
Book reviews



Handbook of the geometry of Banach spaces Vol. 2

Edited by W. B. Johnson
and J. Lindenstrauss
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There is much merit in limiting one's projects to a scale at which they might actually be completed. Mathematics, and Banach space theory in particular, has any number of multi-volume publications where the authors and/or the publishers ran out of steam before providing the details enticingly promised in Volume 1. In the case of Banach space theory, such problems were often intimately connected to the perceived lack of momentum in the subject in the 1970s and 1980s. Fortunately (at least for those of us who work in this area), the preparation of obituaries was a little premature and much deep and interesting research continues to be done.

I am pleased to say that the *Handbook of the Geometry of Banach Spaces* project has not at all suffered from any loss of momentum. The editors, Bill Johnson and Joram Lindenstrauss have continued the steady hand they applied to the first volume to this second and final part of the *Handbook*. (Volume 1 was reviewed in *Aust. Math. Soc. Gazette* **30** (2003), 168–169.)

The style of the *Handbook* is to provide carefully written articles, each devoted to an important and current topic in Banach space research. Each essay is written by between one and three of the top experts in the field. Almost without exception it

would be hard to find a better choice of authors for each topic than the ones provided. Some evidence for the vitality of this field is that two of the authors in the *Handbook*, Jean Bourgain (1994) and Tim Gowers (1998), were awarded Fields Medals, at least in part for their Banach space research. Volume 2 contains 19 essays, bringing the total over the two volumes to 41. The split between volumes is essentially along alphabetical lines. Volume 1 contains first authors Abramovitch to König; Volume 2 contains Ledoux to Zizler, as well as a few articles that missed the Volume 1 deadline.

As the editors admit, there are a number of topics that might have warranted their own essay, but which were not included. Four of these are listed in the preface, and it wouldn't be too challenging to think up a couple more. Looking at the final list of 41 essays however, there is ample justification for their claim that the *Handbook* contains a 'reasonably comprehensive and accessible view of the present state of the subject'.

Indeed the *Handbook* is refreshingly democratic. Starting from the shared basic concepts which are covered in the introductory essay in Volume 1, the essays can be tackled in any order and so the reader is left to make their own choice as to what is interesting or important. The typical essay is around 50 pages, which provides enough room to give some details, without asking for an excessive investment of time from the reader in order to get somewhere interesting.

The essays themselves sit in varying points on the scale between classical and cutting edge. The material in Maurey's essay 'Type, cotype and K -convexity',

for example, generally contains mathematics developed in the 60s, 70s and 80s. Others, such as Mankiewicz and Tomczak-Jaegermann's 'Quotients of finite-dimensional Banach spaces; Random phenomena', contains extra sections added just before publication covering the latest (September 2002) developments. Volume 2 also contains four short addenda/corrigenda updating essays from Volume 1.

Even for the more classical topics, such as interpolation theory, the essays tend to lean away from covering material for which there are excellent alternative sources, and concentrate on the newer material, or material which is more central to Banach space practitioners. On the other hand, many of the essays contain excellent expositions of material that is well-understood by experts, but for which there is few accessible sources for the beginner. Pisier and Xu's article 'Non-commutative L^p -spaces' is an example here.

Throughout, there is a common theme of trying to get the reader up to the point where they can see what is happening currently at the research front. Open problems, as well as helpful heuristic explanations, occur frequently in many of the essays. Each article contains a substantial list of references. This is backed up by comprehensive author and subject indexes which cover both volumes. The author index alone runs to 24 pages.

The emphasis on recent research means that it is inevitable that some of the essays will end up ageing better than others. Nevertheless, it is clear that these two volumes will become a standard and important reference for both graduate students and more experienced Banach space researchers. The *Handbook* is a must for any serious mathematics library and highly recommended for researchers in this and neighbouring fields.

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Combinatorics and Graph Theory

John M. Harris, Jeffrey L. Hirst
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Undergraduate Texts in Mathematics

Springer New York 2000

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There are quite a few textbooks on combinatorics, or graph theory, or both. This book attempts to give a grounding in three areas: graph theory, enumerative combinatorics and infinite set theory. In a sense the book is like a sampler, giving the reader a taste of each topic and listing references for further reading. The book is aimed mainly at third/fourth year undergraduate students, but the authors note that graduate students looking for an introduction to these topics may find the book helpful.

