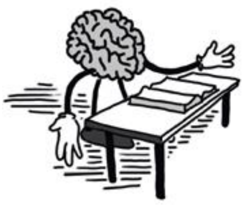
Book reviews



Analysis for Applied Mathematics

Ward Cheney
Graduate Texts in Mathematics **208**
Springer New York 2001
ISBN 0-387-95279-9

Writing a mathematics book imposes a number of constraints on an author, arising primarily out requirements of sequential development and caveats about points of detail. It not infrequently happens as a result that a mathematically careful book is scarcely readable: rigour has become uncomfortably like *rigor mortis*.

Ward Cheney's book provides a refreshing reminder that things don't have to be this way. This is a top–down book. Effort is made not just to be correct but to elucidate. Attention is given to motivation and to the relations between topics. Overview is not lost despite the presence of considerable detail. In the interests of overview, the author has not hesitated in some proofs to make forward references to material coming later in the book. Sometimes, as with the converse part of the Eberlein–Smulyan Theorem, the reader is referred to another text. The book enjoys a clear visual layout and has excellent references and index. There is a rich supply of exercises. The style of presentation is pleasant.

There are eight chapters as follows.

Chapter 1 (normed linear spaces) includes convexity, convergence, compactness, completeness, Zorn's Lemma, Hamel bases, the Hahn–Banach, Baire category, interior mapping and closed–graph theorems, weak convergence and reflexive spaces.

Chapter 2 (Hilbert spaces) includes orthogonality, orthonormal bases, adjoints of bounded linear operators, spectral theory and Sturm–Liouville theory.

Chapter 3, on calculus in Banach spaces, includes Fréchet and Gâteaux derivatives, the chain rule and mean–value theorems, the Kantorovich theorem on Newton's method, implicit function theorems and the basis of the calculus of variations.

Chapter 4 deals with approximate methods of analysis, and treats methods for solving operator equations, including iteration, Neumann series, projection and methods based on homotopy and continuation.

Chapter 5, on distributions, includes convergence, convolutions, applications to differential operators and distributions with compact support.

Chapter 6 concerns the Fourier transform. The Schwartz space is introduced and inversion theorems, the Plancherel theorem, tempered distributions and Sobolev spaces treated.

Chapter 7 (additional topics) includes fixed–point theorems, selection and separation theorems, the Arzelà–Ascoli theorems, compact operators and Fredholm theory, topological and linear topological spaces, and analytic pitfalls.

The concluding Chapter 8 (measure and integration) addresses measurable functions, Egorov's theorem, the monotone convergence theorem, Fatou's lemma, the dominated convergence theorem, the Radon–Nikodym theorem and Fubini's theorem.

The author states in the preface that the book evolved from a course for beginning graduate students. In view of the differences between graduate training programmes in Australia and the United States, the book will have a smaller than proportional natural audience here. While it is user–friendly, I believe most graduate students would find it heavy going without the intercession of a teacher. The graduate student will find some of the exercises fairly challenging without additional reading.

The coverage is greater than many graduate students or staff in applied mathematics in either Australia or the U.S.A would find necessary, though it is very well suited for the more theoretical end of the spectrum in applied probability, optimisation and control theory. It is clear from the list of contents above that there is ample material for several substantial courses.

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Computer Algebra and Symbolic Computation: Elementary Algorithms

Joel S. Cohen
A.K. Peters Natick 2002
ISBN 1-56881-158-6

Like most mathematicians these days I routinely use a Computer Algebra System (CAS) as part of my daily mathematical life. I am not, however, a power user of such systems and I have relatively little knowledge of how this sort of software actually works. When I was asked to review this book I jumped at the chance, hoping to gain some insight into the inner workings these incredible tools. Thus this review is presented from the perspective of an interested end-user rather than an expert in the field of CAS's.

This book is the first of two books written by the author on Computer Algebra and Symbolic Computation. The second book, entitled *Computer Algebra and Symbolic Computation: Mathematical Methods* unfortunately wasn't available to me at the time of writing this review. According to the author, the first book "is concerned with the algorithmic formulation of solutions to elementary symbolic mathematical problems" while the second book "is an introduction to the mathematical techniques and algorithmic methods of computer algebra". Thus this first book describes how mathematical expressions are handled by a CAS and shows how some elementary operations from algebra (e.g. expanding and factorizing polynomials), trigonometry (e.g. expanding trigonometric functions), calculus (e.g. integration) and differential equations (e.g. solving first order d.e.'s) can be implemented. Cohen claims that the material from this book could be used at an undergraduate level. He also claims that much of the material of this book is not available elsewhere. The second book goes on to discuss, in terms of efficiency and effectiveness, more advanced algorithms for many other mathematical problems. According to Cohen, the material covered in the second book is more difficult and requires a greater mathematical sophistication. The author describes his overall target audience by saying that "these books serve as a bridge between texts and manuals that show how to use a CAS and graduate level texts that describe algorithms at the forefront of the field".

