non-mathematician. They have an anticipation that the proposed model will involve fancy formulas about which they have no understanding, and, moreover, that it will answer every question that will arise in the future. For the mathematician there is a need to explain that one *"formulates mathematical models"* to answer questions. Often the challenge is not the formulation of the model, but the identification of the question. Indirectly, this is a feature of the nature of mathematics. The structure of the solutions of (ordinary differential) equations are often acutely sensitive to certain small changes in their structure (e.g., just add a non-linear term to the equation for simple harmonic motion).

Initially, however, one must identify the question under examination! Models are formulated to answer questions. Knowing and understanding the question usually assists in keeping the model simple, and assists with the implementation of the von Bekesy rules.

The agreement between Hardy and Littlewood about their collaboration [8]

> **HL-1.** If one wrote to the other, one did not need to worry that one might make a mistake.

How very, very deep and seminal. It brings one back to the "Lets Guess!" paradigm. This is not how mathematicians often behave towards each other's work, and other people's understanding of mathematics. In general, mathematics is not taught in schools from this perspective.

What is the larger of the two volumes formed by a rectangular sheet? Young children in general say that they are the same. When asked why, they respond that the areas are the same. This is a wonderful mistake - the associated learning experience builds new circuitry within the brain.

> **HL-2.** When one received a communication from the other, there should be no urgency in replying.

This brings one back to von Bekesy's rules (i) and (ii).

> **HL-3.** They could each work on the same aspect, but that would be inefficient.

Successful collaboration involves a mixture of complementary skills. The bigger the pool of knowledge in which a solution is sought, the higher the probability of success.

> **HL-4.** Priority would never be an issue.

So very, very important and wise. In solving a problem, the insight that opens a new door involving a deeper understanding of the situation is equally important as the technical gymnastics to put the insight on a rigorous footing. One does not know in advance from where that initial key insight will come.

Great successful collaborations are based on such rules, though they may not be explicitly understood. The same rules apply in any collaboration, including personal.

3 Some snapshots of some of my research

Let me share with you some aspects of some of the research which have a direct connection to the award of the George Szekeres medal.

The Stuart & Sons Piano

To establish a new musical aesthetic, one must create a significantly new and special sound that catches the attention of the general public as well as musicians and composers. In general, this will only occur when something truly innovative is achieved, as with the Stuart & Sons' pianos manufactured by Piano Australia. The innovation was to replace the traditional horizontal zig-zag clamping of the strings to the bridge on the soundboard by a *vertical* zig-zag clamping. To understand the relevance and significance of this achievement, one must review the history of the development of the piano, and understand the role of aesthetics in achieving stability of sound for composers, musicians and audiences.

The enhanced "singing" sound of the Stuart & Sons' pianos relates to psychoacoustics of hearing as well as the nature of the vibration of piano strings. Experimentally, the proof is simple that there is a difference between horizontal and vertical zig-zag clamping – just play C two octaves above (or below) middle C on different pianos and listen to the differences. Musically, to establish a new aesthetic, it is not only a matter of adapting the performance practice of existing works to the new genre, but also composing music, with new sound effects, that cannot be produced on traditional instruments and which catches the attention of composers, musicians and audiences.

Prior to 1800, the physical arrangement of the vibrating length of piano strings copied that of the harpsichord. One end of the vibrating length was determined by an 11^o bend through a horizontal zig-zag clamp fixed to the bridge on the soundboard. The other end was determined by an 11^o bend around a fixed pin located between the Capo de Astra bar and the tuning pin. This primitive arrangement was effective for the relatively gentle plucking motion of the harpsichord, but was unable to retain the strings in the proper position under the more powerful blows of the piano's hammer action.

During the first decade of the 1800s, the famous piano and harp maker Erard of Paris invented a brass stud (called an *agraffe* (staple)) with holes drilled through it to accommodate the triple, double or single strings of the tricords, bicords and the monocords. A thread on the base of the *agraffe* allowed it to be rigidly attached to the piano frame. By firmly resisting the upward motion of the hammer blow, the agraffe was an immediate success and central to the subsequent popularity of the piano. It allowed the vibrations of the speaking length to radiate a clearer tone and a fuller sound with reduced impact noise.

During the 19th and early 20th centuries, various attempts were made to duplicate this more rigid clamping system on the bridge. Expensive, but cumbersome, vertical zig-zag systems were found to give superior results. However, the cheaper and simpler historic system retained its dominance. The Australian piano maker Wayne Stuart commenced his search for a less expensive and more functional solution in the mid-1970s which resulted in his vertical zig-zag innovation of the late 1980s. (This is a excellent example of how long the innovative step that solves the specifics of a problem can lag behind the good idea on which it is based.) The new (grand) pianos that utilize the Stuart clamping are manufactured by Piano Australia Pty Ltd in Newcastle with the brand name Stuart & Sons. As a direct consequence of the clamping, these pianos have extraordinary clarity of tone, increased sustain and lower inharmonicity, when compared with traditional grand pianos. Interestingly, for a rigorous explanation of the difference between horizontal and vertical zig-zag clamping, one must turn to an analysis of the non-linear vibrating string equation.