Chapter 1, "Graph theory", covers trees, planarity, colourings, matchings and Ramsey theory, and contains a discussion of the four-colour problem. Chapter 2, "Combinatorics", looks at inclusion-exclusion, ordinary generating functions, Pólya's theory of counting, and a couple of other topics. Chapter 3, entitled "Infinite combinatorics and graphs", covers infinite pigeonhole theorems, the ZFC axioms of set theory, ordinals and cardinals, and other topics including a discussion of Gödel's incompleteness theorem.

The authors motivate their topics with concrete applications or examples. They also strive to "enliven" the book with jokes and quotations. Depending on your taste, this may be entertaining or distracting. There are exercises at the end of every section, and each chapter concludes with some references for further reading. I was surprised that some well-known graph theory textbooks (such as Diestel's "Graph Theory") were not mentioned in the reference list of Chapter 1, while the reference list of Chapter 2 is not very user-friendly since references are only cited by number (such as [122]) without giving the authors' names. I felt that the material in Chapter 3 was more difficult than the rest of the book, though

this may just be because I am much less familiar with it than the material of the first two chapters.

By treating three areas in one book, the authors are necessarily unable to cover these areas in as much depth as someone writing an entire textbook on one of the areas. But there are a lot of interesting and useful results gathered together in this book, and I believe that it would make a good introductory text for anyone who wanted to know a little about any of these three topics.

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Lie Groups, Lie Algebras and Representations: An Elementary Introduction

Brian C. Hall

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ISBN 0-387-40122-9

This book differs from most of the texts on Lie Groups in one significant aspect. Instead of dealing with Lie groups defined as differentiable manifolds with a group structure, it develops the whole theory on matrix Lie groups. This approach results in a less abstract approach and will be appreciated by those who find differential geometry difficult to understand. I have taught for many years an honours course on differential geometry which included a basic introduction to Lie groups and from my experience I can testify that many students grasped the fundamental concepts only after working through examples which usually involved matrix Lie groups. Thus there seems to be some merit in such an approach. A matrix Lie group is defined as a closed subgroup of the complex general linear group and immediately after the definition the author systematically proceeds through all classical examples :

special linear, orthogonal, special orthogonal, unitary, special unitary, generalized orthogonal, Lorentz, and Heisenberg groups. Physics students will appreciate a detailed discussion of the relationship between $SU(2)$ and $SO(3)$. Even though concentrating on matrix groups makes the development of the theory more accessible, mathematical rigour is not compromised. This book is no Lie groups for pedestrians, it is a rigorous mathematical text. Even in the main text an example of a Lie group which is not a matrix Lie group is mentioned, while Appendix C gives a precise brief introduction to the manifold approach. The exponential map is defined using the standard exponential of a matrix, but the connection between Lie algebras and Lie groups is again studied in detail, including examples where the map is not onto. The abstract definition of a Lie algebra is also given and the connection between Lie group homomorphisms and Lie algebra homomorphisms is discussed thoroughly. After about one third of the main text the author begins the presentation of the representation theory. The exposition is heavily based on examples. There is a whole chapter on representations of $SU(3)$, again very useful for students of Physics. The second half of the book is all about semisimple Lie algebras and Lie groups leading the reader through roots and weights and Dynkin diagrams to the classification of semisimple Lie algebras.

About fifty pages of the book are taken by five appendices. Appendix A is a short review of the theory of groups, appendix B is a review of linear algebra, the above mentioned appendix C describes the standard approach to Lie groups including the Lie algebra of left-invariant vector fields. Appendix D is again useful to physicists as it explains Clebsch-Gordan theory and the Wigner-Eckart theorem, while the last appendix discusses fundamental groups and gives even a definition of a fibre bundle. Readers who study both the main text as well as the appendices will get a substantial introduction to Lie groups as well as a brief one to differential geometry.

Finally, one should mention that each of the eight chapters plus appendix A contain a good collection of exercises. Solutions are not included, but questions are well structured and often include hints. As the exercises often include additional examples, they should be attempted by every serious reader.

I believe that the book fills the gap between the numerous popular books on Lie groups on one side and highly abstract texts on the other side and as such it is a valuable addition to the collection of any mathematician or physicist interested in the subject.