As mentioned above my only experience with CAS's is as an end-user. Despite this, I found this book very easy to read. The writing is clear, there are plenty of examples and the material is developed at a steady pace. All of the concepts and examples used in the book are discussed with reference to the CAS's Maple, Mathematica and MuPad and the text and exercises encourage the reader to verify and experiment using the CAS of their choice. Each section of each chapter includes a selection of exercises but no answers are provided. Each chapter concludes with a "Further Reading" section for those wanting more detail about the topics covered. The overall tone of the book is very much suited to an undergraduate target audience. The book comes with a CD on which implementations of many of the examples and algorithms are given for each of CAS's Maple, Mathematica and MuPad. My institution has standardized on Maple and so I only accessed the Maple files from the CD which I had no trouble doing. Since all of these files worked fine I presume that the same would also hold true for the Mathematica and MuPad files. The CD also contains an electronic version of the text but I did not access this.

While I cannot comment on Cohen's claim that much of the material of this book won't be found elsewhere I can say that I found the content of this book very interesting. I was surprised, for example, to find that the way mathematical expressions are handled within a CAS is based on the tree structures we teach in our first year discrete mathematics course. I was intrigued to find that the idea of recursion plays a central role in the algorithmic structures in a CAS and of course I was curious to see how the algorithms actually work. As a result of reading this book I am not only much more knowledgeable about the way a CAS works but I am also a much more knowledgeable user of my own CAS.

Perhaps the best way to indicate what I think of this book is that, as a result of reading it, I am thinking of including a topic on CAS's in one of my undergraduate courses next year and I will be using Cohen as the reference. Thoroughly recommended.

Table of Contents: Introduction to Computer Algebra, Elementary Concepts of Computer Algebra, Recursive Structure of Mathematical Expressions, Elementary Mathematical Algorithms, Recursive Algorithms, Structure of Polynomials and Rational Expressions, Exponential and Trigonometric Transformations.

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Numerical Methods for Ordinary Differential Equations

J.C. Butcher
Wiley Europe 2003
ISBN 0-471-96758-0

This book should be in the working collection of every serious numerical analyst. It is concerned with the analysis of discretization methods, principally Runge-Kutta methods, applied to the solution of ordinary differential equations. At the top level it is a particular example of a generic approach to computational algorithms characterized by a sequence of local approximations or discretizations to a target system with the local approximations being glued together in an overall solution process. It has a general importance because this class of algorithms is important. A key factor in the attainable accuracy of such computations is the quality of these local approximations. Typically this is calculated by formally substituting the exact solution into the local approximation and using Taylor series expansions to evaluate the defect. This approach can be constructive because parameters in the approximation can be selected to control the number of terms in the resulting expansion that vanish (the order of consistency). Frequently the expansions involve Taylor series evaluations of functions of functions and the resulting algebraic calculations rapidly become complex even with the assistance of symbol manipulation programs. Here the main application is the development of Runge-Kutta single step integration methods for solving the initial value problems of first order systems of ordinary differential equations. It follows that the glue is a simple recursion in this case, and suitability comes down to the question of stability. Runge-Kutta methods proved popular in early computer implementations when the Runge-Kutta-Gill formula implemented on EDSAC1 was shown to make minimal demands on the limited computer memory available. However, they tended to fall out of favour as computer architectures became less restrictive because the general opinion tended to regard them as expensive in the sense of taking more function evaluations per step than the rival linear multistep methods superbly analysed in the 1956 thesis of Germund Dahlquist. It took years of patient endeavour by a number of somewhat isolated enthusiasts to show that this was not necessarily the case. It is generally agreed that John Butcher has been the main protaginist whose seminal contributions have led to this re-evaluation. His approach has been to associate rooted trees with the partial Taylor expansions, to identify equivalence classes of formulae with mappings from rooted trees to the reals, and to deduce the properties of these classes from the structure of groups of such transformations. Within this framework, questions of the order of consistency of local approximations become algebraic questions capable of general answers, and information about the suitability of the glue can be derived by adding further algebraic restrictions. The point not to be missed is that questions relating to the development and implementation of a class of numerical algorithms have led to a substantial, profound, and highly original essentially algebraic investigation.

