Real world mathematics in action 2002!

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Abstract

In this paper, we describe a one-day event which changed student perceptions about the relevance of mathematics in today's world. We discuss in detail a competition held during this event in which Year 11 and 12 mathematics students were given an opportunity to experience consulting in the area of promotions and marketing.

1 Introduction

"*I saw areas I didn't realize Maths was important to.*" and "*I learnt plenty. In particular, about lateral thinking.*" are typical student comments on the 2000 inaugural fair *Real world mathematics in action! I'd like to see that!* [1, 2] that inspired the follow-up event on June 19th, 2002. Interest from Victorian schools in *Real World Mathematics in Action 2002!* was so great that registered numbers quickly reached capacity and, unfortunately, many missed out. In the end, the Department of Mathematics and Statistics at the University of Melbourne played host to three hundred and thirty students and staff from twenty-eight schools across Victoria.

Designed for year 11–12 students and their teachers, this fair engages and informs participants through two key events, *Mathematicians Exposed!* and the *Mathematics in Industry and Technology (MIT) Challenge*. The aims of the fair are to:

- promote the importance and broad applications of mathematics and problem solving skills;
- raise awareness of career opportunities for mathematics and statistics graduates.

The survey of the fair showed the event to be an overwhelming success with 91% of respondents saying that it was both enjoyable and informative. The personal interactions with various research students and staff in the Department, combined with the opportunity to meet graduates from a wide range of areas and to learn about where their mathematical training has taken them, have served as a great inspiration to many students. Apart from having accomplished our objectives, this event has clearly proven to be an effective means of recruiting students. In fact, a number of students who attended the Maths Fair are now studying mathematics at the University of Melbourne.

2 Mathematicians Exposed!

The fair began with the morning session of *Mathematicians Exposed!*, an event designed to showcase career profiles of mathematics and statistics graduates. Various industry representatives, from the areas of botany, medicine, business consulting, finance, manufacturing and telecommunications, gave a series of short presentations. The speakers discussed how and where mathematics and statistics are used in their daily work, and the level of mathematical training needed to pursue careers in their profession. All of the guest speakers have international experience in their chosen fields. To illustrate the breadth and expertise of the speakers, we include the career profiles of two of the speakers.

Dr John Carlin is the Deputy Head of the Clinical Epidemiology and Biostatistics Unit at the Murdoch Children's Research Institute, Royal Children's Hospital. John has worked in health and medical research at the Royal Children's Hospital since 1991, after doing Honours in mathematics and statistics and a PhD in statistics. He is involved in a wide range of projects concerned with improving the clinical care of sick children and preventing the development of diseases, injuries and unhealthy behaviours.

Dr Les Trudzik is the Managing Director of KPMG Consulting. Les has extensive experience assisting organisations achieve world-class operations through managing business transformation and change programs, through reviewing and reengineering core business processes and through general management, business strategy and operations research activities. Les is also National Coordinator for Knowledge Management activities within KPMG Consulting and has extensive experience designing and implementing improvements to knowledge management and business intelligence capabilities, including the rollout of KWorld, KPMG Consulting as international knowledge sharing environment. Les has worked with a large number of public and private sector organizations, particularly within the Justice, Defence, Government Administration and Manufacturing sectors.

A new addition to the Maths Fair program in 2002 was the graduate and student profiles. This session provided an opportunity to get "up close and personal" with new mathematics graduates and current undergraduate and postgraduate students who talked about the trials and tribulations of doing mathematics, and their mathematical adventures. Below is the career profile of one of the recent graduates who spoke during the session.

Dan Boulton is a supply chain analyst at Carlton and United Breweries (CUB). In 1995, Dan completed an Honours degree in mathematics, specializing in operations research. Before joining CUB Dan worked in strategy and operations planning in places such as the US, Ireland and Adelaide. In his current role at CUB, Dan makes operations decisions about how to best utilise 5 breweries and 31 distribution centres to minimise the cost of fulfilling customer demand across Australia and New Zealand for hundreds of products such as Victoria Bitter, Carlton Cold, Cascade and Subzero.