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**Sir James Cockle:
First Chief Justice of Queensland
1863-1879**

J. M. Bennett

Federation Press Annandale 2003

ISBN 1-86287-485-9

This book is the seventh and most recent in the series *Lives of the Australian Chief Justices*. As this information might indicate, its focus is legal rather than mathematical, although its subject achieved eminence and recognition in both fields. [He was the first Chief Justice of the (then) Colony of Queensland, and was knighted in consequence; he was also elected to a fellowship of the Royal Society and was for two years president of the London Mathematical Society.]

The author of this latest and most extensive biography is an eminent authority on the law, and here he is concerned to rebut the judgement that Cockle made little contribution in that area. This certainly is the clear message of earlier biographies [1], [2], which imply that his mathematical contributions overshadowed his legal legacy. My

own assessment [3] tends to take the side of those who hold the opposite view.

Although the author expresses some reservations about my brief article [3], he and I are not in real disagreement on the point. Rather, the Mathematics is mentioned, but rarely described in any detail. Indeed, there is an active disclaimer: “This is not the place to attempt any informed assessment of Cockle’s mathematical achievements.” (p 116). The description of Cockle’s involvement with Mathematics occupies only some seven pages (3-7, 115-116) of the total text of 117, although perhaps we might add to these a few *obiter dicta* found scattered through the rest of the text.

Even so, what assessment of Cockle’s Mathematics is offered is somewhat ingenuous, and would seem to be based on the public recognition it received, rather than on any detailed analysis. There are even some strange judgements. To describe George Birch Jerrard as a “lesser luminary [than Cockle]” (p 6) is a notable instance. [Jerrard has an entry in the Dictionary of Scientific Biography; Cockle does not. This reflects the contemporary assessment of the relative contributions of the two men.] It is also most unlikely that the current eclipse of Cockle’s mathematical reputation is due to the “cynicism bred of the computer age”.

The Foreword (by Mr Justice B. H. McPherson, Judge of Appeal in Queensland) supplies, *inter alia*, a pithy and accurate summary of the work. The opinion there given is that Cockle merits a biography in the series on account of “the strength of the man’s personal character and his achievement sustained against powerful odds”.

The background is related in my earlier study [3], but here the matter is dealt with in much greater detail. Cockle was appointed Chief Justice of Queensland over the head of the only resident judge in the new colony, Alfred Lutwyche, who had fallen out with the government of the day. Cockle not only curbed Lutwyche’s inappropriate political aspirations, but formed

a firm friendship with his brother judge as well.

Cockle's lack of influence on the subsequent course of the law is partially explained on p 60. " 'We must be careful' he said in a typical instance, 'that we do not, in deciding this case, decide any collateral matter'. Because of that narrowness [and also for other reasons], it is difficult to find many judgements of his authorship that still stand as leading authorities."

The impression given is one of strictly literal interpretation, and indeed Bennett makes this point explicitly (on p 90): "In interpreting statutes, the Chief Justice inclined towards literalism". This is perhaps an example of a mathematically trained mind at work, although it is not flagged as such. In other instances, there is explicit comment along such lines, taking the form of auctorial *obiter dicta*. One example will suffice. On p 78 an excerpt from an address in summation to a jury is presented. It takes the form of a combinatorial argument, and one readily accepts the comment that "he probably spoke over [the jurors'] heads".

Cockle's principal mathematical correspondent was the Reverend Robert Harley, and Bennett has located their correspondence in the Cambridge University Library. It covers the years before Cockle's residence in Brisbane, and Bennett makes little use of it, but the information may perhaps be of interest to other researchers.

References

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3. M.A.B. Deakin, *Sir James Cockle, FRS*, Aust. Math. Soc. Gazette **29** (2002), 7–12.

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Radical Theory of Rings

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In the beginning, radicals were perceived as obstacles to decent algebraic structure theorems. For example, while every linear representation of a finite group G over the complex numbers is a direct sum of irreducible representations, linear representations over a field of characteristic p have no such nice decomposition whenever G contains an element of order p . The reason is that the corresponding group ring has a nilpotent ideal that lies in every maximal ideal, and hence the group ring itself does not have a nice decomposition as a direct sum of simple ideals.