The difference in layout of the strings in traditional grand pianos and the Stuart & Sons' pianos is illustrated in Figure 1. The side views are essentially identical. It is the plan views that are different, though the difference is minor and relates to whether the strings are horizontally or vertically zig-zag clamped to the bridge on the soundboard.

Any modelling of the difference between horizontal and vertical zig-zag clamping will remain unfocussed until appropriate data are identified. Such data have been published by Weinreich [17], [18] and give direct proof that, in pianos with horizontal zig-zag clamping, the strings start vibrating vertically, but gradually develop elliptical polarization to eventually vibrate parallel to the face of the soundboard. Though the Weinreich experiments [17] have not been repeated for vertical zig-zag clamping, the circumstantial evidence indicates that, once the strings start vibrating vertically, they continue to vibrate vertically.

To explain the vibrational difference between horizontal and vertical zig-zag clamping, one must examine the difference between the harmonicities that such vibrating strings radiate. The linear wave equation is inappropriate as an explanatory model because it predicts perfect harmonicity independent of the type of clamping. This happens because, among other things, the linear wave equation does not model the changing tension along the string as it vibrates.

As in the solution of many mathematical problems, one must go into a more general framework (the non-linear wave equation) to find a solution before an adequate explanation can be proposed [1]. This extended framework not only involves mathematical modelling but also the psychoacoustics of music and, in particular, the concept of *virtual pitch* [15, section 2.3.2.].

Wheat-flour dough rheology

In the 1930s, the invention of recording mixers placed cereal science on a firm scientific footing: the resulting pen recordings of wheat-flour dough mixing contained sufficient qualitative information to allow end-product performance of different wheat-flours to be consistently assessed. In part, this was because the pen recordings clearly captured the features associated with the protein content of different wheat-flours.

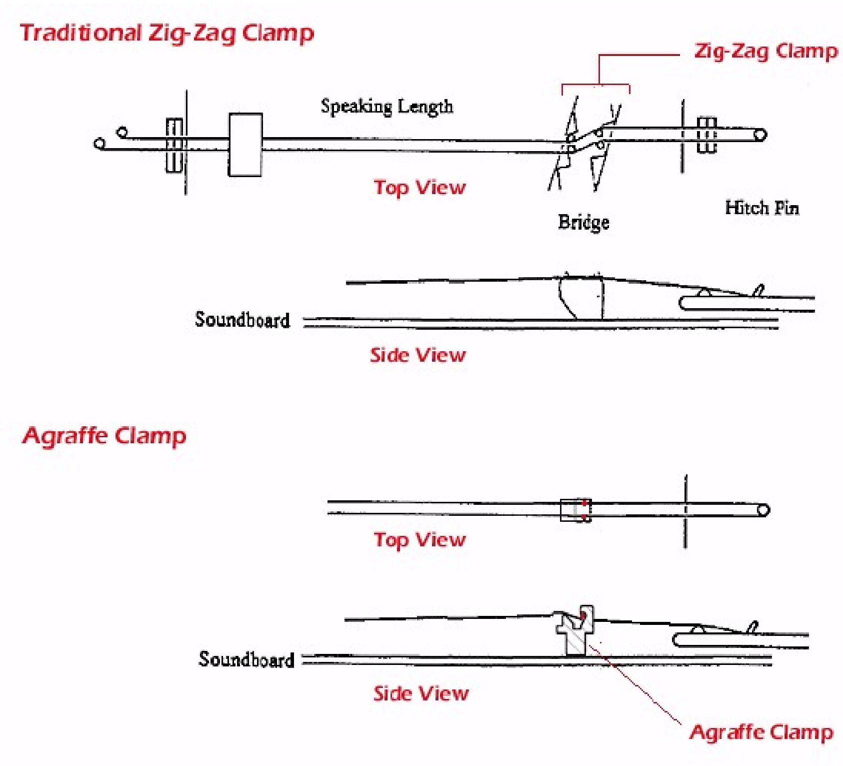

Figure 1. Layout of the strings in the traditional grand pianos and the Stuart & Sons' pianos.

Early electronic recording mixers aimed to emulate the traditional pen-recording instruments and used a sampling rate of 10 Hertz [10]. Subsequently, using a much higher data rate of 250 Hertz, it was possible to record and compute the changing stress-strain patterns of the individual elongate-rupture-relax events as they occurred during the mixing of a dough.