3 A crash course on mathematical modelling

Following the industry presentations was an interactive session on the art of mathematical modelling. Dr. David Stump, the Manager of Quantitative Risk Analysis at ANZ Banking Group, conducted this session. The aim of this presentation was to give a crash course on the techniques of mathematical modelling. David demonstrated how to develop and analyse a mathematical model for a loan portfolio. The specific problem he discussed was: If there are 100 loans in a portfolio, how much money do you have to set aside to cover a 1 in 100 year loss? For those competing in the *MIT Challenge*, this was a crucial introduction for what was to follow, while the rest learned how businesses use mathematics to maximize their profits.

4 For those not participating in the MIT Challenge

Two parallel events took place in the afternoon: the *MIT Challenge* competition and the second session of *Mathematicians Exposed!*. In the latter, the teachers, along with students who chose not to compete in the *MIT Challenge*, tried their hand at solving short maths problems (Analyze This!) and met people who use mathematics and statistics in their jobs (Exhibits).

Analyze this! Students and teachers competed in a 1-hour problem solving competition organized by the Melbourne University Mathematics and Statistics Student Society. Around 45 teams consisting of either 3–4 students or 3 teachers were asked a series of short mathematical problems. The winning teams received movie passes or chocolates, and certificates.

Exhibits There was a wide range of interactive exhibits and displays of current research and consulting projects undertaken by postgraduate students and research staff from the Department of Mathematics and Statistics including the Statistical Consulting Centre. These exhibits covered a wide range of areas such as medicine, commerce, genetics, telecommunications, transportation, bio-informatics, colloid science, nanotechnology, defence, operations research, geometry, atmospheric sciences, optics, statistical mechanics, solid-state physics, and food manufacturing. The exhibits highlighted the use of mathematical and statistical skills and techniques to solve real-world problems in these areas as well as provide information about careers for mathematics and statistics graduates.

5 Mathematics in Industry and Technology (MIT) Challenge

Forty-two teams consisting of four Year 11–12 students competed in the *Mathematics in Industry and Technology (MIT) Challenge*. The main objective for the *MIT Challenge* was to give students a taste of real world consulting in the format of a competition. With this in mind we designed a team competition that would engage students in all aspects of consulting: meeting the client, understanding the client's problem, working together as a team, planning a solution strategy, communicating ideas to the client in both written and oral presentations.

The format of the competition was as follows: an industry representative ("the client") presents a real world problem for the teams ("the consultants") to establish a solution approach within 3 hours. The teams then submit a written report of their proposed solution approach to a panel of judges who evaluate the written reports, selecting the top 5 ranked teams. A representative from each of the 5 top ranked teams gives a 5-minute oral presentation of their team's findings.

A key aspect of organising the *MIT Challenge* was to find a real life problem that was accessible to Year 11 and 12 mathematics students so that they could make a reasonable attempt at solving the problem in under 3 hours. Students were not expected to solve the problem completely; the main aim was for teams to demonstrate that they understood the key aspects of the problem and to develop a solution approach or strategy to solve the problem.

During the *MIT Challenge*, students were able to consult with the client about their proposed solution approach. In fact, this consultation process was necessary to successfully complete Part 1 of the *MIT Challenge*, which required teams to determine whether or not they had been given sufficient information to solve the problem. Teams were required to ask the client for any information that was missing.

The *MIT Challenge* problem was developed and presented by Professor Peter Taylor (Department of Mathematics and Statistics, University of Melbourne). In recent years, Peter was Director of the Teletraffic Research Centre at the University of Adelaide. The

MIT Challenge stems from a problem that was presented to the Teletraffic Research Centre by a marketing company on behalf of one of its food manufacturing company clients. Peter gave a broad overview of the Teletraffic Research Centre, and then discussed the particular problem the teams would be considering in the *MIT Challenge*. The following sections give the actual *MIT Challenge* problem and possible solution strategies for this problem.