During the first half of the 20-th Century, when rings began to be studied as mathematical structures in their own right, radicals were recognized as important structural components of a ring. For example, Wedderburn in 1908 and Köthe in 1930 recognized that a ring A might contain a 'bad' ideal $r(A)$ such that the factor ring $A/r(A)$ is 'good' in the sense that it has an essentially unique decomposition into indecomposable components. The 'bad' ideal studied by Köthe was the *nil radical*, the maximal ideal containing only nilpotent elements. In the 1940's and 1950's, other radicals with similar properties were recognized, named after their discoverers such as Baer, Brown and McCoy, Divinsky, Jacobson and Levitzky. The main problem in Ring Theory during this period was to describe the inclusion relations among the various flavours of radicals for various classes of rings.

A third stage of progress began in the 1950's when Amitsur and Kurosh independently showed that the classical radicals shared certain ring theoretic properties and used these properties to axiomatise the notion of *radical class of rings*. For example, a Kurosh–Amitsur radical class is a class of rings closed under homomorphic images,

unions of ascending chains and extensions. Then for any radical class \mathcal{C} there is a corresponding *semi-simple class* \mathcal{D} such that any ring A (in a suitable class of rings) has an ideal $r(A)$ in \mathcal{C} with factor ring $A/r(A)$ in \mathcal{D} . Dually, one can define *semi-simple class* to be a class of rings \mathcal{D} such that a ring A is in \mathcal{D} if and only if every non-zero ideal of A has a non-zero factor ring in \mathcal{D} . Then there exists a corresponding radical class \mathcal{C} such that any ring A (in a suitable class of rings) has an ideal $r(A)$ in \mathcal{C} with factor ring $A/r(A)$ in \mathcal{D} . These definitions, and variants introduced by mathematicians such as Hoehnke and Plotkin, included not only the classical radicals mentioned above, but also such surprises as von Neumann regular rings.

The fourth stage, which began in the 1960's and shows no sign of senescence, was to apply the abstract definitions of radical and semi-simple classes to more general algebraic objects than associative rings, for example universal algebras and categories. This development is particularly associated with the names of Andrunakievich and Ryabukhin. As an example, in 1966 Dickson defined a *torsion theory* in an abelian category \mathcal{A} to be a pair (T, F) of disjoint classes of objects for which T is closed under cokernels, F is closed under kernels and every object A has a kernel $r(A)$ in T with cokernel $A/r(A)$ in F .

The book under review tells the story, and tells it well. Both authors have connections to Australia, Gardner at the University of Tasmania and Weigandt, from Budapest, as a frequent visitor to these shores. Both played important rôles in the fourth stage described above. Before launching into a description of the book itself, it is appropriate to mention its predecessors. The first monograph on the general theory of radicals was 'Rings and Radicals' by N J Divinsky, University of Toronto Press, 1964. The next was 'Radicals of Rings' by F A Szasz, Wiley, 1981 and then came 'Rings, Modules and Preradicals' by L Bican, T Kepka and P Nemeč, Volume

75 in the Dekker Series Lecture Notes in Pure and Applied Algebra, 1982. The last such predecessor was 'Radical Theory' by Gardner himself, Longman, 1989. There has been one undergraduate text on the subject, 'A Radical Approach to Algebra', by Mary Gray, Addison-Wesley, 1970. The present work subsumes all of these, as well as much of the research which has appeared in journal articles and Conference Proceedings during the past 25 years.

Now for a detailed analysis of the Contents: after a brief introductory chapter introducing the notation and the major properties and examples of rings, the authors present in Chapter 2 the general theory of radicals following the method of Amitsur and Kurosh. In Chapter 3, the major classical radicals, which the authors refer to as concrete radicals, are introduced. Chapter 4 is a comprehensive study of concrete radicals and the resulting structure theories of various classes of rings. Finally, Chapter 5 concerns the basic features of radical theory in more general categories, including nonassociative rings, rings with involution and near-rings.

The book as a whole is well-designed in the usual Dekker-LaTeX format and is free of obtrusive typographical errors. The index is reasonably complete. Occasionally the list of references fails to follow standard mathematical practice of listing books and papers by the same authors in chronological order of publication, but this hardly detracts from its usefulness.

To summarise, this is an essential book for the specialist in Radical Theory, a useful handbook for algebraists and students of ring theory, and a book which should be in the library of every Institution which regards itself as a centre for the study of Algebra.

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