In order to "see and characterize" what is happening at the molecular level within a dough during the mixing, it is necessary to analyse the resulting stress-strain patterns. This has involved mathematical modelling from a number of independent perspectives including: hysteretic modelling of the elongation-rupture-relaxation events [2], [3]; viscoelasticity and the Boltzmann causal integral equation [11] and [5]; modelling the fading memory of the relaxation modulus in the Boltzmann model of linear viscoelasticity [4], [6]. Interestingly, in rheological and physical contexts, the "fading memory" of the relaxation modulus $G(t)$ is often modelled as a completely monotone function of the form

$$G(t) = \int_0^\infty \exp(-t/\tau)\frac{H(\tau)}{\tau}d\tau.$$

To date, however, the existence has been ignored of alternative representations of the form

$$G(t) = \int_0^\infty \exp(-\theta(t)/\tau)\frac{H(\tau)}{\tau}d\tau,$$

where $\theta(t)$ is such that $\theta(0) = 0$ and its derivative is completely monotone. The importance of this generalization for the analysis of viscoelastic processes, at least theoretically, is that it allows for different types of fading memory for the same molecular configurations, as well as other possibilities [4], [6].

Both theoretically and scientifically, the most important special case for $\theta(t)$ is t^β, $0 < \beta < 1$, which is the generator for the Kohlrausch (stretched exponential) function [13] and [5].

Pattern formation in plants

The future importance of mathematics and statistics will depend more and more heavily on how they contribute to the solution of the problems of science, technology and industry. The traditional basic research of mathematics and statistics will continue to have a significant impact, but the funding of such research will rely increasingly on the visibility that mathematics and statistics establish through their support of applications.

In a way, this is the start of a new era for mathematics. Mathematical knowledge has reached the stage where there is a plethora of structures, properties and techniques from which to choose when formulating mathematical models for real-world processes. However, in terms of von Bekesy's third rule, this is best done jointly by mathematicians and statisticians, in collaboration with the scientists, engineers and/or industrialists who conceptualize the initial framework within which the need for mathematical and statistical expertise arises. In this way, applications become a driving force for new mathematics.

In terms of biological research, some examples are:

(i) Biological Model Systems.

Biologists, like mathematicians, use the structure and properties of simpler model systems (organisms) to understand the the properties (genetics) of more complex organisms. Because the simpler model organisms, such as *Arabidopsis*, have fully sequenced genomes and can be bred from seed to seed a number of times throughout the year, it is much easier to determine basic genomic and genetic properties and pathways. Among other things, the goal is to establish rules for the migration of information through different levels of organisms such as

$$\textit{Arabidopsis} \rightarrow \text{Rice} \rightarrow \text{Wheat}.$$

(ii) Pattern Formation in Biology.

The partial differential equation models for morphogenesis, formulated and analysed by Turing [16] and others (cf. [12]), have played a key historical role in the study of macroscopic patterning in plants. In such situations, the link to the genetics is through the coefficients in such equations, which thereby limits interpretations to macroscopic inhibition and activation. On the other hand, observational data often fits the positional information concept of Wolpert (cf. [12]). However, genetic experiments have reached the stage where some of the individual genes that control the development are known specifically. From this molecular perspective, appropriate mathematical models have yet to be formulated. The possibility that the genetic control is through some switching signalling needs to be explored. Even though the key hormones are know in some context, such as the role of auxin in phyllotaxis, the genetics of the basic geometry remains open [9].

(iii) Bioinformatics and Microarrays.

Though the popular view of the analysis of microarray data is one of statistics, the role of the hybridization in determining the colour of the spots can not always be ignored [7].

Problem ownership

At the 8th International Conference on Plant Growth Substances, held in Canberra recently, one of the world experts on the genetic control of phyllotaxis, Professor Cris Kuklemeier, when discussion mathematical modelling, commented:

"The most talented and impressive people in the world are mathematicians. However, be careful that they do not snow you. They love to formulate models and normally do an excellent job. However, be aware, situations occur where the mathematician is more interested in the mathematics of the modelling than its relevance to the problem under examination or the available data."

In my view, when mathematicians/statisticians are not sympathetic to, and respectful of, the person who brought the problem to them, they weaken the excellent image of mathematics and statistics. Great difficulty can and often does arise when the mathematician/statistician tries to take over the ownership from the original formulator. The original owner should be treated diplomatically, as an equal, using the Hardy-Littlewood rules as the guide.

A thankyou to George!

The three letter acronym for the George Szekeres medal is GSM. Among others, Rosemount makes a great wine called GSM. As my thanks to George, I tried to buy him a bottle of GSM 2002 to mark the year when the George Szekeres medal was first awarded. Only Rosemount GSM 2000 was on sale. But, it has won a medal, and, therefore, is quite appropriate to be my thanks.

Acknowledgements

I do greatly appreciate the support that I have received, in various ways over the years, from the Australian mathematics community and various overseas mathematical colleagues–even the colleagues who made matters a little more challenging than expected. In one way or another they gave me growth. I have many great colleagues and friends who I value greatly and to whom I owe a grateful and appreciative thanks. As much as I would like to mention individual names, it would clearly be inappropriate. They know who they are, and no more needs to be said.

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CSIRO Mathematical and Information Sciences, GPO Box 664, Canberra, ACT 2601
E-mail: Bob.Anderssen@csiro.au