5.1 The MIT Challenge problem

Imagine that you are working for a mathematical consulting company which was recently approached by a marketing company AA Marketing on behalf of one of its clients Brand X Foods.

AA Marketing wishes your company to determine the value of prizes that Brand X Foods might be liable to pay in a supermarket docket competition that it is planning to conduct. The competition works as follows. The backs of the rolls of paper used for printing supermarket dockets have a repeating series of advertisements. One of these advertisements is for Brand X Foods. If a shopper presents a docket in which the printout for a Brand X Food item on the front coincides with the advertisement for Brand X Foods on the back, then that shopper will win $50 for each such advertisement. This situation is illustrated in Figure 1. The left hand side of the figure depicts the back of the roll, with the repeating series of advertisements. The right hand side depicts the front of the roll with the purchase information.

AA Marketing has supplied the following information:

- The distance B between the beginning of one Brand X Foods advertisement and the beginning of the next Brand X Foods advertisement on the back of a roll of docket paper is 1190mm.
- The height C of a Brand X Foods advertisement is 35mm.
- The baseline-to-baseline height a of the printout a single item on the front of a shopping docket is 5mm.
- The length l of non-item information on a single shopping docket is assumed to be constant with a value of 105mm.
- The average length d of the purchase item information on a shopping docket is 85mm.
- One in 30 items purchased by the shopper is a Brand X Food item.

To complete this problem you need to undertake two tasks:

1) Determine whether any extra information is needed and, if so, ask for it.
2) Calculate how much money in prizes Brand X Foods can expect to pay per 59.5 metre docket roll.

5.2 Solution to the MIT Challenge problem

On each roll there are fifty Brand X Food advertisements, so the expected amount that Brand X Foods might have to pay out per roll is fifty times the expected amount that it might have to pay out per advertisement. Brand X Foods will have to have to pay out on an advertisement if:

- The advertisement on the back of the docket lines up with at least some, let's say n, purchases on the front of the docket.
- Of the n purchases, at least one is a Brand X Food item.
- The shopper with the winning docket would have to notice that he/she had won and claim the prize.

5.2.1 *Probability of claiming the prize.* Let the probability that a Brand X Food item lines up with a Brand X Food Advertisement be $p_{\text{line-up}}$. Then, if every shopper claimed the prize, Brand X Foods would have to pay the \$50 prize with probability $p_{\text{line-up}}$. This leads to an expected liability of

$$L = 50 \times p_{\text{line-up}}$$

However, not all shoppers would notice that they have won. Let p_{claim} be the probability that a shopper with a winning docket will claim the prize. A shopper would both win and claim the prize with probability $p_{\text{line-up}} \times p_{\text{claim}}$. Therefore, the actual expected liability will be

$$L = 50 \times p_{\text{line-up}} \times p_{\text{claim}} \tag{1}$$

5.2.2 *Calculation of $p_{\text{line-up}}$ given n.* We know that n purchases line up with the advertisement. Each one is a Brand X Foods item with probability 1/30. The only way that there will be no Brand X Food item lining up with the advertisement is if every item is *not* a Brand X Food item. Assuming independence of the items, this occurs with probability $(29/30)^n$. Thus we have

$$p_{\text{line-up}}(n) = 1 - \left(\frac{29}{30}\right)^n$$

Since the baseline-to-baseline height of an item on the docket is 5mm and the height of the Brand X Foods advertisement is 35mm, the highest value that n can be is 7. Thus, assuming that we know the value $\pi(n)$ of the probability that n items line up with the advertisement, we can calculate $p_{\text{line-up}}$ via the equation

$$p_{\text{line-up}} = \sum_{n=0}^{7} \pi(n)\ p_{\text{line-up}}(n) = \sum_{n=0}^{7} \pi(n) \left[1 - \left(\frac{29}{30}\right)^n\right] \tag{2}$$

5.2.3 *Extra information needed.* There are two pieces of information that are needed to solve the problem, but have not been given by AA Marketing. They are:

(1) The probability that a shopper with a winning docket will claim the prize. Students who realized this fact and requested this information were asked to consider two scenarios:
   - A conservative scenario, where we assume $p_{\text{claim}} = 0.2$;
   - A "best-guess" scenario obtained by looking at data from past competitions, where we assume $p_{\text{claim}} = 0.05$.