The subject matter of this book is a reworking of Butcher's 1987 text "Numerical Analysis of Ordinary Differential Equations" which provided the definitive account of his algebraic approach. The problem domain considered is exclusively the initial value problem for systems of ordinary differential equations, and the examples regarded as difficult are the so called stiff equations which are stable initial value problems supporting solutions with widely differing time scales. Here a stiffly stable formula is one which permits the computation of slowly varying solutions on coarse grids adapted to these rather than requiring the fine grids needed to follow the rapidly decaying solutions. The problem of super stable formulae, which have the unfortunate property of mapping increasing solutions to (generally) slowly decaying ones, is mentioned obliquely in a brief discussion of the Van der Pol equation. This narrow focus provides the basis for the minor criticism that this is not the whole field that the book title pretends to address. Reference is made in the two introductory chapters to recent application areas which are major growing points of the field. These include methods for the integration of differential equations on manifolds, an area where the rooted tree technology has had a significant application in the hands of Iserles and Norsett, and special methods for Hamiltonian and related problems which include major contributions by Australasian researchers McLachlan and Quispel. However, these are not followed beyond initial references. No mention is made of boundary value problems where the stability properties are distinctly more complex - so the quality of the glue becomes a deeper question, nor of collocation methods, a subclass of Runge-Kutta methods, which are typically the methods of choice in the boundary value context because of distinctive structural features which set them apart from the general class. Thus, as would be expected, the algebraic theory of Runge-Kutta methods dominates, the dedicated third chapter occupying over 170 pages of the text. The tools developed are shown to have significant utility in the analysis of linear multistep methods in an insiteful penultimate chapter, and a final chapter treats a unifying class of methods under the heading of general linear methods. There is no real conclusion. As others have shown, the Butcher approach has led to significant advances also in neighbouring fields and this is an important point to bear in mind when assessing this material. Certainly the objections of nearly fifty years ago have long been put aside. However, I suspect that implementors of major ODE packages now put more of an emphasis on broadening applications scope rather than being overly concerned with incremental improvements in integration formulae in the classical subject area - even if these come under the catchy heading DIMSIM. This is beside the point for the serious numerical analyst. This book provides an account of a major mathematical investigation and should be appreciated as such. Because the approach is open ended he needs this book.

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Essential Mathematical Biology

N. Britton
SUMS series
Springer Heidelberg 2003
ISBN 1-85233-536-X

It was a great pleasure reading Essential Mathematical Biology. The introduction implies that the book was written primarily as a text for a two semester third year mathematics or joint mathematics-biology students. The book formally requires no more mathematical background than would be available to students who have covered a standard second year applied maths course in Australia. However, while the book is very well written without large jumps in the mathematical reasoning, it is also quite concise and covers a large amount of material. My guess is that a course that tried to cover the bulk of the material in this book might be more appropriate at honours level rather than third year level, though selections from the book would be a very good basis for a one semester third year course. The writing and style are very clear. The mathematical steps are laid out neatly with clear definitions and notation and these steps are not smothered in unnecessary prose. On the other hand, there are very nice insightful discussions of the biological implications of the mathematical models and the reverse implications of the biology for the refinement of the mathematical models. The mathematical material is also interspersed with interesting snippets of biological and historical background and one or two humorous comments.

The first half of the book covers the use of ordinary difference equations and differential equations to model the population dynamics of single species and interacting species, including host-parasite systems and hence infectious disease epidemiology. Various biologically important complications are incorporated into the basic models and it can be seen how the match or mismatch between the mathematical model and the biological system enhances understanding of the system. The first part of the book also has a chapter covering population genetics and evolution. This is the only chapter of the book for which I could say that there is a clearer exposition elsewhere (JC Frauenthals Mathematical Epidemiology) though this latter book is much less ambitious in its coverage and is written for students at a slightly lower level of mathematics ability).

The second half of the book covers situations where models based on partial differential equations, particularly those for diffusion, may be relevant. Topics covered include biological motion, tumour modelling and the mathematics behind the initial differentiation of embryos from uniform blobs to the beginnings of complex animals. The appendices cover a range of topics. There is material on the stability of fixed point solutions of difference equations including discussion of Hopf bifurcation theory. There is also material on methods for the solution of differential equations. Finally, there are reasonably detailed solutions to the many problems set through the book.

The only fault one could pick with the book is that in a few places, there needs to be more reference to material in the appendix. Indeed some of the material in the appendix would perhaps be better incorporated into the main text. There appear to be very few errors, though in one place a blood cell count is misstated by a factor of 1000. There is apparently more material at an associated website – but on the only occasion I tried, I was not able to access this site.