(2) Although AA Marketing has given us the average length d of the purchase item information on a shopping docket, they have told us nothing about the variability of this information. The value of $\pi(n)$ depends heavily on this variability. To solve this problem, we need to know the distribution of the lengths of the purchase item information or equivalently, the numbers of items each customer buys. Students who realized this fact and requested this information were given an Excel table with the lengths of the purchase item information on dockets purchased by 500 different customers.

5.2.4 *Calculation of $\pi(n)$.* One possible method of calculating $\pi(n)$ is to run a simulation based on the purchase length data. This can be done by picking docket lengths randomly from the data and "constructing" the front of the roll. You could then record the proportion of advertisements on the back of the roll that match up with n items as n varies from 0 to 7. This will give estimates for $\pi(0)$ to $\pi(7)$, which can be inserted in equation (2) to obtain the

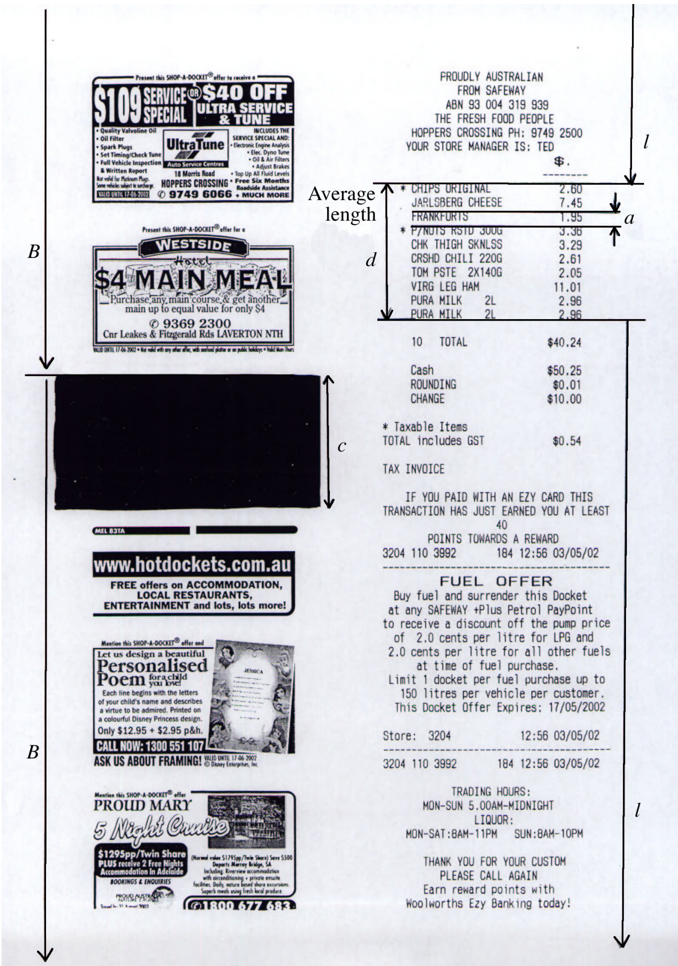

Figure 1. Diagram of the front and back of a typical supermarket docket with relevant measurements labelled.

answer. This is the method that the organisers hoped that the best student teams would use.

An analytical solution can be derived using *renewal theory*. In this problem, we think of *time* as going along the front of the docket and the renewal points as the points $t_1, t_2, \ldots$ where the start of the purchase items for the successive customers occurs. The first part of each interval between renewal points is taken up with purchase item information and the last 105mm is taken up with the non-item information.

For each value of x, we can use the data on docket lengths to work out the distribution of these intervals between the renewal points

$$F(x) = P(t_j - t_{j-1} \leq x). \tag{3}$$

Now think of the beginning of a Brand X Food advertisement on the back of the docket as defining a random point s along the docket. Let t_j be the next point after s (see Figure 2 for details).

Consider the three cases of renewal point locations depicted in Figure 3. If $t_j - s \leq 105$, the entire 35mm of the advertisement will line up with non-item information and so the number of items lining up will be 0. On the other hand, if $t_j - s > 140$, the entire 35cm of the advertisement will line up with purchase item information and the number of items lining up will be 7. In between, if $105 < t_j - s \leq 110$, then one item will line up, if $110 < t_j - s \leq 115$, then two items will line up and so on. Thus, for $n = 0, \dots, 7$, we can work out the probability of n items lining up if we can work out the probability that $t_j - s$ lies in the various intervals $0 - 105, 105 - 110, \dots, 135 - 140, 140 - \infty$. Renewal theory tells us that

$$P(a < t_j - s < b) = \frac{1}{190} \int_a^b [1 - F(x)]\, dx, \tag{4}$$

where $F(x)$ is given by equation (3).

Using equation (4) for the data given, we can calculate that $\pi(n)$ is given by

$$\pi(n) = \begin{cases} .578947 & \text{if } n = 0 \\ .024768 & \text{if } n = 1 \\ .023311 & \text{if } n = 2 \\ .021940 & \text{if } n = 3 \\ .020649 & \text{if } n = 4 \\ .019434 & \text{if } n = 5 \\ .018291 & \text{if } n = 6 \\ .292660 & \text{if } n = 7 \end{cases} \tag{5}$$

Substituting equation (5) into equation (2), we get $p_{\text{line-up}} = 0.0753$ and, using equation (1), we derive a value of 0.753 for L, assuming that $p_{\text{claim}} = 0.2$. Similarly, if we assume $p_{\text{claim}} = 0.05$ then $L = 0.1883$. Since there are 50 advertisements per roll, the liability

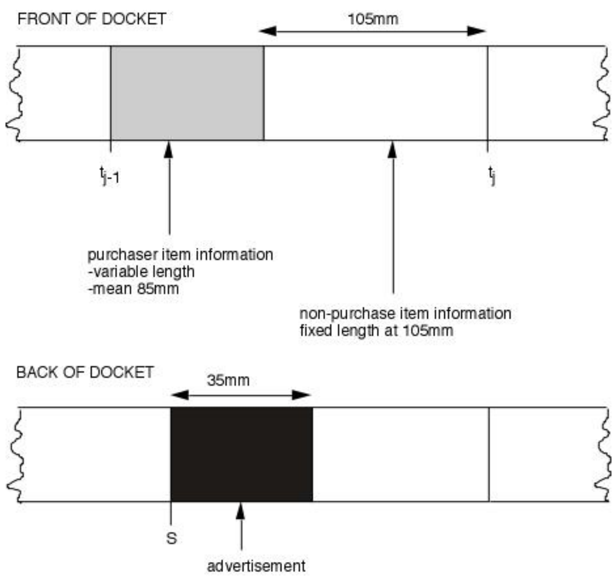

Figure 2. Diagram of the front and back of a typical supermarket docket with renewal points and random point labelled.

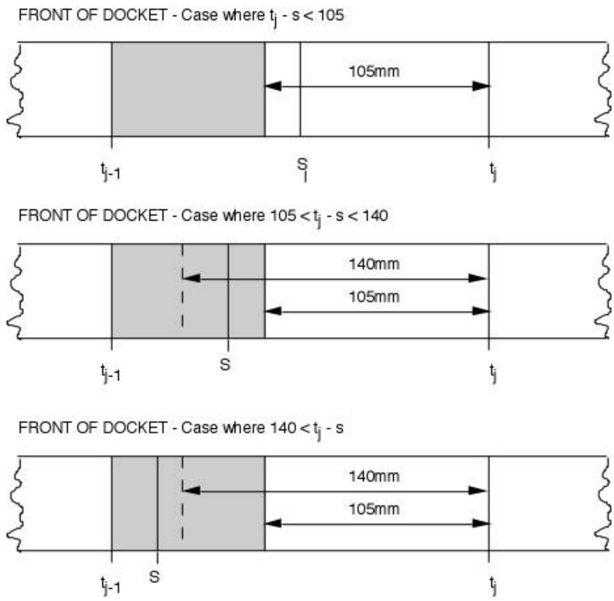

Figure 3. Diagram of the front of a typical supermarket docket depicting three different random point locations.