I have the impression that some mathematicians used to think of mathematical biology as a peripheral and easy area of applied mathematics that is of little use. Likewise biological scientists have in the past seen little use for mathematical modelling. Anything can be modelled, but the model may have little predictive ability and may not lead to any new insights. I recall a quote whose source I cant remember: "Give me 6 parameters and I'll draw you an elephant, a 7th and it will wag its tail". This book is yet another testament to the fact that these adverse attitudes are now entirely inappropriate. The book is a great contribution to students interested in mathematical biology – an area rich in intellectual challenges for mathematicians and a source of important insights for biological scientists.

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Elements of Number Theory

John Stillwell
Springer Heidelberg 2003
ISBN 0-387-95587-9

Stillwell has taken very careful aim at his target audience and hit them squarely between the eyes. His book is written for budding number theorists in universities everywhere and the author has done a wonderful job of conveying his obvious love of the subject to the reader.

The author sticks carefully to his claim that

> *"Solving equations in integers is the central problem in number theory,..."*

and covers basic classical number theory. The reader will find no reference to the last thirty years of algebraic geometry, he even avoids mentioning the term elliptic curve, though he does make a tangential reference to them in one of the early exercises.

All the usual players are present: namely the integers, Euclid's algorithm, modular arithmetic, Pell's equation, the Gaussian integers, the four square theorem, quadratic reciprocity, rings and ideals. Alongside these one finds some ring-ins, including the RSA cryptosystem, Conway's graph theoretic approach to Pell's equation. This reader was particularly enamoured of Stillwell's decision to include Rousseau's recent Chinese Remainder Theorem-based proof of quadratic reciprocity—which in many ways helps to demystify the latter.

This leads me to mention the one minor detraction in the book, the chapter on the RSA Cryptosystem, which has the look and feel of a last minute addition. It is far smaller than any other chapter and may have once formed part of the previous one on congruence arithmetic. It is misleading in a number of fundamental ways—like suggesting that

> *"... no known method is substantially better than dividing the 200-digit number* p_1p_2 *by most of the approximately* 10^{100} *numbers less than its square root ..."*

which seems to deny the existence of the number field sieve and its sub-exponential work. In all other chapters Stillwell has gone to great pains to portray the deeper truths accurately.

Each chapter opens with a substantial preview stating the central concepts which are about to appear. Stillwell has no qualms about stating the important points more than twice—but he has the ability to do it in such a way that one sees it from many angles, for example the composition of quadratic forms contrasted with the composition of the corresponding ideals. The exercises have been particularly well crafted, forming an integral part of the text, rather than just being an ad-hoc collection of problems.

There are mercifully few errors. The minor typos that survived include mention of $Q[n]$ rather than $Q[\sqrt{n}]$ in Exercises 5.4.1 and 5.4.2, and the incorrect ideal description $2n + (1 + \sqrt{-5})n$ on page 207. An unfortunate error slipped through on the proof of cancellation of ideals on page 229 where $AB \supseteq AC$ should imply $(\alpha)B \supseteq (\alpha)C$.

Stillwell has chosen to make quadratic forms the underlying theme of the book. In particular, he notes repeatedly that mathematicians from at least Fermat onward were interested in primes of the form

$$x^2 + ny^2$$

for various integers, n. While he does not prove results in a completely general setting (say for the algebraic integers of an arbitrary number field) he does prove them in restricted cases (e.g. quadratic Euclidean domains) which ought to be sufficient to allow the more gifted students room to explore. The obvious connection to quadratic fields is made and the examples of failure of unique factorization in $Z[\sqrt{-5}]$ is resolved as is the ambiguity of primes of the form $x^2 + 5y^2$ which is the books climax.

In short, this book is a delight to read and is ideally suited to a beginning course in number theory.

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Lengths, Widths, Surfaces: a Portrait of Old Babylonian Algebra and Its Kin

Jens Høyrup
Springer Heidelberg 2002
ISBN 0-387-95303-5

The present book is a valuable addition to the literature on the mathematics of the ancient Babylonians (c 2000-c1600BCE), the subsequent Seleucid mathematics and their relations with other (later) mathematics. Drawing on studies of the culture in which the texts were developed, it contains extensive "conformal translations" of the most important texts, as well as analyses of them and interpretations of the pictures that emerge.

The author clearly declares his perspective with the book's title "Lengths, Widths, Surfaces: a Portrait of Old Babylonian Algebra and Its Kin": ie the material can be considered, in some sense, as algebra. But it is also geometric, involving lengths, widths etc.