per 59.5 metre docket roll is $50L$. This gives a final liability per docket roll of \$37.65 if $p_{\text{claim}} = 0.2$ and \$9.41 if $p_{\text{claim}} = 0.05$.

5.3 How the students fared

Most of the teams realized that they needed at least one of the two extra pieces of information to solve the problem completely. As expected no team presented a correct fully worked solution for the liability per docket roll. However, many teams correctly identified some of the key elements required in the calculation. Whilst their solutions did not produce the correct numerical result, the winning teams presented a number of the correct problem elements and displayed good mathematical intuition and insight in their solution approach. Some of the student oral presentations were of a high standard.

Prizes were awarded according to the quality of the proposed written solution and oral presentation. Teams from the following schools won the major prizes in the *MIT Challenge*:

1st Prize: Korowa Anglican Girls School (\$800 for team)

2nd Prize: Thomas Carr College (\$400 for team)

3rd Prize: The King David School (\$200 for team)

Other short listed teams were awarded certificates of merit.

6 The closing show and student feedback

The closing show was performed by Dr Burkard Polster (Monash University) or Burke the Mathematical Juggler as he is better known. Burkard entertained the audience with his superb juggling skills while explaining the mathematics behind the juggling.

The short problem solving competition *Analyze this!* was the highlight of the fair with nearly 96% of participants saying they enjoyed the event. Removing the trivia questions and focusing on mathematics calculations made the competition accessible to all Year 11 and 12 students. The new format worked well and the questions were of suitable difficulty for the time allowed.

The *MIT Challenge* was also popular, with 88% of participants saying they enjoyed the event. Competing teams found the *MIT Challenge* thought provoking, challenging and fun. Some of the teams comprising weaker Year 11 students found the problem too difficult for their current mathematical skill level and consequently did not enjoy the competition as much we hoped. The format of the *MIT Challenge* worked very well except that teams had to wait for a long time to speak with the client.

The talks by guest speakers were met with mixed responses. Burke the mathematical juggler was a hit with students. The short talks (7 minutes) by recent graduate and current University students were very popular with students and teachers. However, some of the longer talks (15-20 minutes) were unsuitable for year 11-12 students as they were too technical and too detailed. In future fairs, the emphasis will be on having shorter talks by recent graduates covering a wide range of career opportunities.

Those participants attending the exhibits found them to be very informative and enjoyable. However, attendance at the displays was very disappointing; many students chose to explore the University grounds instead of attending the displays.

Some of the specific comments made on the survey were:

- "I didn't know that you could have so many jobs that involved maths."
- "The whole day was terrific! I hope there will be many more."
- "It was interesting and opened my eyes to other career opportunities."
- "Very informative. May have to change my Uni preferences to include maths."
- "It was interesting and challenging."
- "A unique experience."
- "It was fun!"
- "Broadened my horizon."
- "A great opportunity for my students to meet other mathematicians and students and to get an insight into University Maths and beyond."

Following the overwhelming success of the first two Maths Fairs, the Department of Mathematics and Statistics has decided to hold a Maths Fair every two years. We look forward to seeing more students and teachers in the upcoming Maths Fair to be held on Wednesday June 16th, 2004. For more details of past Maths Fairs and information about the upcoming fair, please see the website: http://www.ms.unimelb.edu.au/msaction/

7 Acknowledgements

We thank the Department of Mathematics and Statistics of the University of Melbourne, ANZIAM Victorian Branch, Apple and Analytica for their financial support of the Maths Fair. Without their support the event would not have been possible.

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Received 15 March 2004, accepted 29 March 2004.