The book is not for raw beginners. For instance it skims lightly not only over base 60 mathematical notation but also over the "standard interpretation" (the basically algebraic picture sketched by Neugebauer, Thureau-Dangin and others in the 1930s) of Babylonian mathematics. Without some familiarity with the latter and the ways in which it has been challenged in the intervening years over the intervening years it may be hard to understand this book and to see what constitutes its originality.

Høyrup supports the broad thrust of the "standard interpretation" but refines it: in his view

> "*the Old Babylonian operation with lines and areas really was* an algebra, *if this be understood as analytical procedures in which unknown quantities are* represented *by functionally abstract entities - numbers in our algebra, measurable line segments and areas in the Old Babylonian technique*" (Author's emphases).

Høyrup's translation is "conformal", ie it tries to "conserve the structure of the original, rendering always a given expression by the same English expression, rendering different expressions differently"(p41). The translation is word for word and conserves word order where at all possible. This leads to undoubtedly awkward English such as "My confrontation inside the surface I have torn out: 14'30 is it " (p52). A persevering reader will get the hang of it after a bit of practice but it is not light reading. A further strength of the book is that it draws on up to date linguistic, particularly etymological information about the languages of the texts.

I consider the absence of any surviving original diagrams indicating cut-and-paste techniques a significant weakness in the author's view that operations were considered geometrically by the Babylonians. There appears to be no hard primary evidence that such geometrical techniques were used. Not even the rough work tablets from Ur and Nippur identified by Eleanor Robson [1999] have included any diagrams. However, Høyrup presents a persuasive array of secondary evidence, for instance the etymology of the words used for various mathematical operations, such as 'tear out' for 'subtract,' or 'append' for 'add'.

An argument based only on such etymology would not be entirely convincing. For instance a future archaeologist finding, in mathematical texts from our era, references to "fields" and "extracting square roots", could not validly conclude that the twentieth century was preoccupied with growing square carrots, nor that "drawing conclusions" involved drawing instruments. One might further object that the first people to reflect on, say, addition had either to use an existing word or to coin a new one; 'append' might well have come to mind as an appropriate metaphor.

But Høyrup does rather more: he argues that all the language used is derived from physical operations and, in addition, the sequence of operations reinforces a cut and paste interpretation: the cut is made ("torn out") before the cut material is pasted ("appended") (eg p 57), even though in other contexts addition usually precedes subtraction.

Høyrup accommodates even the apparent inhomogeneity of Babylonian problems and the apparently chaotic ordering of problems in compilation texts in his geometric picture. Under the standard interpretation there are many occasions when Babylonian mathematicians appear to add sides and areas, for instance Neugebauer's Mathematische Keilschrift-Texte Vol3 p5 has "I have added the area and the side of my square" (my translation from the German). Høyrup translates this as "the surface and my confrontation I have accumulated" where he explains that, throughout the book, he uses "confrontation" to mean an extension of unit length whose width is the confronted edge. And when analysing the structure of compilation texts containing anthologies of mathematical problems, the author considers that the Babylonians based the ordering of their problems on the configurations they involved rather than, as we might, on the underlying mathematical principles.

All in all, this is an important book for those wanting to look at Babylonian mathematics in depth, but for a first introduction it would still be worth reading the relevant sections in generic History of Mathematics or a classic such as Neugebauer's The Exact Sciences in Antiquity first.

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MathMedia

"Outside the equation"

The Age, December 13, 2003, published an interview by Michelle Griffin with Robyn Arianrhod, author of the "pop maths" book *Einstein's Heroes*. The book uses the discoveries of Isaac Newton, Michael Faraday and James Clerk Maxwell to explain mathematical principles. Arianrhod "has spent years comparing Einstein's equations with the equations of electromagnetism devised by James Clerk Maxwell. And it was following the path backwards from Einstein to his heroes that Arianrhod found the form of the book. It began when Arianrhod decided she needed to find out how Maxwell arrived at his theories of electromagnetism. She started reading the copies of his correspondence kept at Monash library and became fascinated by the man, a shy but ever-curious gentleman, son of a Scottish laird, who kept company with the greatest minds of the early 19th century. Ultimately, it would be Maxwell's work that established mathematics as the best language for unlocking the new physics."

See also http://www.smh.com.au/articles/2003/11/09/1068329423959.html and
http://www.theage.com.au/articles/2003/12/11/1071125592157.html?from=storyrhs

Quoted from The Age, 13 Dec 2003, by Michelle Griffin